

Selected updates on semileptonic B decays and $|V_{xb}|$ determination

Giulia Ricciardi*

Dipartimento di Fisica E. Pancini, Università di Napoli Federico II, and INFN, Sezione di Napoli, Complesso Universitario di Monte Sant'Angelo, Via Cintia, I-80126 Napoli, Italy

E-mail: giulia.ricciardi@na.infn.it

We summarize selected up-to-date results related to semileptonic B meson decays, namely results on the exclusive determination of the parameter $|V_{cb}|$ of the CKM matrix, on the inclusive determination of $|V_{ub}|$, and on $R(D^{(*)})$.

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1. Introduction

The parameters $|V_{xb}|$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix play a central role in the analyses of the unitarity triangle and in testing the Standard Model (SM). The long-standing tension among their values, depending on whether they are extracted using exclusive or inclusive semi-leptonic B decays, also motivates more and more precise theoretical and experimental investigations¹. We briefly discuss recent results on the exclusive determination of $|V_{cb}|$, on the inclusive determination of $|V_{ub}|$, and on $R(D^{(*)})$.

2. Exclusive $|V_{cb}|$ determination

We can express the differential ratios for the semi-leptonic CKM favoured decays $B \rightarrow D^{(*)}\ell\nu$ in terms of the recoil parameter $\omega = p_B \cdot p_{D^{(*)}}/m_B m_{D^{(*)}}$, which corresponds to the energy transferred to the leptonic pair. For negligible lepton masses ($\ell = e, \mu$), each ratio depends on one form factor, $\mathcal{F}(\omega)$ for $B \rightarrow D^*\ell\nu$ and $\mathcal{G}(\omega)$ for $B \rightarrow D\ell\nu$, and the phase space vanishes at the no-recoil point $\omega = 1$ in both cases. Summarizing, we can write

$$\begin{aligned} \frac{d\Gamma}{d\omega}(B \rightarrow D^*\ell\nu) &\propto G_F^2(\omega^2 - 1)^{\frac{1}{2}}|V_{cb}|^2 \mathcal{F}(\omega)^2 \\ \frac{d\Gamma}{d\omega}(B \rightarrow D\ell\nu) &\propto G_F^2(\omega^2 - 1)^{\frac{3}{2}}|V_{cb}|^2 \mathcal{G}(\omega)^2 \end{aligned} \quad (2.1)$$

In the heavy quark limit, both form factors are related to a single Isgur-Wise function, $\mathcal{F}(\omega) = \mathcal{G}(\omega) = \xi(\omega)$, which is normalized to unity at zero recoil, that is $\xi(\omega = 1) = 1$. There are non-perturbative corrections to this prediction, expressed at the zero-recoil point by the heavy quark symmetry under the form of powers of Λ_{QCD}/m , where $m = m_c$ and m_b . Other corrections are perturbatively calculable radiative corrections from hard gluons and photons.

In order to extract the CKM factors, we need not only to compute the form factors, but also to measure experimental decay rates, which vanish at zero-recoil. Therefore, experimental points are extrapolated to zero recoil, using a parametrization of the dependence on ω of the form factor. Several recent extrapolations to zero recoil adopt a parametrization where ω is mapped onto a complex variable z via the conformal transformation $z = (\sqrt{\omega + 1} - \sqrt{2})/(\sqrt{\omega + 1} + \sqrt{2})$. The form factors may be written in form of an expansion in z , which converges rapidly in the kinematical region of heavy hadron decays. The coefficients of the expansions are subject to unitarity bounds based on analyticity. Common examples are the CLN (Caprini-Lellouch-Neubert) [7], the BGL (Boyd-Grinstein-Lebed) [8] and the BCL (Bourrely-Caprini-Lellouch) [9] parameterizations. They are all constructed to satisfy the unitarity bounds, but the CLN approach differs mostly in its reliance on next-to-leading order HQET relations between the form factors. Because of such additional constraints, the accuracy of the CLN approach has been questioned in both $B \rightarrow D\ell\nu$ [10] and $B \rightarrow D^*\ell\nu$ [11, 12] channels. The way out of parametrization dependence is to compute the form factors at nonzero recoil values. In the case of $B \rightarrow D\ell\nu$ decays, the form factors in the unquenched lattice-QCD approximation are available for a range of non-zero recoil momenta since 2015. Lastly, we remark that combined fits to $B \rightarrow D^{(*)}\ell\nu$ differential rates and angular distributions can help constraining the form factors and allow not only the extraction of $|V_{cb}|$ but also of form factor ratios under various scenarios [13].

¹For brief overviews see for example [1, 2, 3, 4, 5, 6] and references therein.

2.1 $B \rightarrow D^* \ell \nu$ channel

The form factor $\mathcal{F}(1)$ at zero recoil for the $B \rightarrow D^* \ell \nu$ channel, in the lattice unquenched $N_f = 2 + 1$ approximation, has been estimated first by the FNAL/MILC collaboration, which used Wilson fermions for both c and b heavy quarks [14]

$$\mathcal{F}(1) = 0.906 \pm 0.004 \pm 0.012 \quad (2.2)$$

The first error is statistical and the second one is the sum in quadrature of all systematic errors. The total uncertainty is around the (1-2)% level. The largest error is the heavy quark discretization error related to the Fermilab action. The LANL/SWME collaboration is working to reduce it by using an improved version of the Fermilab action, the Oktay-Kronfeld action [15]. Other preliminary results concern analyses at non-zero recoil [16] and with Möbius domain-wall quarks, at zero and non-zero recoil, from $N_f = 2 + 1$ QCD [17]. A recent value of the form factor $\mathcal{F}(1)$ has been presented by the HPQCD collaboration, which used the fully relativistic HISQ (Highly improved staggered quarks) action for light, strange and charm quarks and the NRQCD (Nonrelativistic QCD) action for the bottom quark [18]

$$\mathcal{F}(1) = 0.895 \pm 0.010 \pm 0.024 \quad (2.3)$$

Both the results in Eq. (2.2) and Eq. (2.3) are in good agreement. The form factor in Eq. (2.2) has been used by the latest $|V_{cb}|$ determinations from the Heavy Flavour and Lattice Averaging Groups, HFLAV and FLAG respectively. Using the CLN parametrization, the 2016 HFLAV average [19] gives

$$|V_{cb}| = (39.05 \pm 0.47_{\text{exp}} \pm 0.58_{\text{th}}) \times 10^{-3} \quad (2.4)$$

where the first uncertainty is experimental and the second error is theoretical (lattice QCD calculation and electro-weak correction). The 2016 FLAG $N_f = 2 + 1$ $|V_{cb}|$ average value yields [20]

$$|V_{cb}| = (39.27 \pm 0.49_{\text{exp}} \pm 0.56_{\text{latt}}) \times 10^{-3} \quad (2.5)$$

This average employs the 2014 HFLAV experimental average [21] $\mathcal{F}(1)\eta_{\text{EW}}|V_{cb}| = (35.81 \pm 0.45) \times 10^{-3}$ and the value $\eta_{\text{EW}} = 1.0066$ [22].

In 2017, for the first time, the unfolded fully-differential decay rate and associated covariance matrix have been published, by the Belle collaboration [23], prompting several independent determinations of the value of $|V_{cb}|$ under different theoretical approaches [12, 11, 24, 25, 26, 18]. Most notably, using the Belle measurement [23], it has been argued that previous determinations using the CLN form factor parameterisation could be affected by underestimated uncertainties, and therefore form factor analyses based on BGL should be preferred [12, 11, 24]. The previous analyses, in the BGL framework, appear consistent with each other and give, using t lattice zero recoil form factors, the values

$$\begin{aligned} |V_{cb}| &= (41.9^{+2.0}_{-1.9}) \times 10^{-3} \quad [12] \\ |V_{cb}| &= (41.7^{+2.0}_{-2.1}) \times 10^{-3} \quad [11] \end{aligned} \quad (2.6)$$

The central values are higher than the corresponding values in the CLN parametrization, and closer to the values from the inclusive approach. On the other side, it has also been argued that fits that

yield the higher values of $|V_{cb}|$ suggest large violations of heavy quark symmetry and tension with lattice predictions of the form factor ratios [26].

This year for the first time the Belle Collaboration has performed fits with both the CLN and BGL form factor parameterizations, obtaining [27]

$$\begin{aligned}\mathcal{F}(1)\eta_{EW}|V_{cb}| &= (35.06 \pm 0.15 \pm 0.54) \times 10^{-3} && \text{CLN fit} \\ \mathcal{F}(1)\eta_{EW}|V_{cb}| &= (38.73 \pm 0.25 \pm 0.60) \times 10^{-3} && \text{BGL fit}\end{aligned}\quad (2.7)$$

The first uncertainty is statistical, the second one is systematic. This is the experimentally most precise determination performed with exclusive semileptonic B decays. Taking the value for the form factor in Eq. (2.2) and $\eta_{EW} = 1.006$ [22], they find [27]

$$\begin{aligned}|V_{cb}| &= (38.4 \pm 0.2 \pm 0.6 \pm 0.6) \times 10^{-3} && \text{CLN + LQCD} \\ |V_{cb}| &= (42.5 \pm 0.3 \pm 0.7 \pm 0.6) \times 10^{-3} && \text{BGL + LQCD}\end{aligned}\quad (2.8)$$

The value of branching fraction is found to be insensitive to the choice of the parameterization. Both sets of fits give acceptable χ^2/ndf : therefore the data do not discriminate between the parameterizations.

Form factor estimates via zero recoil sum rules [28, 29, 30] give, in general, relatively higher values of $|V_{cb}|$ and a theoretical error more than twice the error in the lattice determinations. Recently, form factors determinations from B meson light-cone sum rules beyond leading twist have been presented [31].

2.2 The $B \rightarrow D\ell\nu$ channel

For $B \rightarrow D\ell\nu$ decay, form factors are available for a range of recoil momenta by both the FNAL/MILC collaboration and the HPQCD Collaboration [32, 33]. By parameterizing the dependence on momentum transfer using the BGL parameterization, the former collaboration has determined $|V_{cb}|$ from the relative normalization over the entire range of recoil momenta [32]

$$|V_{cb}| = (39.6 \pm 1.7_{\text{exp+QCD}} \pm 0.2_{\text{QED}}) \times 10^{-3}\quad (2.9)$$

The average value is almost the same than the one inferred from $B \rightarrow D^*\ell\nu$ decay by the same FNAL/MILC collaboration, see Eq. (2.4). The HPQCD Collaboration has performed a joint fit to lattice and 2009 BaBar experimental data [34] and extracted the CKM matrix element $|V_{cb}|$ using the CLN parametrization [33]

$$|V_{cb}| = (40.2 \pm 1.7_{\text{latt+stat}} \pm 1.3_{\text{syst}}) \times 10^{-3}\quad (2.10)$$

The first error consists of the lattice simulation errors and the experimental statistical error and the second error is the experimental systematic error.

In 2015 the decay $B \rightarrow D\ell\nu$ has also been measured in fully reconstructed events by the Belle collaboration [35]. They have performed a fit to the CLN parametrization, which has been used to determine $\eta_{EW}\mathcal{G}(1)|V_{cb}|$. By using the form-factor normalization $\mathcal{G}(1)$ found by the FNAL/MILC Collaboration [32], and $\eta_{EW} \simeq 1.0066$ [22], they obtain [35]

$$|V_{cb}| = (39.86 \pm 1.33) \times 10^{-3}\quad (2.11)$$

The Belle Collaboration has also obtained a slightly more precise result (2.8% vs. 3.3%) by exploiting lattice data at non-zero recoil and performing a combined fit to the BGL form factor. It yields [35]

$$|V_{cb}| = (40.83 \pm 1.13) \times 10^{-3} \quad (2.12)$$

Global fit results in the BGL parametrization are in agreement with the previous determinations, giving the value [10]

$$|V_{cb}| = (40.49 \pm 0.97) \times 10^{-3} \quad (2.13)$$

Here the lattice results [32, 33], as well as Belle [35] and Babar [34] data, have been used.

3. Inclusive $|V_{ub}|$ determination

In order to extract $|V_{ub}|$ from semileptonic $B \rightarrow X_u \ell \nu$ decays one has to reduce the $b \rightarrow c$ semileptonic background through experimental cuts. Such cuts enhance the relevance of the so-called threshold region in the phase space, jeopardizing the use of operator product expansion techniques. In order to face this problem, that is absent in the inclusive determination of $|V_{cb}|$, different theoretical schemes have been devised, which are tailored to analyze data in the threshold region, but differ in their treatment of perturbative corrections and the parametrization of non-perturbative effects. In Table 1 we present the results for four theoretical different approaches, as analyzed by BaBar [36, 37], Belle [38] and HFLAV [19] collaborations, that is: ADFR [39, 40, 41], BNLN [42, 43, 44], DGE [45] and GGOU [46]². The 2016 averaged HFLAV determinations [19] are the most recent HFLAV averages, and have been reported also in the 2019 update of PDG [48]. Although conceptually quite different, all these approaches lead to roughly consistent results when the same inputs are used and the theoretical errors are taken into account. Generally, these values

Table 1: Status of inclusive $|V_{ub}|$ determinations (Refs. are in the text).

Inclusive decays	$ V_{ub} \times 10^3$			
	ADFR	BNLN	DGE	GGOU
HFLAV 2016 [19]	$4.08 \pm 0.13^{+0.18}_{-0.12}$	$4.44 \pm 0.15^{+0.21}_{-0.22}$	$4.52 \pm 0.16^{+0.15}_{-0.16}$	$4.52 \pm 0.15^{+0.11}_{-0.14}$
BaBar 2011 [36]	$4.29 \pm 0.24^{+0.18}_{-0.19}$	$4.28 \pm 0.24^{+0.18}_{-0.20}$	$4.40 \pm 0.24^{+0.12}_{-0.13}$	$4.35 \pm 0.24^{+0.09}_{-0.10}$
Belle 2009 [38]	$4.48 \pm 0.30^{+0.19}_{-0.19}$	$4.47 \pm 0.27^{+0.19}_{-0.21}$	$4.60 \pm 0.27^{+0.11}_{-0.13}$	$4.54 \pm 0.27^{+0.10}_{-0.11}$

are higher compared to corresponding exclusive determinations. The tension is of the order of $2 - 3\sigma$, depending on the chosen framework. When averaged, the ADFR value is lower than the ones obtained with the other three approaches, and closer to the exclusive values. By taking the arithmetic average of the results obtained from these four different QCD predictions of the partial rate, the Babar collaboration gives [36]

$$|V_{ub}| = (4.33 \pm 0.24_{\text{exp}} \pm 0.15_{\text{th}}) \times 10^{-3} \quad (3.1)$$

²Artificial neural networks have also been used to parameterize the shape functions and extract $|V_{ub}|$ in the GGOU framework [47]. The results are in good agreement with the original paper.

The latest analyses and theoretical progresses in semi-leptonic heavy to light decays involve power corrections [49, 50], global fits to inclusive rare and semileptonic data [47, 51] and, most recently, exploratory lattice calculations of inclusive B meson semileptonic decay [52]. At Belle II prospects are good for improvements of the $|V_{ub}|$ determination, with both inclusive and exclusive approaches, thanks to more data and better reconstruction performance [53].

4. Exclusive $b \rightarrow c$ decays into heavy leptons

In the SM the couplings to the W^\pm bosons are universal for all leptons and processes showing non-universality would indicate physics beyond the SM. This universality can be tested in semileptonic B meson decays involving a τ lepton, through the ratio of branching fractions

$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu_\ell)} \quad (4.1)$$

The denominator is the average for $\ell \in \{e, \mu\}$. This ratio is typically used instead of the absolute branching fraction of $B \rightarrow D^{(*)} \tau \nu_\tau$ decays, in order to cancel uncertainties common to the numerator and the denominator. These include the CKM matrix element and several theoretical uncertainties on hadronic form factors and experimental reconstruction effects.

In the SM, values for $R(D^*)$ have been calculated already in 2012 by means of HQE, giving [54]

$$R(D^*) = 0.252 \pm 0.003 \quad (4.2)$$

A later lattice estimate yields [33]

$$R(D^*) = 0.300 \pm 0.008 \quad (4.3)$$

The discussion on different parameterizations, briefly outlined in Sect. 2, has prompted in 2017 new SM determinations in the BGL parameterization [24, 25, 13]. These are generally consistent with the old predictions for $R(D^*)$. Their arithmetic average, as given by HFLAV [55] is

$$R(D^*) = 0.258 \pm 0.005 \quad (4.4)$$

As far as $R(D)$ is concerned, lattice SM predictions [32, 33] have been averaged by the FLAG collaboration [20], yielding

$$R(D) = 0.2300 \pm 0.008 \quad (4.5)$$

As before, recent calculations have been performed in the BGL parameterization [10, 25, 13], and their arithmetic HFLAV average [55] is

$$R(D) = 0.299 \pm 0.003 \quad (4.6)$$

Exclusive semi-tauonic B decays were first observed by the Belle Collaboration in 2007 [56]. Subsequent analysis by Babar and Belle [57, 58, 59] measured branching fractions above, although consistent with, the SM predictions. In 2012-2013 Babar has measured $R(D^{(*)})$ by using its full data sample [60, 61], and reported a significant excess over the SM expectation, confirmed in 2016 by the first measurement of $R(D^*)$ using the semileptonic tagging method (Belle [62]). In 2015 a

confirmation came also by the LHCb collaboration, who has studied the decay $\bar{B} \rightarrow D^{*+} \tau \bar{\nu}_\tau$ with $D^{*+} \rightarrow D^0 \pi^+$ and $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$ in pp collisions [63].

In 2016, the Belle collaboration reported a new measurement in the hadronic τ decay modes [64], statistically independent of the previous Belle measurements, with a different background composition. These results are consistent with the theoretical predictions of the SM in Ref. [54] within 0.6σ standard deviations. They also reported the first measurement of the τ lepton polarization in the decay $\bar{B} \rightarrow D^* \tau^- \bar{\nu}$ [64], which is again compatible with SM expectations [65]. Last year, the LHCb collaboration has measured $R(D^{*-})$, in agreement with the SM prediction [66, 67].

By averaging the most recent measurements [60, 61, 59, 63, 62, 64, 66, 67], the HFLAV Collaboration has found [55]

$$R(D^*) = 0.306 \pm 0.013 \pm 0.007 \quad (4.7)$$

$$R(D) = 0.407 \pm 0.039 \pm 0.024 \quad (4.8)$$

where the first uncertainty is statistical and the second one is systematic. R_D and R_{D^*} exceed the SM values by about 2σ and 3σ , respectively. If one consider both deviations, the tension rises to about 4σ . At Belle II a better understanding of backgrounds tails under the signal and a reduction of the uncertainty to 3% for R_{D^*} and 5% for R_D is expected at 5 ab^{-1} [53].

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