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The mixing of Hybrids with Quarkonia

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We report on an effective field theory calculation of the lower lying heavy hybrid spectrum, which includes mixing with heavy quarkonium states as a novel feature. Spin zero (one) hybrids turn out to mix with spin one (zero) quarkonia, which is intrumental to explain apparent spin symmetry violating decays of certain XYZ resonances that have been identified as hybrid states. We also present some model independent results for the hiperfine splittings.

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1. Introduction

The interest on exotic hadrons has recently experienced a revival due to the pletora of charmonium, and some bottomonium, resonances, the so called XYZ states, discovered in the last decade that do not easily fit in the quark model spectrum (see [1]). We shall focus here on charmonium and bottomonium hybrids, namely $c\bar{c}$ and $b\bar{b}$ states with a non-trivial gluon content, with the aim to understand at least part of the spectrum of isospin zero XYZ states.

Since m_c, $m_b \gg \Lambda_{\rm QCD}$, the heavy quarks move slowly so that they see an instantaneous potential as the effective interaction. The Born-Oppenheimer (BO) approximation has been revisited in relation with the XYZ states in [2]. It has also been incorporated into an effective field theory framework in [3] elaborating on weak coupling pNRQCD [4, 5], and in [6, 7] elaborating on strong coupling pNRQCD [5, 8, 9]. The following sections are based on refs. [7, 10].

2. Quarkonium

In order to set the scale of the hybrid spectrum it is important to have the quarkonium spectrum calculated in the same framework. Since lattice calculations exist for both the quarkonium and the hybrid potentials [11], we shall fix the single arbitrary constant of all these potentials by fitting to the charmonium and bottomonium spectrum. The shape of the quarkonium potential evaluated on the lattice (Σ_g^+) is well described by the Cornell potential, which has the short and long distance behavior expected from QCD perturbation theory and the QCD effective string theory (EST) [12] respectively. The spectrum obtained with the potential above is displayed in Tables V of [7].

3. Hybrids

The hybrid potentials and the quarkonium potential (Σ_g^+) are displayed in Fig. 2 of [11] (see [13] for more recent lattice calculations) . They are labeled according to the representations of the $D_{\infty h}$ group, the group of a diatomic molecule. At short distances all the hybrid potentials must approach the repulsive Coulomb potential of the color octet configuration, as perturbation theory dictates. Furthermore, they should gather in short distance multiplets of the rotational group [5].

$$(\Sigma_g^+) \quad , \quad (\Sigma_u^-, \Pi_u) \quad , \quad (\Sigma_g^-, \Pi_g, \Delta_g) \quad , \quad (\Sigma_g^{+\prime}, \Pi_g^\prime) \quad , \quad (\Sigma_u^+, \Pi_u^\prime, \Delta_u) \quad , \quad \dots \tag{3.1}$$

At long distance they must approach the behavior dictated by the QCD EST, namely to the same linear potential as the quarkonium case (Σ_g^+) with a subleading $1/r$ behavior that depends on the string state [12]. They enjoy the degeneracies of the Nambu-Goto string,

$$(\Sigma_g^+) \quad , \quad (\Pi_u) \quad , \quad (\Sigma_g^{+\prime}, \Pi_g, \Delta_g) \quad , \quad (\Sigma_u^+, \Sigma_u^-, \Pi_u^\prime, \Pi_u^{\prime\prime}, \Delta_u, \Phi_u) \quad , \quad \dots \tag{3.2}$$

Notice that the hybrid potentials, unlike the quarkonium one, have a classical minimum, which must sit at $r \sim 1/\Lambda_{\rm QCD}$ (there is no other scale available). Hence the small energy fluctuations about this minimum have a size $\sqrt{\Lambda_{\rm QCD}^3/m_Q}$, which is parametrically smaller than $\Lambda_{\rm QCD}$. Hence, if we are only interested in the lower lying states for each potential, we are in a situation similar to the strong coupling regime of pNRQCD [5], in which the energy scale $\Lambda_{\rm QCD}$ can be integrated out.

We restrict ourselves to the lowest lying hybrid multiplet, namely that formed by (Σ_u^-, Π_u). At short distances, these states correspond to a quark-antiquark in a color octet configuration together with a chromomagnetic field that makes the whole operator color singlet [5]. Hence, we choose a vectorial wave function matrix $\mathbf{H}(\mathbf{0}, \mathbf{r}, t)$ with the same symmetry transformations as that operator. As a consequence, the P and C quantum numbers of a hybrid state with quark-antiquark orbital angular momentum L and quark-antiquark spin S are, $P = (-1)^{L+1}$, $C = (-1)^{L+S+1}$. The Hamiltonian at leading order (BO approximation) is chosen such that the projection of $\mathbf{H}$ to $\mathbf{r}$ evolves with $V_{\Sigma_u^-}$ and the projection orthogonal to $\mathbf{r}$ with V_{Π_u} in the static limit. That is,

$$\mathscr{L} = \mathrm{tr}\left(H^{i\dagger}\left(\delta_{ij} i\partial_0 - h_{Hij}\right) H_j\right) \tag{3.3}$$
$$h_{Hij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r)\right)\delta_{ij} + (\delta_{ij} - \hat{r}_i\hat{r}_j)\left[V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)\right] .$$

In addition to L and S defined above, we characterize the states with J, the total angular momentum of the gluons plus the orbital angular momentum of the quark-antiquark, $\mathscr{J}$, the total angular momentum of the system, and $\mathscr{M}$, its third component.

3.1 Spectrum

Spin symmetry implies that states with the same J and L are degenerate. They form quadruplets except for the case $J = 0$ that they form a doublet. In fact, L is not a good quantum number because the potential in (3.3) is not central. This leads to coupled eigenvalue equations for $L = J \pm 1$, whereas a single (uncoupled) eigenvalue equation remains for $L = J$. We use NL_J ($N = 1, 2, \ldots$, $L = s, p, d, \ldots$, $J = 1, 2, \ldots$) to label the states in the spin symmetric limit, where N is the principal quantum number. The results for the charmonium and bottomnium spectrum, both for hybrids and quarkonium, are displayed in the Tables I and II of ref. [7] respectively.

The lightest hybrid multiplet corresponds to the $1\,(s/d)_1$ quantum number (4011 MeV for charmonium and 10690 MeV for bottomonium). The remaining hierarchy from lighter to heavier reads $1\,p_1$, $1\,(p/f)_2$, $1\,d_2$, $2\,(s/d)_1$, ... The XYZ states that fit in our hybrid spectrum are displayed in Table 3. C-parity implies that only spin zero hybrids would have been observed, except for $X(4350)$. However, decays to spin one quarkonium states have been observed for all spin zero 1^{--} states above, except for $X(4630)$ and $Y(4390)$, which disfavors the hybrid interpretation due to spin symmetry. We show in the following section how this problem can be overcome.

3.2 Mixing

Mixing with quarkonium is in principle an $1/m_Q$ suppressed effect. However, if there is a quarkonium state with mass close to a hybrid one's, it may become a leading order effect. The symmetries of the quarkonium and the hybrid fields imply that the mixing at order $1/m_Q$ is controlled by a single term,

$$\mathscr{L}_{\text{mixing}} = \mathrm{tr}\left[S^\dagger V_S^{ij}\left\{\sigma^i, H^j\right\} + \text{h.c.}\right] \quad , V_S^{ij} = (\delta^{ij} - \hat{r}^i\hat{r}^j)V_S^\Pi + \hat{r}^i\hat{r}^j V_S^\Sigma . \tag{3.4}$$

The term (3.4) mixes spin zero (one) hybrids with spin one (zero) quarkonium, and may be the source of large spin symmetry violations. V_S^{ij} is unknown at the moment, but it can be easily evaluated on the lattice. Explicit formulas can be found in ref. [7]. We can work out, however,

State	M	J^{PC}	XYZ	J^{PC}_{exp}
$1(s/d)_1$	4011	1^{--},$(0,1,2)^{-+}$	Y(4008)	1^{--}
$1p_1$	4145	1^{++},$(0,1,2)^{+-}$	Y(4140)	1^{++}
			X(4160)	$?^{??}$
$2(s/d)_1$	4355	1^{--},$(0,1,2)^{-+}$	X(4320)	1^{--}
			X(4350)	$?^{?+}$
			Y(4360)	1^{--}
			Y(4390)	1^{--}
$1p_0$	4486	0^{++},1^{+-}	X(4500)	0^{++}
$3(s/d)_1$	4692	1^{--},$(0,1,2)^{-+}$	Y(4660)	1^{--}
			X(4630)	1^{--}
$2(s/d)_1$	10885	1^{--},$(0,1,2)^{-+}$	$Y_b(10890)$	1^{--}

Table 1: Hybrid states in our spectrum with masses and quantum numbers compatible with charmonium (above) and bottomonium (below) XYZ resonances.

its short distance behavior from the weak coupling regime of pNRQCD [14], and its long distance one from the QCD EST [12]. V_S^{Π} and V_S^{Σ} are proportional to the NRQCD matching coefficient of the chromomagnetic interaction c_F, which we approximate by its tree level value $c_F = 1$. Then, we obtain for the short and long distance behavior,

$$V_S^{\Pi}(r) \underset{r\to 0}{\sim} V_S^{\Sigma}(r) \xrightarrow[r\to 0]{} \pm\frac{\lambda^2}{m_Q} \quad , V_S^{\Sigma}(r) \xrightarrow[r\to\infty]{} -\frac{\pi^2 g\Lambda'''}{m_Q\kappa r^3} \, , V_S^{\Pi}(r) \xrightarrow[r\to\infty]{} \sqrt{\frac{\pi^3}{\kappa}}\frac{g\Lambda'}{2m_Q r^2} \, , \tag{3.5}$$

where λ is a constant $\mathscr{O}(\Lambda_{\rm QCD})$, $\kappa = 0.187$ GeV^2 the string tension , and Λ' and Λ''' are constants $\mathscr{O}(\Lambda_{\rm QCD})$ that also appear in the long distance behavior of the quarkonium spin-orbit and tensor potentials [15]. We obtain from fits to the lattice data for those potentials in ref. [16], $g\Lambda' \sim -59\,$MeV$\,$, $g\Lambda''' \sim \pm 230\,MeV\,$.. We model the mixing potential with simple interpolations that reproduce the correct short and long distance limits. We fix the parameter λ and the possible signs by requiring the mixing to be large for the $S=0$ charmonium hybrids. With this choice, $Y(4008)$, $Y(4360)$ and $Y(4660)$ contain 29%, 35% and 17% of spin one quarkonium. Hence, the spin symmetry violating decays of $Y(4008)$, $Y(4360)$ and $Y(4660)$ are qualitatively explained. The complete results are presented in Tables IV and VI-XII of ref. [7]. We only display here, in Table 4, the XYZ states that can be identified in our spectrum.

4. Hyperfine splittings

Hyperfine splittings appear at $\mathscr{O}(1/m_Q)$ in hybrids rather than at $\mathscr{O}(1/m_Q^2)$ as in quarkonium. Furthermore, at leading order, they are controlled by a single term in the Lagrangian [7],

$$i\varepsilon^{ijk}V^S(r)\,\mathrm{tr}\left(H^{i\dagger}\left[\sigma^k, H^j\right]\right) \tag{4.1}$$

This allows to put forward results that are independent of the form of $V^S(r)$ [17]. Let us specify first the quantum mechanical Hamiltonian, H_{hf}, following from (4.1) in the $|SLJ\mathscr{J}\mathscr{M}\rangle$ basis. In

Resonance	J^{PC}_{exp}	Assignement	Mass (MeV)
X(3823)	2^{--}	$1d$	3792
X(3872)	1^{++}	$2p$	3967
X(3915)	$0\,\text{or}\,2^{++}$	$2p$	3968
X(3940)	$?^{??}$	$2p$	3968
Y(4008)	1^{--}	$1s/d_1$	4004
X(4140)	1^{++}	??	??
X(4160)	$?^{??}$	$1p_1$	4146
Y(4220)	1^{--}	$2d$	4180
X(4230)	1^{--}	$2d$	4180
Y(4274)	1^{++}	$3p$	4368
X(4350)	$?^{?+}$	$2(s/d)_1$ or 3p	4355 or 4369
Y(4320)	1^{--}	$2(s/d)_1$	4366
Y(4360)	1^{--}	$2(s/d)_1$	4366
X(4390)	1^{--}	$2(s/d)_1$	4366
X(4500)	0^{++}	$1p_0$	4566
Y(4630)	1^{--}	$3d$	4559
Y(4660)	1^{--}	$3(s/d)_1$	4711
X(4700)	0^{++}	$4p$	4703
$\mathrm{Y_b}$(10890)	1^{--}	$2(s/d)_1$	10890

Table 2: The identification of isospin zero XYZ charmonium (above) and bottomonium (below) resonances with states in our spetrum. The last column shows the masses obtained in our calculation.

this basis, H_{hf} contains diagonal and off-diagonal terms, and it is non-vanishing on $S=1$ states only. The detailed structure of H_{hf} can be found in [17, 10]. At first order in perturbation theory, only the diagonal terms contribute. They lead to the following mass formulas for a quadruplet J

$$\frac{M_{1\,J+1}-M_{0\,J}}{M_{1\,J}-M_{0\,J}}=-J\,, \quad \frac{M_{1\,J-1}-M_{0\,J}}{M_{1\,J}-M_{0\,J}}=J+1\,, \tag{4.2}$$

where $M_{S\,\mathscr{J}}$ denotes the mass of a given state in the quadruplet ($\mathscr{J}=J$ for $S=0$, and $\mathscr{J}=J, J\pm 1$ for $S=1$). The formulas above imply $M_{1\,J+1}+M_{1\,J-1}=M_{1\,J}+M_{0\,J}$, which can be checked against recent lattice data from HSC collaboration [18] for the $1(s/d)_1$, $1p_1$ and $1(p/f)_2$ states. The difference between the lhs and rhs is 14 MeV, 45 MeV and 39 MeV respectively. These figures fall within the expected size of the $\mathscr{O}(\Lambda^3_{\rm QCD}/m_Q^2)$ corrections, and hence this formula is consistent with the data of ref. [18]. Note, however, that the off-diagonal terms in H_{hf} mix different J multiplets, and may lead to enhanced $\mathscr{O}(\Lambda^3_{\rm QCD}/m_Q^2)$ contributions to the hyperfine splittings if there are multiplets with similar masses, in an analogous way as the mixing with quarkonia leads to enhanced spin symmetry violations.

5. Discussion

All the isospin zero XYZ resonances fit well in our spectrum, either as a quarkonium or hybrid states, except for $X(4140)$. This resonance could in principle be assigned to the $1p_1$ spin zero charmonium hybrid. However the selection rule found in refs. [6, 7] that $L = J$ hybrids do not decay to quarkonium at leading order, disfavors this assignment. We end up assigning the $X(4160)$ to the $1p_1$ spin zero hybrid, and since there is no other state with 1^{++} quantum numbers nearby in our spectrum, $X(4140)$ is left with no assignment.

The mixing of spin zero hybrids with spin one quarkonia is important for the assignments of $Y(4008)$, $Y(4220)$, $X(4230)$, $Y(4320)$, $Y(4360)$, $Y(4660)$ and $Y_b(10890)$ as hybrid states, since otherwise spin symmetry violating decays of these resonances would have been observed. It is also important in order to understand why the spin symmetry violating transitions of $\Upsilon(10860)$ to P-wave states are of the same order as the ones that respect spin symmetry.

In the charmonium spectrum there are too many 1^{--} resonances. The wide $Y(4008)$ resonance was observed by Belle [19], but it has not been confirmed by Babar [20] or BESIII [21]. If we assume that this resonance is not there, then there is no assignment for $1(s/d)_1$ spin zero state, unless we assign it to the $\psi(4040)$. The fact that the $1(s/d)_1$ spin zero state has about a 30% mixing with spin one quarkonium may justify the assignment. Then the $3s$ state would be the $\psi(4160)$ and the $2d$ state the $X(4230)/Y(4220)$. The $X(4230)$ and the $Y(4220)$ resonances have compatible parameters and must be identified for them both to be compatible with our spectrum. This is consistent with the recent analysis of ref. [22]. For the spin zero $2(s/d)_1$ state, there are three competing resonances that should be identified, $Y(4320)$, $Y(4360)$, and $Y(4390)$. This is also consistent with the recent analysis of ref. [22]. If this picture is correct, below the $Y(4660)$ resonance there would only be the $3d$ state to be discovered, around 4560 MeV.

Concerning 1^{--} bottomonium resonances, all of them fit in our spectrum. In addition there should be three states to be discovered below the $\Upsilon(10860)$, the $2d$, the $1(s/d)_1$ and the $3d$, around 10440 MeV, 10690 MeV and 10710 MeV respectively.

Regarding the hyperfine splittings, notice that formula (4.2) holds for the splittings induced by the spin-orbit term in quarkonium as well (by changing J by L). Hence the ultrafine splitting introduced in [23] turns out to vanish for hybrids. Let us finally mention that the short distance behavior of $V^S(r)$ together with that of the $1/m_Q^2$ potentials contributing to the hyperfine splitting has been recently calculated in [24]. Predictions for the bottomonium hyperfine splittings at NLO were put forward by approximating the full potentials by their short distance behavior and fitting the unknown parameters to the hybrid charmonium spectrum of ref. [18]. We would like to remark that unlike quarkonium, in which the typical value of r lies between $1/m_Q\alpha_s$ and $1/(m_Q\Lambda_{\rm QCD}^2)^{1/3}$ and hence it is parametrically smaller than $1/\Lambda_{\rm QCD}$, the typical value of r in hybrids is of the order of $1/\Lambda_{\rm QCD}$. Hence the short distance behavior need not be a good approximation for the relevant distances in the hybrid system. Notice also that, at NLO, the mixing with quarkonium states studied in Sec. 3 contributes to the hyperfine splitting as well [7].

6. Conclusions

We have calculated the charmonium and bottomonium hybrid spectrum in a QCD based approach, including for the first time the mixing with quarkonia. The latter leads to enhanced spin symmetry violations, which are instrumental to identify a number of XYZ states as hybrids. Most of the isospin zero XYZ states fit well in our spectrum, either as hybrids or as standard quarkonium states. We have also provided model independent formulas for the hyperfine splittings.

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