

Rare FCNC radiative leptonic $B_{s,d} \rightarrow \gamma l^+ l^-$ decays in the SM

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We report our recent results [1] for long-distance QCD effects in the flavour-changing neutral current radiative leptonic decays $B \rightarrow \gamma l^+ l^-$, $l = \{e, \mu\}$. One encounters here two distinct types of long-distance effects: those encoded in the $B \rightarrow \gamma$ transition form factors induced by the $b \rightarrow q$ quark currents, and those related to the charm-loop effects. We calculate the $B \rightarrow \gamma$ form factors in a broad range of the momentum transfers making use of the relativistic dispersion approach based on the constituent quark picture which has proven to provide reliable predictions for many weak-transition form factors. Concerning the description of the charm-loop contributions, we point out two observations: First, the precise description of the charmonium resonances, in particular, the relative phases between ψ and ψ' , has impact on the differential distributions and on the forward-backward asymmetry, A_{FB} , in a broad range of $q^2 \geq 5 \text{ GeV}^2$. Second, the shape of A_{FB} in $B \rightarrow \gamma l^+ l^-$ and in $B \rightarrow V l^+ l^-$ (V the vector meson) in the q^2 -region between ψ and ψ' provides an unambiguous probe of the relative phases between ψ and ψ' . Fixing the latter will lead to a sizeable reduction of the theoretical uncertainties in A_{FB} at $q^2 = 5 - 9 \text{ GeV}^2$ where it has the sensitivity to physics beyond the SM.

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1. Introduction

Rare FCNC decays of B -mesons are forbidden at the tree level in the Standard Model and occur only via loop diagrams. Respectively, their branching ratios are very small, of order $10^{-8} - 10^{-10}$ [2]. On the other hand, new particles can propagate in the loops and hence such processes are expected to be sensitive to the possible effects of New Physics. Therefore, FCNC processes are in the focus of efforts of many experimental collaborations. Several FCNC decays have been observed experimentally, exhibiting a few deviations from the Standard Model at the level of $2 - 3 \sigma$ (see discussion in [3, 4]).

An appropriate framework for the theoretical description of FCNC B -decays is the effective field theory: virtual heavy particles of the SM with the masses much greater than m_b (i.e., W , Z , and t -quark) are integrated out yielding the $b \rightarrow q$ effective Hamiltonian [5–7]

$$H_{\text{eff}}^{b \rightarrow q} = \frac{G_F}{\sqrt{2}} V_{tq}^* V_{tb} \sum_i C_i(\mu) \mathcal{O}_i^{b \rightarrow q}(\mu). \quad (1.1)$$

The basis operators $\mathcal{O}_i^{b \rightarrow q}(\mu)$ contain only dynamical light degrees of freedom (u , d , s , c , and b -quarks, leptons, photons and gluons). These light particles appear as dynamical degrees of freedom in the diagrams for the amplitudes of rare FCNC B -decays. The Wilson coefficients $C_i(\mu)$ absorb the contributions of the heavy particles (W , Z , and t in the SM) given by the box and the penguin diagrams; taking into account hard gluon exchanges in the Feynman diagrams leads to the dependence of C_i on the scale μ . Finally, the top-quark contribution to the amplitude of the radiative leptonic B -decay is given by

$$A(B_q \rightarrow \gamma ll) = \langle \gamma ll | H_{\text{eff}}^{b \rightarrow q} | B_q \rangle. \quad (1.2)$$

The presence of the B -meson in the initial state leads to the necessity of treating nonperturbative QCD effects; the amplitudes of FCNC rare leptonic B -decays involve a great variety of such nonperturbative QCD contributions. Next sections briefly review such contributions.

2. Top-quark contributions

The diagrams generated by the t -quarks (as well as other heavy particles of the SM) in the loops are presented in Fig. 1 and Fig. 2. The B -decay amplitudes corresponding to these diagrams are described in terms of $B \rightarrow \gamma$ transition form factors. In order to take into account nonperturba-

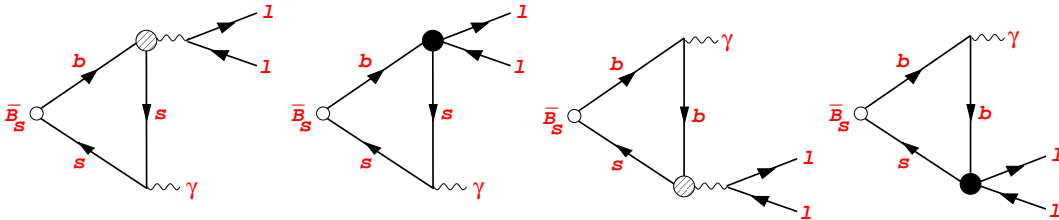


Figure 1: Real photon emission from the valence quark. Dashed circle denotes the $b \rightarrow s\gamma$ operator $C_{7\gamma}$. Solid circle denotes the $b \rightarrow sl^+l^-$ operators O_{9V} and O_{10A} .

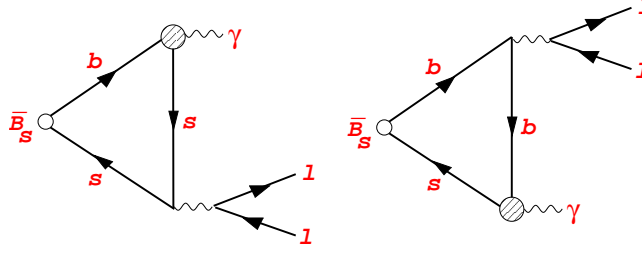


Figure 2: Real photon emission from the penguin FCNC vertex. Dashed circle denotes the $b \rightarrow s\gamma$ operator $C_{7\gamma}$.

tive QCD contributions related to the B -meson in the initial state, one needs to make use of an appropriate approach.

The diagrams of Fig. 1 yield contributions that have no singularities in the physical decay region. For these contributions, we made use of the dispersion approach based on the relativistic constituent quark picture [8–12]: within this approach, the form factors are given by relativistic spectral representations via the meson relativistic wave functions. The transition form factors from the dispersion approach satisfy all rigorous constraints known from QCD for these quantities [9, 13]. The model parameters (i.e., the constituent quark masses and the hadron wave functions) have been fixed by the requirement to reproduce the known values of meson weak decay constants [1]. The left diagram of Fig. 2 leads to the contributions to the form factors that do have singularities (resonances and cuts) in the physical decay region. To calculate these contributions, we combined the results from the dispersion approach at $q^2 < 0$ with the gauge-invariant version of vector meson dominance [14].

The form factors from the dispersion approach describing both types of the contributions shown in Figs. 1 and 2 and the convenient parametrizations of these form factors have been reported in [1] (see also [15]). Recently, very close numerical results for the form factors have been obtained within a conceptually rather different approach [16], thus providing additional support for the reliability of our predictions for the form factors.

Finally, Fig. 3 displays the Bremsstrahlung contribution to the $B_{s,d} \rightarrow \gamma\ell^+\ell^-$ amplitude; it is given in terms of the B -meson decay constant f_B and is proportional to the mass of the lepton in the final state [17].

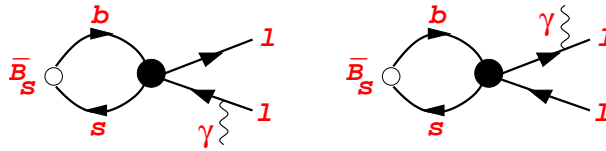


Figure 3: The Bremsstrahlung contribution. Solid circle here denotes the $b \rightarrow s\ell^+\ell^-$ operator O_{10A} , as the O_{9V} operator does not contribute to the Bremsstrahlung amplitude.

The diagrams of Figs. 1, 2, and 3 fully describe the contributions of the top-quark and other heavy degrees of freedom of the SM, that have been integrated out and are being described via the effective Hamiltonian.

3. Charm-quark contributions

Heavy degrees of freedom (t , W , Z) are described by the effective Hamiltonian, but light degrees of freedom, e.g., c and u quarks, remain dynamical and run in the loops. Fig. 4 shows charm contribu-

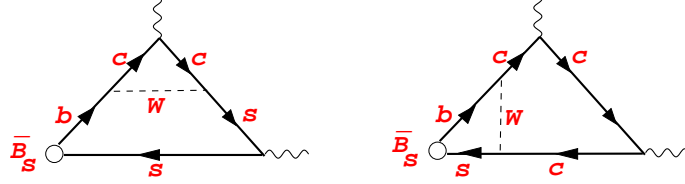


Figure 4: Diagrams with c -quarks in the loops: penguin diagram (left), weak annihilation diagram (right)

tions: the numerically dominant penguin diagram (left) and the weak-annihilation diagram (right). Shrinking the W -boson line to a point, and taking into account soft-gluon exchanges between the charm-loop and the B -meson loop, reduces the charming penguin to two classes of diagrams, Fig. 5.

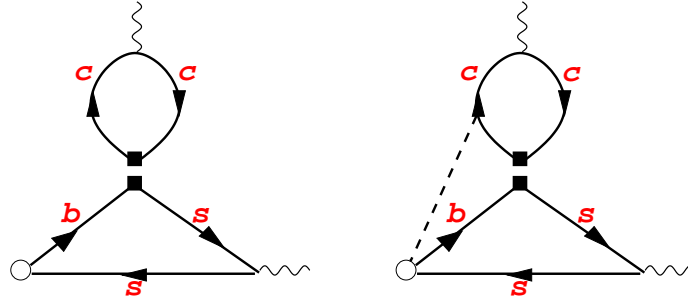


Figure 5: Factorizable (left) and nonfactorizable (right) charming loops. Dashed line is the soft gluon.

The structure of the charm-loop contributions to the $B_s \rightarrow \gamma l^+ l^-$ amplitude has the form:

$$H_{\mu\alpha}(k', k) = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* e \left[\varepsilon_{\mu\alpha k' k} H_V - i (g_{\alpha\mu} k k' - k'_\alpha k_\mu) H_A \right], \quad (3.1)$$

with $0 < q^2 < M_B^2$, including the region of the charmonium resonances. Perturbative QCD cannot be applied here and approaches based on hadron degrees of freedom are necessary [18–25]. For $H_i(q^2, 0)$ one may write dispersion representation in q^2 with two subtractions [20]:

$$H_i(q^2, 0) = a_i + b_i q^2 + (q^2)^2 \left\{ \sum_{\psi=J/\psi, \psi'} \frac{f_\psi \mathcal{A}_{B\psi\gamma}^i}{m_\psi^3 (m_\psi^2 - q^2 - im_\psi \Gamma_\psi)} + h_i(q^2) \right\}, \quad i = V, A, \quad (3.2)$$

where $h_i(q^2)$ describe hadron continuum including broad charmonium states above the DD threshold. At $q^2 > 4M_D^2$, the known broad vector ψ_n ($n = 3, \dots, 6$) resonances are taken into account. The subtraction constants a_i and b_i are determined by matching to the known results from light-cone sum rules at $q^2 \leq 4m_c^2$, including non-factorizable corrections calculated in [20]. The absolute values of the amplitudes $\mathcal{A}_{B\psi\gamma}^i$ may be measured in $B \rightarrow \psi_i \gamma$ decays. However, nonfactorizable gluons

may generate complex relative phases between the contributions of different charmonia [21]. These possible non-universal (i.e., process-dependent and thus different in $B \rightarrow \gamma ll$ and $B \rightarrow Vll$) relative phases cannot be determined by pQCD-based calculations and provide one of the main sources of the theoretical uncertainties for rare radiative leptonic decays. Further details see [1, 26].

4. Results

Here we present some of our predictions for $B_{s,d} \rightarrow \gamma l^+ l^-$ decays reported in [1].

4.1 Differential branching ratios

The differential branching ratios are shown in Fig. 6. The plots in Fig. 6 correspond to the de-

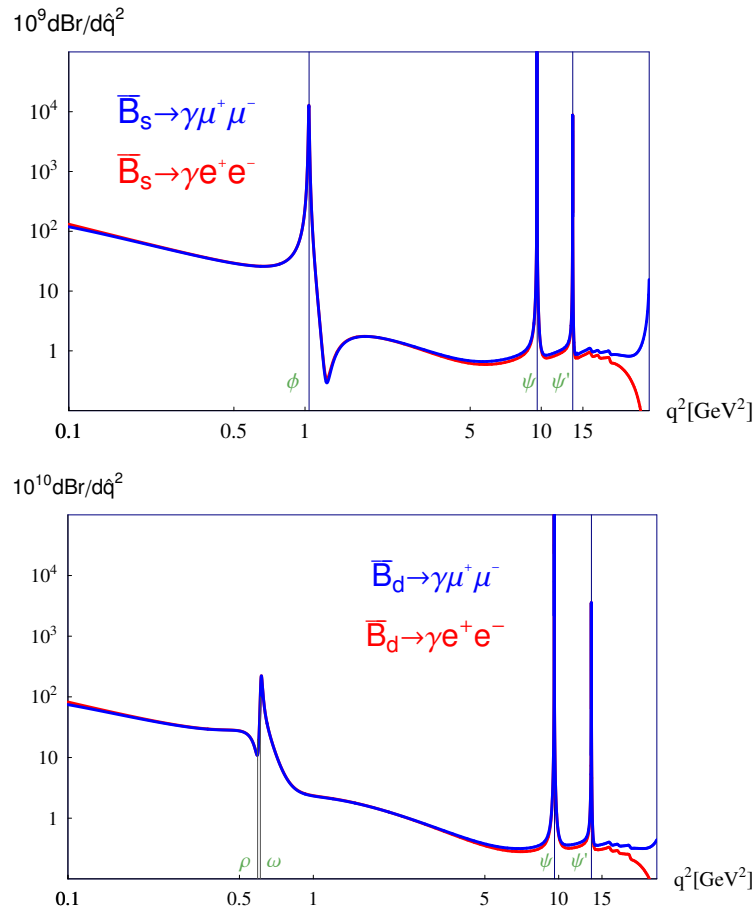


Figure 6: Differential branching fractions for $B_s \rightarrow \gamma l^+ l^-$ (left) and $B_d \rightarrow \gamma l^+ l^-$ (right) decays. Blue lines - $\mu^+ \mu^-$ final state, red lines - $e^+ e^-$ final state.

scription of the charm-loop effects according to Eq. (3.2), and further assuming that all charmonia contribute with the same positive sign (coinciding with the sign of the factorizable contribution).

At $q^2 \leq 6 \text{ GeV}^2$, charming loops contribute at the level of a few percent, so the observables may be predicted with a small error, caused mainly by the form-factor uncertainty. We reported [1]

$$\begin{aligned} \mathcal{B}(\bar{B}_s \rightarrow \gamma l^+ l^-)|_{q^2 \in [1,6] \text{ GeV}^2} &= (6.01 \pm 0.08) 10^{-9} \\ \mathcal{B}(\bar{B}_d \rightarrow \gamma l^+ l^-)|_{q^2 \in [1,6] \text{ GeV}^2} &= (1.02 \pm 0.15) 10^{-11}. \end{aligned} \quad (4.1)$$

For the $B_s \rightarrow \gamma l^+ l^-$ transition, the dominant contribution is given by the ϕ -meson. Its parameters are known well, leading to the small uncertainty in the $B_s \rightarrow \gamma l^+ l^-$ decay rate integrated over $q^2 = [1, 6] \text{ GeV}^2$. For the $B_d \rightarrow \gamma l^+ l^-$ transition, the contribution of the vector resonances is less important, and the branching ratio uncertainty reflects to a large extent the 10% uncertainty in the form factor contributions given by Fig. 1.

4.2 Forward-backward asymmetry

Fig. 7 shows A_{FB} for $\bar{B}_s \rightarrow \gamma \mu^+ \mu^-$.

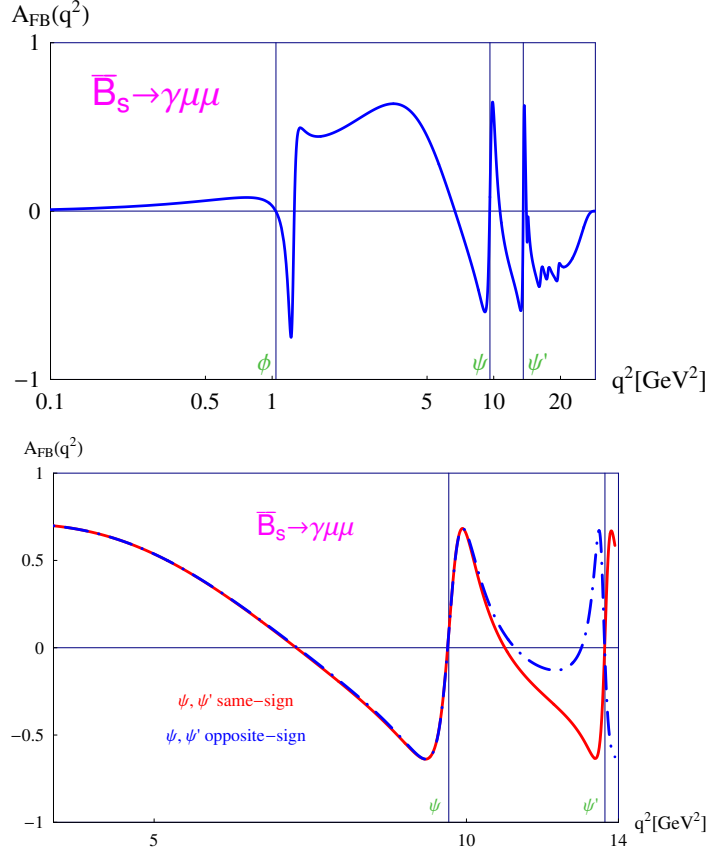


Figure 7: Forward-backward asymmetry for $B_s \rightarrow \gamma \mu \mu$ decays. The upper plot shows the asymmetry for all q^2 for all charmonia contributions taken of the same positive sign, equal to that of the factorizable contribution. The lower plot exhibits the sensitivity of A_{FB} in the region between ψ and ψ' to the relative signs of these states: solid (red) line - both signs positive, dashed (blue) line - the ψ contribution taken positive, whereas the ψ' contribution negative.

Further results see in [1]. A recent critical analysis of the theoretical status of charm-loop effects in FCNF decays is given in [26].

Acknowledgments

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References

- [1] A. Kozachuk, D. Melikhov and N. Nikitin, *Rare FCNC radiative leptonic $B_{s,d} \rightarrow \gamma l^+ l^-$ decays in the standard model*, Phys. Rev. D **97**, no. 5, 053007 (2018) [arXiv:1712.07926]
- [2] A. Ali, *B decays, flavor mixings and CP violation in the SM*, [hep-ph/9606324].
- [3] D. Guadagnoli, D. Melikhov and M. Reboud, *More Lepton Flavor Violating Observables for LHCb's Run 2*, Phys. Lett. B **760**, 442 (2016) [arXiv:1605.05718].
- [4] D. Guadagnoli, *Flavor anomalies on the eve of the Run-2 verdict*, Mod. Phys. Lett. A **32**, 1730006 (2017) [arXiv:1703.02804].
- [5] B. Grinstein, M. J. Savage and M. B. Wise, *$B \rightarrow X(s)e^+e^-$ in the Six Quark Model*, Nucl. Phys. B **319** 271 (1989).
- [6] A. J. Buras and M. Munz, *Effective Hamiltonian for $B \rightarrow X(s)e^+e^-$ beyond leading logarithms in the NDR and HV schemes*, Phys. Rev. D **52**, 186 (1995) [hep-ph/9501281].
- [7] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, *Weak decays beyond leading logarithms*, Rev. Mod. Phys. **68**, 1125 (1996) [hep-ph/9512380].
- [8] D. Melikhov, *Form-factors of meson decays in the relativistic constituent quark model*, Phys. Rev. D **53**, 2460 (1996) [hep-ph/9509268].
- [9] D. Melikhov, *Heavy quark expansion and universal form-factors in quark model*, Phys. Rev. D **56**, 7089 (1997) [hep-ph/9706417].
- [10] D. Melikhov, *Dispersion approach to quark binding effects in weak decays of heavy mesons*, Eur. Phys. J. direct **4**, 2 (2002) [hep-ph/0110087].
- [11] M. Beyer and D. Melikhov, *Form-factors of exclusive $b \rightarrow u$ transitions*, Phys. Lett. B **436**, 344 (1999) [hep-ph/9807223].
- [12] D. Melikhov and B. Stech, *Weak form-factors for heavy meson decays: An Update*, Phys. Rev. D **62**, 014006 (2000) [hep-ph/0001113].
- [13] F. Kruger and D. Melikhov, *Gauge invariance and form-factors for the decay $B \rightarrow \gamma l^+ l^-$* , Phys. Rev. D **67**, 034002 (2003) [hep-ph/0208256].
- [14] D. Melikhov, O. Nachtmann, V. Nikonov and T. Paulus, *Masses and couplings of vector mesons from the pion electromagnetic, weak, and pi gamma transition form-factors*, Eur. Phys. J. **C34**, 345 (2004) [hep-ph/0311213].
- [15] A. Kozachuk, D. Melikhov, and N. Nikitin, *Annihilation type rare radiative $B_{(s)} \rightarrow V\gamma$ decays*, Phys. Rev. D **93**, 014015 (2016) [arXiv:1511.03540].
- [16] S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, A. Liptaj, P. Santorelli, C. T. Tran, *Study of $B_s \rightarrow l^+ l^- \gamma$ decays in covariant quark model*, [arXiv:1808.06261].
- [17] D. Melikhov and N. Nikitin, *Rare radiative leptonic decays $B(d,s) \rightarrow \gamma l^+ l^-$* , Phys. Rev. D **70**, 114028 (2004) [hep-ph/0410146].

- [18] A. Ali, T. Mannel, T. Morozumi, *Forward backward asymmetry of dilepton angular distribution in the decay $b \rightarrow sl^+l^-$* , Phys. Lett. B **273**, 505 (1991).
- [19] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, *Penguins with Charm and Quark-Hadron Duality*, Eur. Phys. J. C **61**, 439 (2009) [arXiv:0902.4446].
- [20] A. Khodjamirian, T. Mannel, A. Pivovarov, and Y.-M. Wang, *Charm-loop effect in $B \rightarrow K^{(*)}l^+l^-$ and $B \rightarrow K^*\gamma$* , JHEP **09**, 089 (2010) [arXiv:1006.4945].
- [21] J. Lyon and R. Zwicky, *Resonances gone topsy turvy - the charm of QCD or new physics in $b \rightarrow sl^+l^-$?*, [arXiv:1406.0566].
- [22] M. Ciuchini, M. Fedele, E. Franco, S. Mishima, A. Paul, L. Silvestrini, and M. Valli, *$B \rightarrow K^*l^+l^-$ decays at large recoil in the Standard Model: a theoretical reappraisal*, JHEP **1606**, 116 (2016) [arXiv:1512.07157].
- [23] S. Brass, G. Hiller, and I. Nisandzic, *Zooming in on $B \rightarrow K^{(*)}l^+l^-$ decays at low recoil*, Eur. Phys. J. C **77**, 16 (2017) [arXiv:1606.00775].
- [24] LHCb Collaboration, R. Aaij *et al.*, *Measurement of the phase difference between short- and long-distance amplitudes in the $B^+ \rightarrow K^+\mu^+\mu^-$ decay*, Eur. Phys. J. C **77**, 161 (2017) [arXiv:1612.06764].
- [25] T. Blake, U. Egede, P. Owen, G. Pomery, K. Petridis, *An empirical model of the long-distance contributions to $\bar{B}^0 \rightarrow \bar{K}^{*0}\mu^+\mu^-$* [arXiv:1709.03921].
- [26] A. Kozachuk and D. Melikhov, *Nonfactorizable charm-loop effects in rare FCNC B -decays*, Phys. Lett. B **786**, 378 (2018) [arXiv:1805.05720].