Jet fragmentation in a dense QCD medium

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We study the fragmentation of a jet propagating in a dense quark-gluon plasma. Using a leading, double-logarithmic approximation in perturbative QCD, we compute for the first time the effects of the medium on the vacuum-like emissions. We show that, due to the scatterings off the plasma, the in-medium parton showers differ from the vacuum ones in two crucial aspects: their phase-space is reduced and the first emission outside the medium can violate angular ordering. We compute the jet fragmentation function and find results in qualitative agreement with measurements at the LHC.

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1. Introduction

One of the main objectives of the experimental programs at RHIC and at the LHC is the characterisation of the quark-gluon plasma (QGP) produced in ultrarelativistic heavy ion collisions. An important class of observables used to study this dense form of QCD matter refers to the physics of jet quenching, i.e. the modifications of the properties of an energetic jet due to its interactions with the surrounding medium.

For example the suppression of the jet cross-section in nucleus-nucleus collision w.r.t proton-proton is an indication of jet energy loss in the plasma [1, 2]. Most recently, substructure observables such that the jet fragmentation function [3, 4], the jet angular shape [5] or the z_g distribution [6] have also shed light on the evolution of a jet in the QGP. This proceeding focuses mainly on the jet fragmentation function which quantifies the particle distribution in energy inside jets.

From a theoretical point of view, high-p_T jets are valuable because it is possible to rely on perturbative QCD to predict their properties. In that context, one of the effects of a dense weakly-coupled quark gluon plasma on virtual partons inside jets is to trigger medium-induced radiations because of the multiple collisions with the medium constituents. This can be computed using the BDMPS-Z formalism [7, 8, 9], recently generalised to include multiple medium-induced branchings [10, 11, 12, 13, 14]. However it is also clear that the overall jet structure should get built via the usual, "vacuum-like" bremsstrahlung through which a virtual parton evacuates its virtuality (until this becomes as small as the hadronisation scale).

Thus, in order to construct the jet evolution in the presence of a medium, one needs to understand the interplay between these two mechanisms: vacuum-like emissions (VLEs) and medium induced radiations. Taken separately, these two mechanisms are by now rather well understood, but it appears as a challenge to construct a unified theoretical picture which consistently encompasses both sources of radiation.

In this proceeding based on the recent paper [15], we emphasize that the double-logarithmic approximation (DLA) of perturbative QCD is the first mandatory step toward a more advanced treatment of the evolution of jets in a QGP. In the vacuum, DLA analysis exhibits the main physical ingredients of jets, namely, the structure of intrajet parton cascades and the role of QCD coherence effects in soft gluon multiplication processes (hump-backed plateau) [16]. With a dense medium, the same analysis can be done and we show that within this approximation the time scales in the evolution factorize.

2. Phase space for vacuum-like emissions with a dense medium

In this section, we present the phase space available for vacuum-like emissions in the presence of a medium.

2.1 Bremsstrahlung spectrum and BDMPS-Z spectrum

In the vacuum, the building block of a jet evolution is the Bremsstrahlung spectrum. A virtual parton produced by a hard scattering can radiate a gluon with a probability which is logarithmically enhanced for soft and collinear emissions. It is a specific feature of the Bremsstrahlung probability

distribution $d\mathscr{P}_{\mathscr{B}}$ for the simple process represented figure 1-left.

$$d\mathscr{P}_{\mathscr{B}} = \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{d\theta^2}{\theta^2}$$

This double-logarithmic enhancement of the emission probability is the essence of the double log-arithmic approximation. In the computation of intrajet observables we resum to all orders only contributions of the form

$$\alpha_s \log(\bar{\theta}^2/\theta_m^2) \log(E/\omega_m) \sim 1$$

for a given cut-off in energy ω_m and angle θ_m, with $\bar{\theta}$ is the opening angle of the jet and E its energy. A loss of a single angular or energy logarithm is sub-leading at DLA.

On top of that, the dense weakly-coupled medium can trigger medium-induced radiations. The underlying hypothesis for such emissions to occur within the BDMPS-Z formalism is that the formation time of the medium-induced parton is much larger than the mean free path of the emitting particle. In the following, the quark-gluon plasma is characterized by only two parameters: the distance L travelled by the jet inside the medium and the jet quenching parameter $\hat{q}$ related to the averaged transverse momentum acquired by multiple collisions during time Δt by $\langle k_\perp^2 \rangle = \hat{q}\Delta t$. In this framework, the probability distribution $d\mathscr{P}_{BDMPS-Z}$ for the process represented figure 1-right is well approximated by the formula [17, 18]

$$d\mathscr{P}_{BDMPS-Z} \simeq \frac{\alpha_s N_c}{\pi} L \sqrt{\frac{\hat{q}}{\omega^3}} d\omega$$

With respect to the Bremsstrahlung spectrum, the obvious property of the BDMPS-Z spectrum is that collinear and soft radiations are not *logarithmically* enhanced. This will have important consequences since working at the double-log accuracy enables to simply ignore such emissions for the intrajet activity.
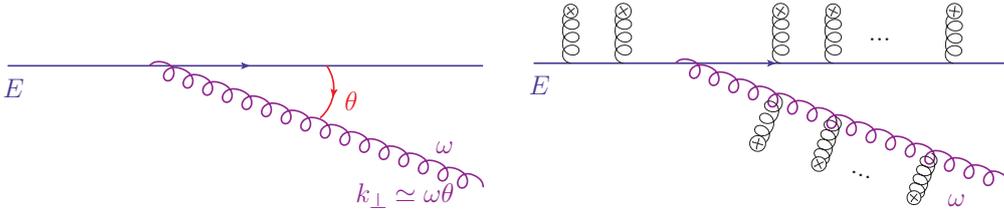
Figure 1: Leading-order Feynman diagrams for the Bremsstrahlung (left) and medium-induced (right) processes. The medium is described by a fluctuating colored classical field whose vertices are represented by the symbol $\otimes$.

2.2 Vacuum-like emissions inside and outside the medium

However, as we show in this subsection, medium induced radiations provide a natural constraint on the phase space available for one emission inside the medium.

Indeed, considering first an emission which occurs inside the medium, its formation time t_f is then smaller than L. t_f is determined by the uncertainty principle, namely the condition that the transverse separation $\Delta r \sim \theta t_f$ between the gluon and its parent parton at the time of emission

to be as large as the gluon transverse wavelength $2/k_\perp$, with $k_\perp \simeq \omega\theta$ its transverse momentum w.r.t its parent. This argument applies to both vacuum-like and medium-induced emissions and implies $t_f \simeq 2\omega/k_\perp^2 \simeq 2/(\omega\theta^2)$. Then, gluons emitted inside the medium have a minimum $k_\perp$ set by the momentum acquired via multiple collisions during its formation, $k_f^2 = \hat{q}t_f$. Gluons produced inside the medium with a transverse momentum smaller than $\hat{q}t_f$ cannot exist. This translates into an upper limit $t_f \leq \sqrt{2\omega/\hat{q}}$ on the formation time of any gluon inside the medium. That said, medium induced gluons for which $k_\perp \simeq k_f$ are excluded because the emission probability is not enhanced by double logarithms. Consequently, at DLA the only contribution to the intrajet activity inside the medium comes from vacuum-like (Bremsstrahlung) emissions with $k_\perp \geq k_f$, inequality which becomes strong at DLA: $k_\perp \gg k_f$ or equivalently $t_f \ll \sqrt{2\omega/\hat{q}}$ meaning that VLEs occur much faster than medium-induced radiations with the same energy.

This is the condition for an emission *inside* the medium to be vacuum-like. For a medium with a fixed finite length, an emission can also occur directly outside if its formation time is larger than L: $t_f \geq L$.

2.3 Lund diagram for one vacuum-like emission

These two conditions for VLEs, either inside with $t_f \ll \sqrt{2\omega/\hat{q}}$ or outside with $t_f \geq L$ can be written in terms of their energies ω and angles w.r.t the emitter θ:

$$t_f \ll \sqrt{2\omega/\hat{q}} \Longleftrightarrow \omega \gg \omega_0(\theta) = (2\hat{q}/\theta^4)^{1/3} \text{ and } t_f \geq L \Longleftrightarrow \omega \leq \omega_L(\theta) = 2/(\theta^2 L)$$

It is enlightening to represent these conditions on a diagram with ω and θ as axis. Compared to the vacuum case for which the phase space is only restricted by the hadronisation line $k_\perp \simeq \omega\theta = \Lambda$, the effect of the medium on the phase space available for VLEs is the presence of a vetoed region [15] where there is no VLE permitted as shown figure 2.

The critical energy $\omega_c = 1/2\hat{q}L^2$ such that $\sqrt{2\omega/\hat{q}} = L$ also appears figure 2. Emissions with larger energies $\omega \geq \omega_c$ behave exactly as in the vacuum: their emission angle can be arbitrarily small and their formation time can be larger than L. We shall assume that $E \geq \omega_c$, which is indeed the case for the high energy ($E \geq 100$ GeV) jets at the LHC.

3. Construction of the parton shower with a dense medium

In the previous section, the phase space for one VLE at DLA has been discussed. Here, we explain how to iterate such emissions and how the medium affects the standard angular ordered vacuum cascade.

3.1 Decoherence

In the vacuum quantum color coherence implies angular ordering: a jet is described in terms of a classical shower picture with successive angular ordered Bremsstrahlung emissions forming the Markov chains of independent elementary radiation events [16]. However a $q\bar{q}$ antenna with opening angle $\theta_{q\bar{q}}$ propagating through a dense QGP can lose its coherence via rescattering off the medium: the quark and the antiquark suffer independent color rotations, hence the probability that the antenna remains in a color singlet state decreases with time [19, 20, 21]. The two legs of
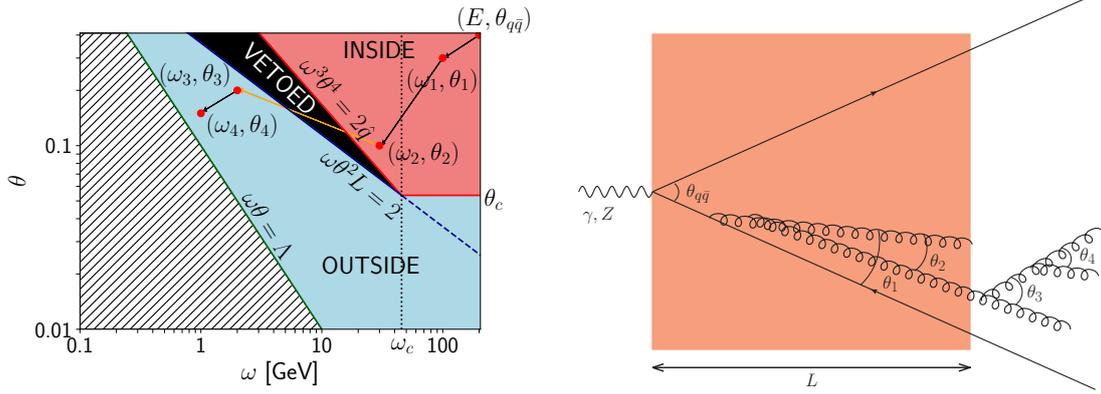
Figure 2: (Left) Phase space for one emission in a jet of energy $E = 200$ GeV and opening angle $\bar{\theta} = 0.4$ in the presence of a medium. The red region corresponds to the condition $t_f \leq \sqrt{2\omega/\hat{q}}$ and $\theta \geq \theta_c$ and the blue region $t_f \geq L$. In the hatched region, partons have hadronised and the perturbative QCD regime is not valid anymore. (Right) Typical cascade which may occur with a medium and not in the vacuum. The position of corresponding successive emissions in the phase space are represented on the left. The first two emissions occur inside the medium and the last two outside. Angular ordering is violated by the third emission, but preserved by all the others.

the antenna start behaving like independent color sources after a time $t_{coh}(\theta_{q\bar{q}}) \equiv (4/\hat{q}\theta_{q\bar{q}}^2)^{1/3}$ [20]. Consequently, angular ordering could in principle be violated inside the medium.

3.2 Iteration of VLEs inside the medium

It is conceptually simpler to consider a jet initiated by a quark-antiquark antenna in a color singlet state with opening angle $\theta_{q\bar{q}} \leq 1$, e.g. produced by the decay of a boosted W/Z boson or a virtual photon. (For a generic jet which is produced by a parton, the role of $\theta_{q\bar{q}}$ would be played by the jet radius $\bar{\theta}$.) The quark and the antiquark are assumed to have equal energies $E/2$. Also, the antenna is assumed to be produced directly inside the medium and to cross the medium along a distance L. Finally, for simplicity we shall work in the limit of a large number of colors $N_c \gg 1$, where a gluon emission can be pictured as the splitting of one dipole into two.

Now one can show that decoherence has no consequences on the development of the shower *inside* the medium. Indeed, decoherence is impossible for VLEs inside the medium. Consider the ith antenna in the evolution: its energy is ω_i and its opening angle is θ_i. It has been emitted by the antenna $i-1$. By definition $\theta_0 = \theta_{q\bar{q}}$ and $\omega_0 = E$. Then it is easy to check that

$$t_f(\omega_i, \theta_i) \geq t_{coh}(\theta_{i-1}) \text{ and } \theta_i \geq \theta_{i-1} \implies t_f(\omega_i, \theta_i) \geq \sqrt{2\omega_i/\hat{q}}$$

so that an incoherent large angle emission is necessarily inside the vetoed region and therefore is not allowed at DLA.

Hence, at DLA, successive in-medium vacuum-like emissions are strongly ordered both in energy and angle [15].

3.3 First emission outside the medium

Actually, the argument presented above does not apply if the antenna $i-1$ is the *last* antenna inside the medium. Indeed, in that case, the condition for the ith antenna to be vacuum-like is not $t_f(\omega_i, \theta_i) \ll \sqrt{\frac{2\omega_i}{\hat{q}}}$ but $t_f(\omega_i, \theta_i) \geq L$. Therefore the good criterion to discuss the coherence property of the antenna $i-1$ is the condition $t_{coh}(\theta_{i-1}) = L$ since the next antenna will require a time larger than L to be emitted. The critical angle $\theta_c = 2/\sqrt{\hat{q}L^3}$ satisfies the equality $t_{coh}(\theta_c) = L$. Thus, if $\theta_{i-1} \leq \theta_c$, the coherence time is also larger than L and angular ordering is preserved. On the other hand, if $\theta_{i-1} \geq \theta_c$ the antenna has lost its coherence during the formation time of the next antenna so there is no constraint on the angle θ_i of the next antenna. The introduction of this angle θ_c leads to a slight modification of the in-medium region in the Lund diagram: this region is now defined by $t_f(\omega) \leq \sqrt{\frac{2\omega}{\hat{q}}}$ and $\theta \geq \theta_c$ so that inside this region, cascades are always angular ordered.

To sum up, decoherence induced by the medium has only one effect on the parton shower at DLA: one violation of angular ordering by the first emission outside the medium is permitted [15] (a similar idea appears in [22]). Nevertheless this effect will have striking consequences on the shape of the fragmentation function.

4. Calculation of the fragmentation function

In this section, the fragmentation function of a jet at parton level is calculated according to the principles that we have just established.

4.1 DLA results

The basic quantity required to compute the fragmentation in our picture is the double differential inclusive probability distribution to find a parton with energy ω and angle θ^2 (w.r.t its emitter) inside the jet of energy E and angle $\theta_{q\bar{q}}$.

$$T(\omega, \theta^2 | E, \theta_{q\bar{q}}^2) \equiv \omega \theta^2 \frac{dN}{d\omega d\theta^2}$$

At DLA in the vacuum, this function satisfies the simple following master equation [16]

$$T_{vac}(\omega, \theta^2 \mid E, \theta_{q\bar{q}}^2) = \bar{\alpha}_s + \bar{\alpha}_s \int_{\theta^2}^{\theta_{q\bar{q}}^2} \frac{d\theta_1^2}{\theta_1^2} \int_{\omega/E}^1 \frac{dz_1}{z_1} T_{vac}(\omega, \theta^2 \mid z_1 E, \theta_1^2)$$

with $\bar{\alpha}_s = \alpha_s N_c/\pi$. The solution to this equation is

$$T_{vac}(\omega, \theta^2 \mid E, \theta_{q\bar{q}}^2) = \bar{\alpha}_s I_0 \left(2\sqrt{\bar{\alpha}_s \log(E/\omega) \log(\theta_{q\bar{q}}^2/\theta^2)} \right)$$

with I_0 the modified Bessel function of rank 0.

In the in-medium region of the phase space or for $\omega \geq \omega_c$, since nothing differs from the vacuum case, the corresponding function with a medium $T(\omega, \theta^2)$ is identical to $T_{vac}(\omega, \theta^2)$. However, in the out-of-the-medium region, T is different because one must take into account "jumps" over the vetoed region with a possible violation of angular ordering. Mathematically, this can be done by convolutions of the function T_{vac} over the disjoint in and out regions of the phase space [15].

Once $T(\omega, \theta^2 | E, \theta_{q\bar{q}}^2)$ is known for every ω and θ^2, the fragmentation function $D(\omega | E, \theta_{q\bar{q}}^2)$ at the parton level is obtained by integrating out the unmeasured angle θ^2 between the $k_\perp = \Lambda$ cut-off and $\theta_{q\bar{q}}$.

$$D(\omega | E, \theta_{q\bar{q}}^2) \equiv \omega \frac{dN}{d\omega} = \int_{\Lambda^2/\omega^2}^{\theta_{q\bar{q}}^2} \frac{d\theta^2}{\theta^2} T(\omega, \theta^2 | E, \theta_{q\bar{q}}^2)$$

The results from [15] are shown figure 3. The left figure refers to the two-dimensional gluon distribution $T(\omega, \theta^2 | E, \theta_{q\bar{q}}^2)$: we more precisely show the ratio $T(\omega, \theta^2)/T_{vac}(\omega, \theta^2)$ between the distribution generated in the presence of the medium and that in the vacuum. This ratio is 1 for all the points either inside the medium or with $\omega \geq \omega_c$ as expected. However, one sees significant deviations from unity for points outside the medium with energies $\omega \leq \omega_c$: for intermediate values of ω and relatively small angles $\theta \simeq 0.1\theta_{q\bar{q}}$, one sees a small but significant suppression compared to the vacuum (up to 15%). For smaller energies and larger angles, $\theta \geq 0.2$, one rather sees a strong enhancement, owing to emissions violating angular ordering.

The right plot in figure 3 shows the ratio $D(\omega)/D_{vac}(\omega)$. One sees a slight suppression (relative to vacuum) at intermediate energies, roughly from 2 GeV up to ω_c, and a substantial enhancement at lower energies $\omega \lesssim 2$ GeV. This enhancement is attributed to small-angle emissions inside the medium, radiating at larger angles outside the medium due to the lack of angular ordering. These results are in qualitative agreement with the respective LHC measurements for the most central PbPb collisions [3, 4].
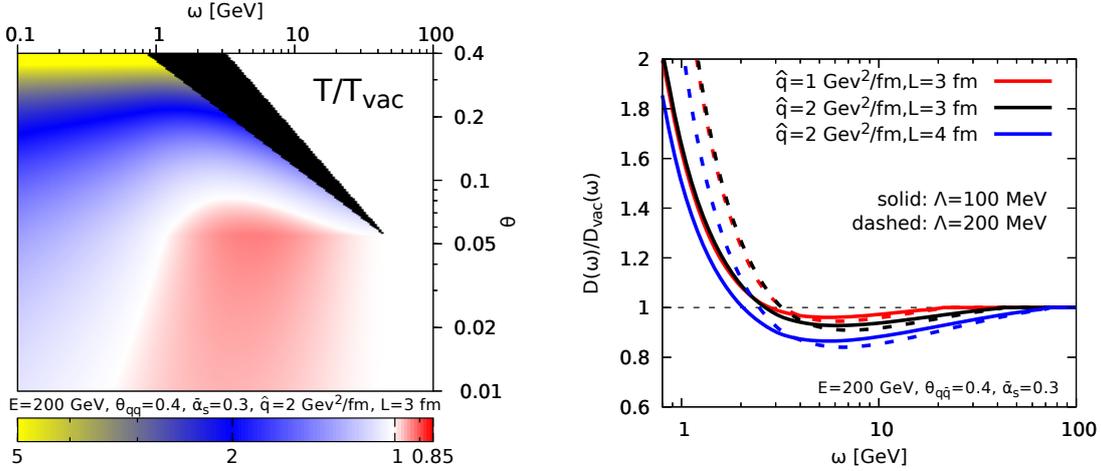
Figure 3: (Left) The ratio $T(\omega, \theta^2)/T_{vac}(\omega, \theta^2)$ between the two-dimensional gluon distributions in the medium and respectively the vacuum, both computed to DLA and for the values of the free parameters E, $\theta_{q\bar{q}}$, $\bar{\alpha}_s$, $\hat{q}$ and L shown in the figure. (Right) The ratio $D(\omega)/D_{vac}(\omega)$ between the fragmentation functions in the medium and respectively the vacuum, for different choices for the medium parameters $\hat{q}$ and L and the hadronisation scale Λ.

4.2 Running coupling and single-log corrections

Previous calculations assumed a strong ordering both in energy and angle and a fixed coupling α_s (an assumption valid in the double-log approximation). It is natural to ask whether our results for

the fragmentation function are robust enough even if we relax one of these hypothesis. For instance, one can relax the fixed coupling approximation or take into account single-log corrections.

This can be done in a straightforward way in this formalism. If only gluons are considered, the general master equation for T_{vac} is indeed [23, 24]

$$T_{vac}(\omega, \theta^2 \mid E, \bar{\theta}^2) = \bar{\alpha}_s \frac{\omega}{E} P_{gg}(\omega/E) + \int_{\theta^2}^{\bar{\theta}^2} \frac{d\theta_1^2}{\theta_1^2} \int_{\omega/E}^1 dz_1 \bar{\alpha}_s(z_1^2 E^2 \theta_1^2) P_{gg}(z_1) T_{vac}(\omega, \theta^2 \mid z_1 E, \theta_1^2)$$

with $P_{gg}(z) = (1-z)\left[z(1-z) + \frac{1-z}{z} + \frac{z}{1-z}\right]$. Then the gluon distribution with a medium T is calculated by convolutions using the new solution T_{vac} of this equation. Numerical results are shown figure 4-left for three cases:

- the double-log approximation corresponds to $\bar{\alpha}_s = cste$ and $P_{gg}(z) \simeq 1/z$.

- the running of the coupling: $\bar{\alpha}_s(\omega^2 \theta^2) = \frac{1}{b} \frac{1}{\log(\omega^2 \theta^2/\Lambda^2)}$, $\bar{b} = 11/12$ and $P_{gg}(z) \simeq 1/z$.

- the next-to-double-log approximation: the running coupling and the finite part of the splitting function are taken into account, namely $P_{gg}(z) \simeq \frac{1}{z} + \int_0^1 dz(P_{gg}(z) - \frac{1}{z}) = \frac{1}{z} - \frac{11}{12}$ [24].

As expected, DLA overestimates the number of soft gluons inside the jet but the enhancement at small energies is still significant if we include the effects of the running coupling and the full splitting function. This shows the robustness of decoherence to generate soft gluons inside jets in the presence of a medium.
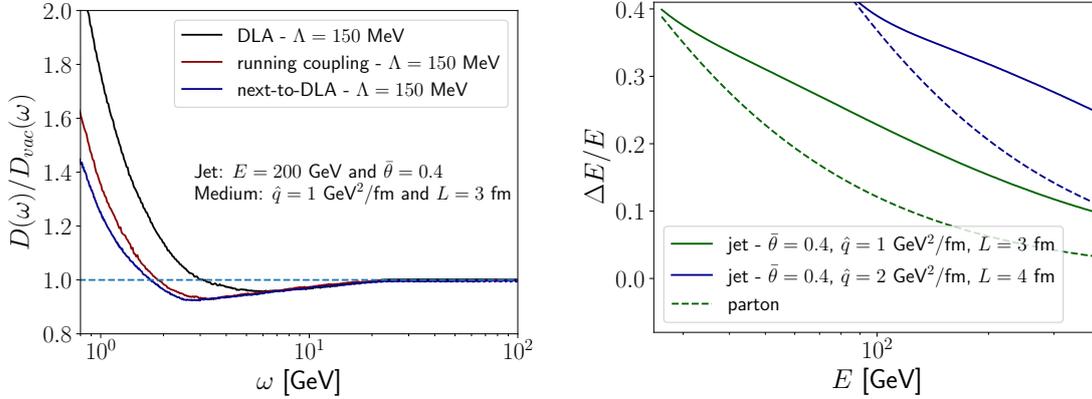
Figure 4: (Left) The ratio $D(\omega)/D_{vac}(\omega)$ between the fragmentation functions in the medium and respectively the vacuum for different approximations in perturbative QCD: DLA, running coupling and single-log corrections from the full splitting function. (Right) The relative energy loss by a jet $\Delta E/E$ due to medium induced radiations as a function of the jet energy E. The dashed lines represent the same quantity for a single parton propagating through the medium.

4.3 Jet energy loss

Even if the next-to-double-log approximation does not conserve the energy since the symmetry $z \Leftrightarrow 1-z$ is broken, one can try to estimate the energy loss for a jet compared to the energy lost by a single parton propagating through the medium.

First of all, at DLA energy loss is negligible for any parton of the cascade inside the medium, except for the *last* one which will propagate through the medium over a distance of order L. Indeed, the maximal energy loss $\omega_{loss}(t_f(\omega_i, \theta_i))$ by the ith parton of the in-medium cascade during its formation time is of the order $\hat{q}t_f^2/2$, the energy of the hardest medium induced emission that can develop during t_f [7, 8]. By the inequality $t_f(\omega_i, \theta_i^2) \ll \sqrt{2\omega_i/\hat{q}}$, one finds that $\omega_{loss}(t_f) \ll \omega_i$.

For an estimation of the energy loss by a jet as a global physical object, the DLA picture has an interesting property: the partons produced inside the medium via DLA cascades act as new sources which will lose energy via medium induced processes. The energy loss by a single source of energy ω propagating over a distance L is estimated from medium induced radiation of BDMPS-Z type iterated using energy-conserving splitting vertex (essential at this stage since we focus on energy) [11]. It is a good approximation to estimate the energy lost at large angles as the energy accumulated by the very soft quanta which thermalize, according to the formula [25]

$$\mathscr{E}(\omega, L) \simeq \omega(1 - e^{-2\pi\omega_{br}/\omega})$$

with $\omega_{br} = \alpha_s^2 \omega_c$ the scale below which multiple medium-induced branchings become important. A source loses typically an energy $\min(\omega, \omega_{br})$. The total energy lost by a jet from an initial parton with energy E is

$$\Delta E = \mathscr{E}(E, L) + \int_{\theta_c^2}^{\theta_{q\bar{q}}^2} \frac{d\theta_1^2}{\theta_1^2} \int_{\omega_0(\theta_1)}^{E} \frac{d\omega_1}{\omega_1} T(\omega_1, \theta_1^2 | E, \theta_{q\bar{q}}^2) \mathscr{E}(\omega_1, L)$$

It would be necessary to impose energy conservation in all vertices of the cascade to have a good quantitative estimation of the energy loss, but even in the result figure 4-right, one sees that jet energy loss decreases with its initial energy more slowly than the parton energy loss. This is a first step toward a precise understanding of the R_{AA} ratio for jets in the data [1, 2] since the vacuum inclusive jet cross-section is not included in this calculation.

5. Conclusion

In this proceeding, we showed that vacuum-like emissions inside the medium can be factorized from the medium-induced radiations within the double-log approximation. This approximation is fine for intrajet multiplicity at small energy but it is not accurate enough for experimental observables relying on energy because energy is not exactly conserved through the jet evolution.

However, DLA gives a new insight on the development of a jet in the quark-gluon plasma. Since based on a probabilistic picture, this approach is suitable for Monte-Carlo implementations, which would allow to go beyond the present approximations and reach more quantitative predictions.

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