PROCEEDINGS OF SCIENCE

Thermal properties of $S = -1$ Hyperons: an S-matrix approach

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I describe how the S-matrix formulation of statistical mechanics can be applied to study a coupled-channel system of $S = -1$ hyperons.

XIII Quark Confinement and the Hadron Spectrum - Confinement2018
31 July - 6 August 2018
Maynooth University, Ireland

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1. Introduction

The central idea of the S-matrix formulation of statistical mechanics by Dashen, Ma, and Bernstein [1] is to compute an effective density of states based on scattering matrix elements. When applied to describe the system of interacting hadrons [2], the approach requires the scattering amplitudes of hadrons as input.

Fortunately a very detailed picture of hadronic interactions has emerged from the impressive volume of experimental data, carefully analyzed by theory such as chiral perturbation theory [3, 4], lattice QCD [5], effective hadron models [6] and time-honored potential models [7, 8]. Consequently we acquire very precise information on particle spectra, production mechanisms and decay properties of typical hadrons and even the exotics [9].

Among the common approaches of computing thermal observables (e.g. the standard imaginary-time formalism), the S-matrix approach offers the most direct connection to the empirical data. This makes the method ideal for tapping into the rich resources of the field of hadron physics for studying heavy ion collisions. Nevertheless, applications thus far are mostly confined to the elastic 2-body scatterings [10, 11, 12]. In this proceeding, we describe a robust method for extracting an effective phase shift and an effective spectral weight for a coupled system of multiple interaction channels. I shall demonstrate the method by studying the thermal system of $S=-1$ hyperons.

2. S-matrix formulation of statistical mechanics

In the S-matrix formalism, the logarithm of the partition function is written as a sum of two terms:

$$\ln Z = \ln Z_0 + \Delta \ln Z, \tag{2.1}$$

where

$$\ln Z_0 = V \times \sum_{i \in \mathrm{gs}} d_i \int \frac{d^3k}{(2\pi)^3}\, e^{-\beta\sqrt{k^2+m_i^2}} \tag{2.2}$$

is the grand potential for an uncorrelated gas of ground-state particles, i.e. particles that do not decay under the strong interaction, such as pions, kaons, and nucleons. The interacting part of the grand potential, $\Delta \ln Z$, can be written in the form

$$\Delta \ln Z = V \times \int \frac{d\sqrt{s}}{2\pi} \frac{d^3P}{(2\pi)^3}\, e^{-\beta\sqrt{P^2+s}}\, B_{\mathrm{eff}}(\sqrt{s}), \tag{2.3}$$

where $\sqrt{s}$ is the invariant mass of the relevant scattering system. The quantity $B_{\mathrm{eff}}(\sqrt{s})$ can be understood as an effective level density due to the interaction. A key step of the formulation is to relate $B_{\mathrm{eff}}(\sqrt{s})$, in the low density limit where only binary collisions are important, to the (derivative of) scattering phase shifts

$$B_{\rm eff}(\sqrt{s}) \to B_{\rm smat}(\sqrt{s}) = \sum_{\rm int} d_{IJ} \times 2 \frac{d}{d\sqrt{s}} \delta_{IJ}(\sqrt{s}). \tag{2.4}$$

Here the sum is over all interaction channels, d_g is the relevant degeneracy factor, and $\delta_{IJ}(\sqrt{s})$ is the scattering phase shift. The corresponding results of the standard Hadron Resonance Gas (HRG) model can also be expressed in this form, with the replacement

$$B_{\rm eff}(\sqrt{s}) \to \rho_{\rm HRG}(\sqrt{s}) = \sum_{\rm res} d_{IJ} \times 2 \frac{d}{d\sqrt{s}} \left(\pi \times \theta(\sqrt{s} - m_{\rm res})\right). \tag{2.5}$$

This establishes the fundamental premise of the HRG model: contribution of resonances to the thermodynamics is approximated by an uncorrelated gas of zero-width particles.

Here we recap some of the key features of the S-matrix approach:

- Effective versus full spectral function:

  While the HRG treatment may be valid for a narrow resonance, Eq. (2.4) is applicable even for a broad resonance. For the case of resonant scattering, the effective spectral function $B(\sqrt{s})$ contains both the contribution from the full spectral function $A(\sqrt{s})$ of the resonance and a non-resonant contribution $\Delta A(\sqrt{s})$ from the correlated pair of the forming constituents [13, 14]:

$$B(\sqrt{s}) = A(\sqrt{s}) + \Delta A(\sqrt{s}). \tag{2.6}$$

  In many statistical models the width of resonances is implemented via an energy-dependent Breit-Wigner (BW) function. This amounts to approximating $A(\sqrt{s})$. On the other hand, the term $\Delta A\,(\sqrt{s})$ is mostly neglected. The importance of this term has been demonstrated in the study of the p_T-spectra of the decay pions coming from ρ-mesons [15, 16], where substantial enhancement is found in the soft momentum region.

  In fact, it can be shown [14] that $\Delta A(\sqrt{s})$ dominates near the threshold. The phase shift, to lowest order in momentum q of the particles in the CM frame, takes the following form

$$\delta(\sqrt{s}) \approx d_S \times (a_S\, q) + d_P \times (a_P\, q^3) + \cdots, \tag{2.7}$$

  The scattering lengths $a_{S,P,\ldots}$ are usually well constrained by the chiral perturbation theory. At this limit, B is dominated by the non-resonant piece:

$$B(\sqrt{s}) \to \Delta A(\sqrt{s}) \approx 2 \frac{dq}{d\sqrt{s}} \times (d_S a_S + 3 d_P a_P q^2 + \cdots). \tag{2.8}$$

- Comparison to other approaches:

  Phase shifts encode a wealth of information about the hadronic interactions. For example, a simple hard-core repulsive force tends to push states away from the low energies into the higher energy region. Thermal observables, such as the pressure, would then receive an overall negative correction, as the low-energy part of the density of state is less Boltzmann-suppressed.

  Nevertheless, multiple physical effects may be at work to produce such a drop in the phase shift. Some other reasons (not mutually exclusive) could be:

  - resonance contributes less than expected in a BW framework,
  - a new channel opens up,
  - t- or u-channel exchanges.

  Thus, a single thermodynamic quantity, say the particle density $n(T)$, is insufficient to resolve the various models of interactions, nor to pinpoint a specific density of state [1]. When the relevant density of state is known (i.e. the phase shift is known), model parameters (e.g. couplings, masses, etc) need to be adjusted to reproduce the known phase shift for consistency [17, 18]. This is automatically fulfilled by the S-matrix approach as the empirical data is used as input. The importance of a reliable S-matrix cannot be overstated.

3. Effective phase shift for coupled-channel system

To describe a coupled-channel system, we consider the following generalization to a single-channel phase shift [14, 19]:

$$\begin{aligned} Q(\sqrt{s}) &\equiv \frac{1}{2}\,\mathrm{Im}\,(\,\mathrm{tr}\ln \mathrm{S}\,) \\ &= \frac{1}{2}\,\mathrm{Im}\,(\,\ln\det \mathrm{S}\,)\,. \end{aligned} \tag{3.1}$$

and the corresponding effective spectral function reads

$$B = 2\frac{d}{d\sqrt{s}}Q(\sqrt{s}). \tag{3.2}$$

Eq. (3.1) has been applied to extract an effective phase shift for the $S = -1$ hyperon system. This problem demands a coupled-channel treatment as inelastic interactions among $(\bar{K}N, \pi\Lambda, \pi\Sigma)$ set in at rather low momentum. In this study the (complex) determinant of a (16×16) S-matrix, contructed by the Joint Physics Analysis Center (JPAC) collaboration [20], is numerically computed at each CM energy $\sqrt{s}$, for each partial wave. The phase shifts and the corresponding spectral functions are shown in Fig. 1.

[1] A differential distribution, such as $\frac{dn}{d\sqrt{s}}$ or some momentum distributions, would be more indicative.

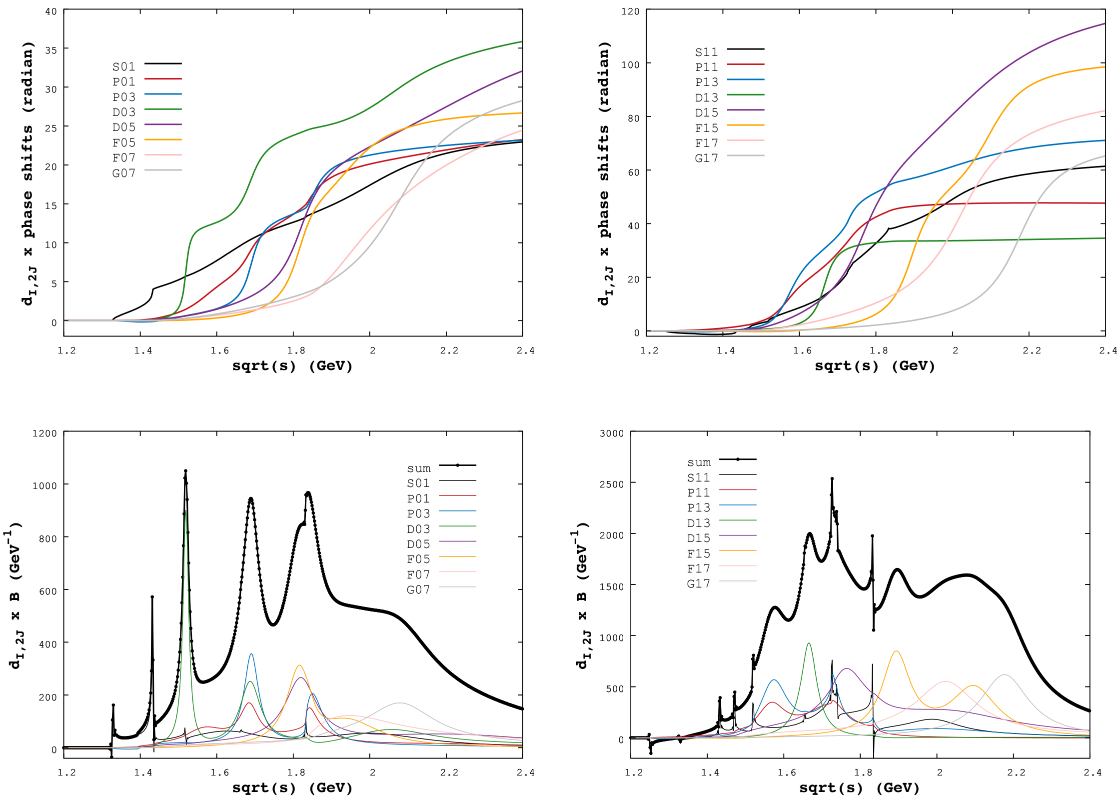

Figure 1: Top: The generalized phase shift function $Q(\sqrt{s})$ (in radians) extracted from a coupled-channel PWA [20] for $I = 0$ (left) and $I = 1$ (right). Bottom: The corresponding effective spectral functions $B(\sqrt{s}) = 2\frac{d}{d\sqrt{s}}Q$. See Ref. [19] for details.

Computing thermal observables based on these empirical phase shifts allow the proper treatment of resonances, and the natural incorporation of additional hyperon states which are not listed in the Particle Data Group. An interesting quantity, also studied by the Lattice QCD [21, 22, 23], is the second order baryon strangeness correlation, χ_{BS}. The result is shown in Fig. 2.

4. To do

Many topics remain to be explored in the S-matrix formulation. One pressing issue is the systematic inclusion of $(N > 2)$-body interactions. The analysis of the $\pi\pi N$ state is particularly interesting for the phenomenology of heavy ion collisions. As the $\pi N \to \pi\pi N$ amplitudes are not as firmly established as the elastic case, a detailed comparison of various PWAs is needed. This would be a good opportunity to revisit the theoretical issues of eigenphases, branchings and the construction of 3-body vertex.

The extension of the S-matrix approach to include the 3-body forces would lead to more reliable theoretical predictions on the thermal yields and the momentum spectra of hadrons in heavy ion collision experiments. It can also help to better understand the lattice QCD results on various

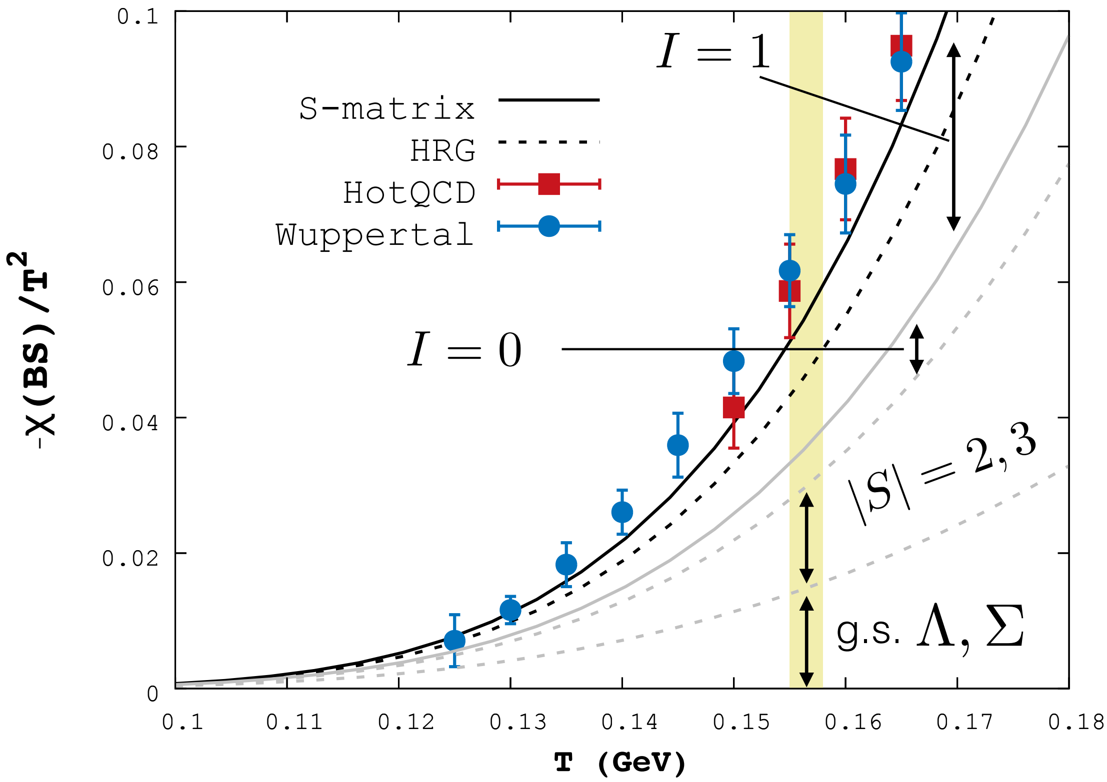

Figure 2: The second order baryon strangeness correlations, χ_{BS} in LQCD [22, 23] compared to the S-matrix computation. Contributions from different sectors are displayed. See Ref. [19] for details.

fluctuation observables. Last, but not least, it would serve as a theory template to study nuclear matter in this approach, where the inclusion of higher-body nuclear forces, on top of NN scatterings, is essential for describing the equation of state. A coupled-channel framework is also the customary way to treat exotics. The S-matrix approach is then the appropriate tool to investigate the thermal properties of these states.

Acknowledgments

I thank Peter Petreczky for the kind invitation to the conference and the fruitful collaboration. I am also grateful to the organizing team, especially the students, who have worked diligently throughout the meeting. This research benefits greatly from the discussion with Hans Feldmeier and Bengt Friman in GSI. This work was partially supported by the Polish National Science Center (NCN), under Maestro grant DEC-2013/10/A/ST2/00106.

References

[1] R. Dashen, S.-K. Ma and H. J. Bernstein, *S-Matrix formulation of statistical mechanics*, *Phys. Rev.* **187** (1969) 345.

[2] R. Venugopalan and M. Prakash, *Thermal properties of interacting hadrons*, *Nucl. Phys.* **A546** (1992) 718.

[3] J. Gasser and H. Leutwyler, *Chiral Perturbation Theory to One Loop*, *Annals Phys.* **158** (1984) 142.

[4] J. A. Oller and E. Oset, *Chiral symmetry amplitudes in the S wave isoscalar and isovector channels and the σ, f_0(980), a_0(980) scalar mesons*, *Nucl. Phys.* **A620** (1997) 438 [hep-ph/9702314].

[5] M. R. Shepherd, J. J. Dudek and R. E. Mitchell, *Searching for the rules that govern hadron construction*, *Nature* **534** (2016) 487 [1802.08131].

[6] R. Rapp and J. Wambach, *Chiral symmetry restoration and dileptons in relativistic heavy ion collisions*, *Adv. Nucl. Phys.* **25** (2000) 1 [hep-ph/9909229].

[7] S. Godfrey and N. Isgur, *Mesons in a Relativized Quark Model with Chromodynamics*, *Phys. Rev.* **D32** (1985) 189.

[8] T. Barnes and E. S. Swanson, *A Diagrammatic approach to meson meson scattering in the nonrelativistic quark potential model*, *Phys. Rev.* **D46** (1992) 131.

[9] R. F. Lebed, R. E. Mitchell and E. S. Swanson, *Heavy-Quark QCD Exotica*, *Prog. Part. Nucl. Phys.* **93** (2017) 143 [1610.04528].

[10] P. M. Lo, B. Friman, K. Redlich and C. Sasaki, *S-matrix analysis of the baryon electric charge correlation*, *Phys. Lett.* **B778** (2018) 454 [1710.02711].

[11] P. Huovinen and P. Petreczky, *Hadron Resonance Gas with Repulsive Interactions and Fluctuations of Conserved Charges*, *Phys. Lett.* **B777** (2018) 125 [1708.00879].

[12] A. Andronic, P. Braun-Munzinger, B. Friman, P. M. Lo, K. Redlich and J. Stachel, *The thermal proton yield anomaly in pb-pb collisions at the lhc and its resolution*, *Phys. Lett.* **B792** (2019) 304 [1808.03102].

[13] W. Weinhold, B. Friman and W. Norenberg, *Thermodynamics of Δ resonances*, *Phys. Lett.* **B433** (1998) 236 [nucl-th/9710014].

[14] P. M. Lo, *S-matrix formulation of thermodynamics with N-body scatterings*, *Eur. Phys. J.* **C77** (2017) 533 [1707.04490].

[15] P. Huovinen, P. M. Lo, M. Marczenko, K. Morita, K. Redlich and C. Sasaki, *Effects of ρ-meson width on pion distributions in heavy-ion collisions*, *Phys. Lett.* **B769** (2017) 509 [1608.06817].

[16] P. M. Lo, *Resonance decay dynamics and their effects on p_T-spectra of pions in heavy-ion collisions*, *Phys. Rev.* **C97** (2018) 035210 [1705.01514].

[17] B. Friman, P. M. Lo, M. Marczenko, K. Redlich and C. Sasaki, *Strangeness fluctuations from $K-\pi$ interactions*, *Phys. Rev.* **D92** (2015) 074003 [1507.04183].

[18] P. M. Lo, B. Friman, M. Marczenko, K. Redlich and C. Sasaki, *Repulsive interactions and their effects on the thermodynamics of a hadron gas*, *Phys. Rev.* **C96** (2017) 015207 [1703.00306].

[19] C. Fernández-Ramírez, P. M. Lo and P. Petreczky, *Thermodynamics of the strange baryon system from a coupled-channels analysis and missing states*, *Phys. Rev.* **C98** (2018) 044910 [1806.02177].

[20] C. Fernández-Ramírez, I. V. Danilkin, D. M. Manley, V. Mathieu and A. P. Szczepaniak, *Coupled-channel model for $\bar{K}N$ scattering in the resonant region*, *Phys. Rev.* **D93** (2016) 034029 [1510.07065].

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[21] A. Bazavov et al., *Additional Strange Hadrons from QCD Thermodynamics and Strangeness Freezeout in Heavy Ion Collisions*, *Phys. Rev. Lett.* **113** (2014) 072001 [1404.6511].

[22] HotQCD collaboration, A. Bazavov et al., *Fluctuations and Correlations of net baryon number, electric charge, and strangeness: A comparison of lattice QCD results with the hadron resonance gas model*, *Phys. Rev.* **D86** (2012) 034509 [1203.0784].

[23] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti and K. Szabo, *Fluctuations of conserved charges at finite temperature from lattice QCD*, *JHEP* **01** (2012) 138 [1112.4416].