

## Phenomenology of minimal seesaw model with $S_4$ symmetry

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We show that a modification of tribimaximal (TBM) mixing matrix accommodating non-zero mixing angle  $\theta_{13}$  and CP violation can be achieved in a minimal seesaw model with discrete symmetry  $S_4$ . This model is very predictive and the undetermined parameters are a common Dirac Yukawa coupling, lightest heavy Majorana neutrino mass and a Majorana phase in the light neutrino sector. The unknown parameters are shown to be constrained through leptogenesis by imposing the recent experimental neutrino data. Based on the constraints obtained from neutrino data and leptogenesis, we predict the branching ratios of the lepton flavor violating processes  $l_i \rightarrow l_j \gamma$  as well as the effective neutrino mass of neutrinoless double beta decays.

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In this work, we show that a minimal modification of TBM mixing matrix,  $U_{\text{TBM}}U_{13}(\theta, \xi)$  [1], is consistent with experiments and originated in a minimal seesaw model (MSM) with  $S_4$  discrete symmetry. We also show that this model is very predictive and undetermined parameters are a common Dirac yukawa coupling, lightest heavy Majorana neutrino mass and a Majorana phase in the light neutrino sector. Possible values of the Dirac-type CP phase  $\delta_D$  can be predicted with regards to two neutrino mixing angles in  $U_{\text{PMNS}}$  [2]. The unknown parameters are constrained through leptogenesis by imposing the recent neutrino data. Based on the constraints obtained from neutrino data and leptogenesis, we predict the branching ratios (BRs) of the lepton flavor violating processes  $l_i \rightarrow l_j \gamma$  as well as the effective neutrino mass of neutrinoless double beta decays.

The Lagrangian for lepton sector of the MSM is given by [3],  $\mathcal{L} = -\bar{l}_{iL}m_{li}l_{iR} - \bar{\nu}_{Li}m_{Dij}N_{Rj} - \frac{1}{2}(N_{Rj})^c M_j N_{Rj}$ , with  $i = 1, 2, 3$ ,  $j = 1, 2$  and the Dirac neutrino mass term  $m_D$  is a  $3 \times 2$  complex matrix. For our purpose, we take a basis where heavy Majorana neutrino mass matrix is diagonal and charged lepton mass matrix is real and diagonal. From the seesaw mechanism, the effective light neutrino mass matrix is given by  $m_{eff} = m_D \frac{1}{M} m_D^T$ , with  $M = \text{Diag.}[M_1, M_2]$ . It is obvious that one of three light neutrino masses is zero in the MSM. For normal (inverted) hierarchical (N(I)H) neutrino mass spectrum,  $m_{1(3)} = 0$ , and thus  $(m_{eff})_{ij} = (U_{\text{PMNS}}^*)_{i2(1)}(U_{\text{PMNS}}^*)_{j2(1)}m_{2(1)} + (U_{\text{PMNS}}^*)_{i3(2)}(U_{\text{PMNS}}^*)_{j3(2)}m_{3(2)}$ , for NH(IH). From two seesaw formula for  $m_{eff}$  given above, one can obtain the relation [4],

$$m_D \frac{1}{\sqrt{M}} O^T = U_{\text{PMNS}}^* \sqrt{m_V^D} \equiv U, \quad (1)$$

where  $\sqrt{m_V^D} = \text{Diag.}[\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}]$ ,  $1/\sqrt{M} = \text{Diag.}[1/\sqrt{M_1}, 1/\sqrt{M_2}]$ .  $2 \times 2$  complex orthogonal matrix  $O$  is parameterized in terms of two complex parameters  $x$  and  $y$  as,  $O = ((x, -y)^T, (y, x)^T)$ . An interesting ansatz for the new form of  $U_{\text{PMNS}}$  must be multiplication of  $U_0^{\text{TBM}}$  by a rotation unitary matrix in the  $(i, j)$  plane with an angle  $\theta$  denoted by  $U_{ij}(\theta)$ ,  $U_0^{\text{TBM}} \cdot U_{ij}(\theta)$  or  $U_{ij}(\theta) \cdot U_0^{\text{TBM}}$ . Among such possible forms, we show that the form  $U_0^{\text{TBM}}U_{13}$  can be generated in the MSM by imposing  $S_4$  discrete symmetry. For our purpose, we introduce a new scalar field  $\phi$  which is  $SU(2)_L$  and triplet under  $S_4$ . We designate  $\nu_i$  and  $N_{Rj}$  to be a triplet and doublet under  $S_4$ , respectively. We allow a phase in  $M$ . Taking symmetric  $\underline{3}$  from  $\underline{3} \otimes \underline{2}$ , the couplings of  $\nu_i$  and  $N_{Rj}$  are given by  $(\nu_2 N_{R1} + \nu_3 N_{R2}, \nu_3 N_{R1} + \nu_1 N_{R2}, \nu_1 N_{R1} + \nu_2 N_{R2})$ . Then, taking  $S_4$  singlet combination of the scalar field  $\phi$  ( $\underline{3}$ ) and  $\nu_i N_{Rj}$  couplings given above, we finally obtain the Yukawa interaction terms given as,  $Y[\phi_1(\nu_2 N_{R1} + \nu_3 N_{R2}) + \phi_2(\nu_3 N_{R1} + \nu_1 N_{R2}) + \phi_3(\nu_1 N_{R1} + \nu_2 N_{R2})]$ . Taking the vacuum of  $\phi$  to be  $(\langle \phi_1 \rangle, \langle \phi_2 \rangle, \langle \phi_3 \rangle)$ ,  $m_D$  is given by

$$m_D^T = \begin{pmatrix} c & a & b \\ b & c & a \end{pmatrix}^T, \quad (2)$$

where  $a = Y \langle \phi_1 \rangle, b = Y \langle \phi_2 \rangle, c = Y \langle \phi_3 \rangle$  and  $T$  stands for transpose. Note that the parameters  $a, b, c$  are complex in general. Then, we obtain from Eq.(1),  $\frac{x}{y} = \frac{U_{13} + q U_{22}}{U_{12} - q U_{23}} = \frac{U_{23} + q U_{32}}{U_{22} - q U_{33}} = \frac{U_{33} + q U_{12}}{U_{32} - q U_{13}}$ , with  $q = \sqrt{M_2/M_1}$ . We note that  $q$  has complex phase that is associated with relative phase of  $M_1$  and  $M_2$ . Also, we can present the parameters  $a, b, c$  in terms of  $M_1, q$  and  $U_{ij}$ . Note that the unknown parameter in  $U_{ij}$  is a Majorana phase in  $U_{\text{PMNS}}$ . Following the method presented in [1],  $\delta_D$  is presented in terms of mixing angles,  $\cos \delta_D = -\frac{1}{2 \tan 2\theta_{23}} \cdot \frac{1 - 2s_{13}^2}{s_{13} \sqrt{2 - 3s_{13}^2}}$ . Thanks to the

formulae for  $\delta_D$ , we can predict  $q$  in terms of the Majorana phase of the light neutrino sector, and the parameters  $a^2, b^2, c^2$  depend on the Majorana phase and  $M_1$ . It would be interesting to examine how those two parameters are constrained from experiments. Physics sensitive to the Majorana phase and  $M_1$  are leptogenesis, radiative LFV decay and neutrinoless double beta decay.

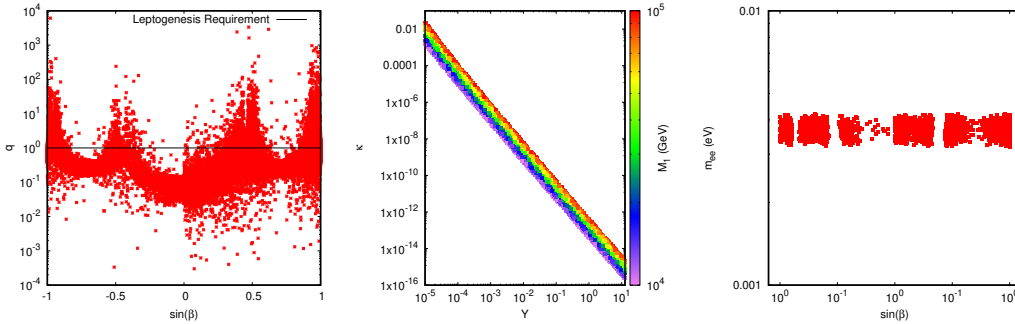
*Leptogenesis*- Using the formulae we derived above, let us estimate lepton number asymmetry and show how the unknown parameters are constrained from experimental results.

The CP asymmetry in this model [5] is given by  $\epsilon_1 = \frac{1}{8\pi v^2} \frac{\sum_{i \neq j} \text{Im}[(m_D^\dagger m_D)_{ij}]^2}{(m_D^\dagger m_D)_{11}} g(x) = \frac{1}{8\pi v^2} \frac{(c^* b + a^* c + b^* a)^2}{|a|^2 + |b|^2 + |c|^2} g(|q|^4)$ . with  $g(x) = \sqrt{x}[1/(1-x) + 1 - (1+x) \ln((1+x)/x)]$  with  $x = |q|^4$  and  $v = 246$  GeV. The matter-antimatter asymmetry is presented by  $\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim \kappa \frac{\epsilon_1}{g_s}$ , where  $n_B$  ( $n_{\bar{B}}$ ) and  $n_\gamma$  are baryon (anti-baryon) number density and photon number density, respectively [6, 7]. For the study of low energy phenomenology, we will consider the resonant leptogenesis [6].

*Lepton Flavor Violating Radiative Decay*- For the radiative LFV processes,  $l_i \rightarrow l_j \gamma$ , the basic expression is given as [8],  $\Gamma_{l_\alpha \rightarrow l_\beta \gamma} = \frac{(m_\alpha^2 - m_\beta^2)^3}{16\pi m_\alpha} (|\sigma_L|^2 + |\sigma_R|^2)$ , where  $\sigma_L = m_\alpha \sum_{a,b} O_{ai} O_{ib} f(t_i, m_{H_i^\pm})$ ,  $\sigma_R = m_\beta \sum_{a,b} O_{ai} O_{ib} f(t_i, m_{H_i^\pm})$  with  $f(t, m) = \frac{i}{16\pi^2 m_\beta^2} \frac{(t-1)(t(2t+5)-1)-6t^2 \log(t)}{12(t-1)^4}$ ,  $t = m_N^2/m^2$  and  $m, m_N$  are the masses of the charged scalar and  $N$ , respectively. The indices  $a$  and  $b$  correspond to the scalars of the  $S_4$  triplet. Note that those processes depend on  $Y$ . For parameter space constrained by experimental neutrino data and baryon asymmetry, we can narrowly predict  $\text{Br}(l_i \rightarrow l_j \gamma)$ .

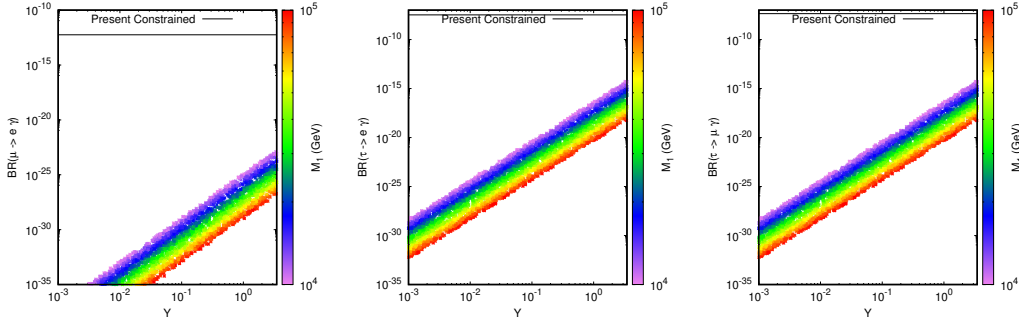
*Neutrinoless Double Beta Decay ( $0\nu\beta\beta$ )*- Since Majorana phase  $\beta$  in  $U_{\text{PMNS}}$  affects  $(0\nu\beta\beta)$ , there may exist a correlation between leptogenesis and  $0\nu\beta\beta$  [9]. The amplitude of  $\nu 0\beta\beta$  is proportional to  $|\sum_i U_{ei}^2 m_i| \equiv |\langle m_{ee} \rangle| = |m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2 e^{-2i(\delta_D + \beta)}|$  for NH [10].

*Numerical results*- For our numerical analysis, we adopt the latest experimental data as inputs taken from Ref. [11]. and the measured value of  $\eta_B$  given by [12]  $\eta_B^{\text{exp}} = (8.65 \pm 0.085) \times 10^{-11}$ . Since  $\kappa$  depends on  $Y$ , we estimate how the allowed regions of  $M_1$  and  $\beta$  are constrained in terms of  $Y$ . In the left panel of Fig. 1, we plot the allowed region  $(\sin\beta, q)$  for NH. The overlapped



**Figure 1:** Allowed regions of  $(\sin\beta, q)$  (left) and  $(Y, \kappa)$  (middle),  $| \langle m_{ee} \rangle |$  vs.  $\beta$  (right).

region between red and black one can lead to the resonant leptogenesis. The middle panel of Fig. 1 shows  $\kappa$  as a function of  $Y$  for  $10^4 \lesssim \text{GeV} M_1 \lesssim 10^5$  GeV, corresponding to  $\eta_B^{\text{exp}}$ . The right panel of Fig. 1 shows how  $| \langle m_{ee} \rangle |$  is predicted in terms of  $\beta$  based on the neutrino data and the allowed regions from leptogenesis. From our numerical analysis, we find that the predicted value of  $| \langle m_{ee} \rangle |$  lies between 0.033 and 0.045 for the NH case in this model. Since the BRs of the radiative LFV decays depend on  $m_D$ , the constraints coming from the measurements of neutrino



**Figure 2:**  $\text{Br}(\mu \rightarrow e\gamma)$  (left),  $\text{Br}(\tau \rightarrow e\gamma)$  (middle), and  $\text{Br}(\tau \rightarrow \mu\gamma)$  (right) vs.  $Y$ .

oscillation parameters and baryon asymmetry lead us to narrowly predict the BRs of the radiative LFV decays. Fig. 2 presents the predictions of  $\text{Br}(\mu \rightarrow e\gamma)$  (left panel),  $\text{Br}(\tau \rightarrow e\gamma)$  (middle panel),  $\text{Br}(\tau \rightarrow \mu\gamma)$  (right panel) with respect to  $Y$  for  $10^4 \text{ GeV} \lesssim M_1 \lesssim 10^5 \text{ GeV}$ . The predictions are quite below the current experimental limits.

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