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Extended neutrinosphere effects on SN nu oscillations

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The neutrino flavour evolution in a supernova can be described either in terms of neutrino fields or as the evolution of individual neutrinos. There is no reason to think that the two approaches should give contradicting results, and both have their advantages. One of the advantages of using individual neutrinos is that it becomes clear that the finite width of the neutrino sphere must lead to averaging over the oscillation phase due to the different emission points, and therefore to a reduction of the effective mixing angle. This significant effect is usually ignored in the literature. In this talk, I will explain the details of the argument and interpret it in terms of the density matrix formalism by taking into account the often neglected collision term.

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1. Introduction

Core-collapse supernovae emit 99% of the released energy as neutrinos. Due to the weak interaction of the neutrinos, they are emitted at the centre of the supernova and free-stream through the outer layers. However, the neutrino flavour is affected by an MSW-like effect due to the changing background density and it has been known for more than a decade that a phenomenon called collective oscillations can lead to a significant flavour conversion as well.

Usually when treating collective oscillations, all neutrinos are assumed to be emitted from an infinitesimal sphere (note the recent exception [1]). The argument is that only a very small amount of mixing occurs in the high density environment of the supernova core, so the flavour state is effectively "frozen" [2]. Although this is true in a certain sense, the finite extend of the neutrinosphere still has an effect on the flavour state emerging at the surface of the production region, and this effect will be briefly discussed in the following.

2. Density matrix formalism

Mixing between electron neutrinos ν_e and e.g. tau- neutrinos, ν_τ with a mass squared difference Δm^2, is described with the density matrix

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\tau} \\ \rho_{\tau e} & \rho_{\tau\tau} \end{pmatrix}, \tag{2.1}$$

and the equations of motion (in absence of collisions) are

$$i\dot{\rho} = [H, \rho], \qquad H = H_{\text{vacuum}} + H_{\text{matter}} + H_{\nu\nu}. \tag{2.2}$$

where the Hamiltonian describing the flavour evolution, H is divided in three parts with different origins. Collective oscillations are closely tied to the neutrino term which has the form

$$H_{\nu\nu} = \sqrt{2} G_F \int dp (\rho - \bar{\rho}). \tag{2.3}$$

One of the most striking features of collective oscillations is that neutrinos with different energies oscillate together, a behaviour that is very much the opposite of what happens when the vacuum term dominates.

3. Linear equations and large conversion

One feature that can be noticed from Eq. (2.2) and (2.3) is that the individual neutrinos are following a linear equation of motion. This is obviously not true for the full system of coupled differential equations, but it is still interesting to see how much of the physics that can be captured by a linear set of equations.

Setting up the equations for a probe neutrino in a general background of neutrinos and matter [3], we make the Ansatz that every large conversion can be described in terms of at least one of the following frameworks: *Resonance*: The diagonal of the Hamiltonian in a properly chosen frame is vanishing. *Adiabatic conversion*: Fast oscillations in both the diagonal and off-diagonal

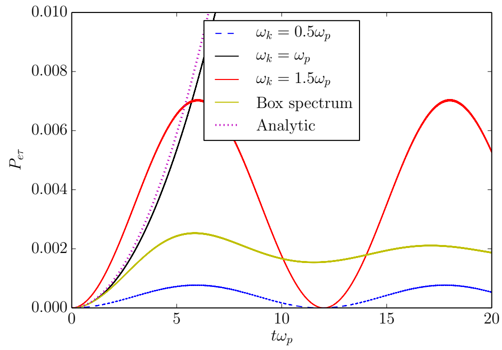

Figure 1: Conversion probability between ν_e and ν_τ in a simple model with a parametric resonance (Figure from [3]).

parts of the Hamiltonian can be removed by going to a rotating frame. The result is a Hamiltonian describing adiabatic evolution. *Parametric enhancement*: The conversion is gradually built up if the period of oscillation equals the period of change of the effective mixing angle.

In Fig. 1 we see the parametric enhancement for a simple model where background neutrinos with $\omega_k = \Delta m^2/4E_k$ travel in one direction and a probe neutrino with $\omega_p = \Delta m^2/4E_p$ travel in a different direction. The parameters are chosen such that the resonance condition is $\omega_k = \omega_p$, but as soon as the energy is slightly different, we see a significant suppression of the conversion amplitude. Taking a simple box spectrum in energy from $0.5\omega_p$ to $1.5\omega_p$, the suppression is significant as well.

One shortcoming of this model is that it does not reproduce the typical energy dependence of collective oscillations. However, it can be used to gauge the averaging effect of an extended emission region. The parametric conversion is driven by the neutrino oscillations in the background, and the amplitude of these oscillations are suppressed when an extended emission region is considered. For the parametric resonance, the conversion time is suppressed linearly when the background oscillation amplitude is reduced [3]. The next question is how collective oscillations are suppressed when feedback is taken into account for the background neutrinos.

4. Extended sources

The radius of the neutrino sphere is typically measured in a few tens of kilometres, but the width is only a factor of a few smaller. Using a conservative value for the width of $\sim$ 1km, the number of oscillations within the production region is $10^6 - 10^8$ [3]. This is also the amount of suppression that can be expected for the oscillation amplitude of the neutrino background.

In order to test how an extended emission region affect collective oscillations, we can use a simplified line model where neutrinos are traveling in the (x,z) plane. The neutrinos are emitted uniformly between $z = 0$ and $z = \Delta z$ at an angle $\beta = \frac{\pi}{4}$. The calculation is done without a collision term, and the neutrinos are introduced in the calculation as they are gradually emitted.

The results for four different sizes of the emission region are shown in the left panel of Fig. 2. The upper panel shows the conversion probability for $\nu_e \to \nu_e$, while the lower panel shows the difference between the normalized off-diagonal of ρ and H. The results show how the conversion is gradually postponed as the emission region becomes larger, but it does not increase linearly.

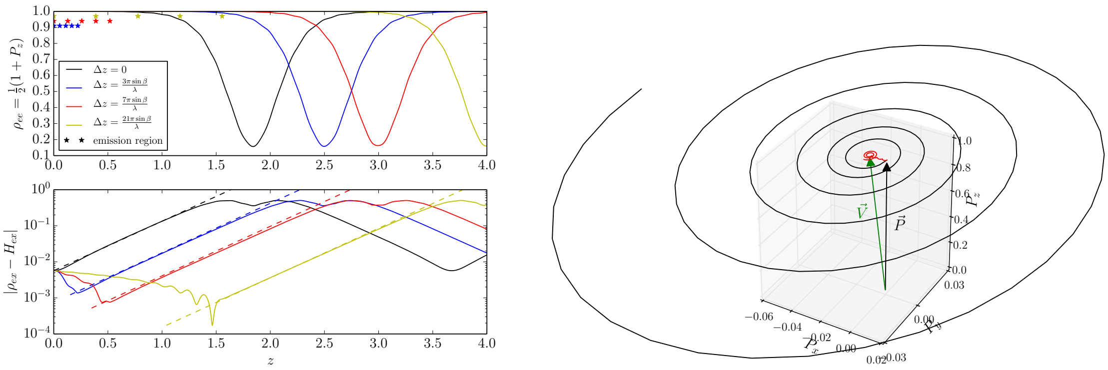

Figure 2: *Upper left*: Conversion probability for four different sizes of the emission region. The size of each emission region is indicated with the stars. *Lower left*: Difference between the normalized off-diagonal term of ρ and H. The dashed lines are the results of a linear stability analysis. *Right*: Density matrix and Hamiltonian written as the two polarization vectors $\vec{P}$ and $\vec{V}$. The colours matches the legend on the figure to the left.

The lower panel in the figure shows how the well known exponential growth is combined with the averaging effect due to the extended emission region. The dashed lines are the result of a linear stability analysis which determines the slope. The initial y-value is determined by averaging the oscillations over the emission region, and reducing the effective initial separation between ρ and H accordingly. The starting point in z is found empirically to be $\frac{2}{3}\Delta z$, and a more refined starting point will be determined in the future.

The right panel of Fig. 2 shows the averaging effect for the polarization vector $\vec{P}$. It is clear that the red curve with the larger emission region at first experiences some averaging, while the black curve starts spiralling out immediately. This can also be interpreted in terms of the collision term that was neglected in Eq. (2.2). One of the effects of the collision term is to push $\vec{P}$ towards the z-axis. As the oscillations at the same time pushes $\vec{P}$ around $\vec{V}$, the net effect for a gradually decreasing collision term is that $\vec{P}$ moves towards $\vec{V}$ - that is what we have called averaging.

5. Conclusions

The finite size of the neutrino sphere leads to an averaging effect, that can suppress the initial condition for collective oscillations by many orders of magnitude. However, due to exponential growth of the solution, flavour conversion is not postponed linearly, but rather logarithmicly.

References

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[3] R. S. L. Hansen and A. Y. Smirnov, *Neutrino conversion in a neutrino flux: Towards an effective theory of collective oscillations*, *JCAP* **1804** (2018) 057 [1801.09751].