Flavour theory & outlook

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Disagreements between experimental results and Standard Model theoretical predictions found in B meson decays, initiated many theoretical studies in flavour physics. The anomalous behaviour observed in B meson decays are known as $R_{D^{(*)}}$ and $R_{K^{(*)}}$ puzzles. The first one refers to the deviations in the decays in the ratio of the decay widths for $B \to D^{(*)}\tau\nu$ and $B \to D^{(*)}\mu\nu$, while the second one is related to the ratio of the decay widths for $B \to K^{(*)}\mu^+\mu^-$ and $B \to K^{(*)}e^+e^-$. Transition. In addition, the measured muon anomalous magnetic moment differs from the SM predictions. I briefly review main results coming from the SM studies and the effective Lagrangian approach containing New Physics for $R_{D^{(*)}}$ and $R_{K^{(*)}}$. Then I discuss basic features of leptoquark models which can resolve both B meson anomalies. If New Physics is confirmed in B decays a number of processes at low and high energies should confirm its presence.
1. Introduction

Standard Model (SM) as a gauge theory of strong and electroweak interactions is very successful in describing physics at low and high energies. Despite its success there are number of reasons, as for example neutrino masses, why we expect to find physics behind the SM. Although LHC did not point out towards any new particle, at low-energies there are few anomalies at level of $\sim 4\sigma$. The experimental results for the muon anomalous magnetic moment [1] deviates from the SM prediction at the level 3 to 4 $\sigma$. The main source of uncertainty comes from the hadronic contributions. In this year lattice QCD made great progress in determination of the vacuum polarization contribution and light-by-light contribution [2, 3]. There are, however new experiments at Fermilab (USA) and J-PARC (Japan), supposed to reach four times better precision. This would motivate theoretical studies to reach highest possible accuracy.

In addition to that long standing puzzle, in the last six years several experiments point towards possible violation of the lepton number universality in B meson semileptonic decays. For the $b \to c l\nu$ process it was found that

$$R_D = \frac{\Gamma(B \to D^\tau\nu_\tau)}{\Gamma(B \to D\nu)} \bigg|_{l \in \{e,\mu\}} = 0.41 \pm 0.05,$$

is $3.9\sigma$ higher than the SM prediction, $R_D^{SM} = 0.286 \pm 0.012$, derived by relying on the lattice QCD data for the vector and scalar form factors, obtained by MILC collaboration [6]. In the case of vector $D^*$ in the final state it was found $R_{D^*} = 0.317 \pm 0.017$, also confirmed by LHCb [5], which appears to be $3.3\sigma$ larger than predicted, $R_{D^*}^{SM} = 0.252 \pm 0.003$ [7, 8, 9]. However, the lattice calculations for the $B \to D^*$ form factors are still lacking. Similar effect has been observed in the ratio $R_{J/\psi} = (\Gamma(B_c \to J/\Psi \tau\nu))/\Gamma(B_c \to J/\Psi \mu\nu)$ in which the experimental value is larger than the theoretical prediction at level of $2\sigma$ [10]. Currently, the first differential decay distribution $B \to D^{(*)}\tau^-\nu_\tau$ was performed in [11], further angular analyses in these processes would help in differentiating between different New Physics scenarios [12, 13].

The form factors used in the SM calculations of $R_{D^*}$, were extracted from the angular distribution of $d\Gamma(B \to D^{(*)}\mu\nu_\mu)/dq^2$, up to a normalization, and of leading order heavy quark effective

**Figure 1:** $R_{D^*}$ and $R_D$ experimental results and the SM predictions by HFLAV, summer 2018.
theory has been used in evaluating the pseudoscalar form factor [8, 9]. For the neutral current transition LHCb collaboration found

\[ R_K = \frac{\Gamma(B \to K\mu^+\mu^-)}{\Gamma(B \to Ke^+e^-)} = 0.745 \pm 0.034 \pm 0.036, \quad (1.2) \]

2.6σ below the Standard Model (SM) prediction, \( R_K^{SM} = 1.00(1) \) [29].

In the case of flavour changing neutral current transition (FCNC) \( b \to s\mu^+\mu^- \) the LHCb experiment has measured ratios \( R_{K^{(s)}} = \mathcal{B}(B \to K^{(*)}\mu^+\mu^-) / \mathcal{B}(B \to K^{(*)}e^+e^-) \) at the low di-lepton invariant mass. Interestingly, these ratios were found to be systematically lower than expected in the SM. In the case of the \( K \) meson in the final state, the ratio \( R_K \) was measured in the kinematical region \( q^2 \in [1.1, 6] \) GeV\(^2\) [25], while \( R_{K^*} \) was measured also in the region \( q^2 \in [0.045, 1.1] \) GeV\(^2\) [28]. The three measured \( R_{K^{(*)}} \) deviate from the SM predictions at \( \sim 2.5 \sigma \) level [29, 30].

![Figure 2: Left: LHCb results for \( R_K \) [25]. Right \( R_{K^*} \) from LHCb measurements [28].](image)

In the SM Lepton Flavour Universality (LFU) results from the basic property of the \( SU(2)_L \times U(1)_Y \) gauge group. The part of SM local gauge invariant Lagrangian for the left-handed fermions is

\[ \mathcal{L} \in \bar{\Psi}_L i\gamma^\mu D_\mu \Psi_L = \bar{\Psi}_L i\gamma^\mu (\partial_\mu - ig_\mu \gamma^5 Y_\mu) \Psi_L \quad (1.3) \]

with \( \Psi_L = Q_L, \ell_L \) and \( Q_L, (\Psi_L = 1/2(1 - \gamma^5)\Psi) \), denoting a weak doublet of quarks and leptons generations

\[ Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad (c_L, s_L), \quad (t_L), \quad \ell_L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \quad (\nu_\mu, \mu_L), \quad (\nu_\tau, \tau_L) \quad (1.4) \]

The coupling \( g \) is the same for all left-handed quarks and leptons and this is a reason why we have universality of the weak coupling constant. In the case of leptons, therefore we talk about lepton flavour universality. As stated in high-energy textbooks for beginners, the Fermi weak coupling constant at low-energies is

\[ \mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} f_\mu \mu^\mu, \quad G_F = \frac{g^2}{4m_W^2}. \quad (1.5) \]

For example, such relation leads to the equality of the decay widths \( \Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = \Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \).
2. Effective Lagrangian approach in B anomalies

In order to explain $(g - 2)_\mu$ and B meson anomalies many proposals of New Physics were done. Most general treatment requires use of effective Lagrangians. This approach respects the symmetries of the SM. After constructing the effective Lagrangian at 1 TeV scale and establishing which operator (operators) might explain observed anomaly (anomalies) one search for an appropriate NP model. The constraints from all possible observables at low energies and the LHC constraints, limit the parameter space and lead towards the full ultra-violat complete theory (meaning that such theory must be well-defined at arbitrarily high energies, should be renormalizable and without Landau poles or at least has a nontrivial fixed point), as sketched in Fig. 3.

2.1 Effective Lagrangian approach in $R_D(\tau)$

The effective Lagrangian for the $b \to c l \nu_l$ decay, with the SM neutrino is given by

$$L_{\text{eff}} = \frac{-4G_F}{\sqrt{2}} V_{cb} \left[ (1 + g_V) (\bar{u}_L \gamma_\mu d_L) (\bar{e}_L \gamma^\mu \nu_L) + g_S (\bar{u}_R d_L) (\bar{e}_R \nu_L) + g_T (\bar{u}_R \sigma_{\mu\nu} d_L) (\bar{e}_R \sigma^{\mu\nu} \nu_L) \right], \quad (2.1)$$

According to studies (e.g. [14, 8, 15]) the favourable solution is just a product of the two left-handed currents with $0.09 \leq g_V \leq 0.13$. There are approaches which include the right-handed neutrino as presented in [17, 18]. If one writes the coefficient

$$\frac{4G_F V_{cb} g_V}{\sqrt{2}} \rightarrow \frac{2}{\Lambda_{NP}^2}, \quad (2.2)$$

then the scale of NP is $\Lambda_{NP} \simeq 3$ TeV. For the scale $\Lambda_{NP} > 3$ TeV theory becomes nonperturbative. However, the $V - A$ form of the NP is not the only solution as suggested in recent publications [19, 20]. These approaches use possibility of using scalar and tensor couplings. The strongest constraints for the pseudoscalar coupling comes from $\Gamma(B_c \to \tau \nu)$ [21]. At the same time in [19] it was noticed that the muon anomalous magnetic moment can be explained by the hierarchical tensor coupling $|C_T^\tau| \gg C_T^\mu > C_T^e$. 

![Figure 3: From effective Lagrangian approach towards the UV-complete theory.](image-url)
2.2 Effective Lagrangian approach in $R_{K^{(*)}}$

The SM processes with the flavour structure $(\bar{s}b)(\bar{\mu}\mu)$ at scale $\mu = \mu_b = 4.8$ GeV are governed by dimension-6 effective Hamiltonian \cite{22,23,24}

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=7,..,10} (C_i(\mu) \mathcal{O}_i(\mu))$$

(2.3)

$$\mathcal{O}_7 = \frac{e}{g^2} m_b (\bar{s}\gamma_i P_R b)(F^\mu \nu),$$

$$\mathcal{O}_9 = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma_\mu \ell),$$

$$\mathcal{O}_{10} = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma_\mu \gamma_5 \ell).$$

(2.4)

Here $P_L/R = (1 \pm \gamma_5)/2$, while $e$ is the electromagnetic and $g$ the color gauge coupling, $F^\mu \nu$ and $G^\mu \nu$ are the electromagnetic and color field strength tensors, respectively. At the scale $\mu_b = 4.8$ GeV the effective SM Wilson coefficients are $C_7^{SM} = 0.29$; $C_9^{SM} = 4.1$ $C_{10}^{SM} = -4.3$ \cite{22,23,24}.

The measurements $R_{K^{(*)}}$ of the LHCb collaboration \cite{25,28} at the low di-lepton invariant mass distribution $q^2$ pointed out that the values of $C_9^{SM}$ and $C_{10}^{SM}$ cannot described experimental results. According to Refs. \cite{26,27}, NP might contribute to the Wilson coefficients $C_i = C_i^{SM} + C_i^{NP}$, the best fit point is $C_9^{NP} = -C_{10}^{NP} = -0.64$, assuming that NP comes from the muonic sector, as presented in Fig. 4. Such fit indicates that NP has following structure

$$R_{K^{(*)}}^{NP} = \frac{1}{\Lambda_{NP}^2} (\bar{s}_L \gamma_\mu \bar{b}_L)(\bar{\mu}_L \gamma_\mu \mu_L).$$

(2.5)

The scale of NP calculated from this Lagrangian $\Lambda_{NP} \simeq 30$ TeV.
2.3 NP explaining both anomalies

The experimental results points towards

$$R_{D^{(*)}}^{\text{exp}} > R_{D^{(*)}}^{\text{SM}} \quad \text{and} \quad R_{K^{(*)}}^{\text{exp}} < R_{K^{(*)}}^{\text{SM}}$$  \hspace{1cm} (2.6)$$

which indicates that the scale of NP is $$\Lambda_{NP}^D \approx 3 \text{ TeV}$$ and $$\Lambda_{NP}^K \approx 30 \text{ TeV}$$. If one expects that the scale of NP is the same in both anomalies and writes down the effective Lagrangian

$$\mathcal{L}_{NP} = \frac{2}{\Lambda^2} (\bar{u}_L \gamma_\alpha d_L)(\bar{\ell}_L \gamma_\alpha \nu_L) + \frac{C_K}{\Lambda^2} (\bar{s}_L \gamma_\alpha b_L)(\bar{\mu}_L \gamma_\alpha \mu_L).$$  \hspace{1cm} (2.7)$$

Obviously, the suppression factor $$C_K \simeq 0.1$$. This means if we expect NP to be at the scale $$\Lambda \approx 3 \text{ TeV}$$ then its contribution to $$R_{K^{(*)}}$$ should be suppressed in comparison with $$R_{D^{(*)}}$$. There are a number of ways how that can be achieved. In the scenario of Ref. [31], NP couples dominantly to the third generation of quarks and leptons. The coupling to the second generation is just a small correction in comparison with the NP coupling to the third generation. The idea to have a NP Lagrangian as in (2.7) was enforced by the number of studies (see e.g. [32, 15, 33]). Another possibility was offered in the proposal of Ref. [34, 35] in which it was suggested that the $$R_{K^{(*)}}$$ anomaly is explained by the contribution of NP at loop level.

Generally, the allowed parameter space is obtained after using constraints coming from the observables listed in Table 1.

<table>
<thead>
<tr>
<th>Flavour observables</th>
<th>LFV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$(g - 2)_\mu$$</td>
<td>$\tau \rightarrow \mu \gamma$</td>
</tr>
<tr>
<td>$B(B_c) \rightarrow \tau \nu$</td>
<td>$\mu \rightarrow e \gamma$</td>
</tr>
<tr>
<td>$B \rightarrow K^{(*)} \nu \nu$</td>
<td>$\tau \rightarrow K(\pi) \mu (e)$</td>
</tr>
<tr>
<td>$B_\mu - B_\tau, D^0 - \bar{D}^0$</td>
<td>$K \rightarrow \mu e$</td>
</tr>
<tr>
<td>$B \rightarrow D \bar{\nu} \mu$</td>
<td>$B \rightarrow K \mu e$</td>
</tr>
<tr>
<td>$K \rightarrow \mu \nu$</td>
<td>$\tau \rightarrow \mu \mu \mu$</td>
</tr>
<tr>
<td>$D_{(s)} \rightarrow \mu (\tau) \nu$</td>
<td>$\tau \rightarrow \mu ee$</td>
</tr>
<tr>
<td>$\tau \rightarrow K \nu, K \rightarrow \pi \mu \nu$</td>
<td>$\tau \rightarrow \Phi \mu$</td>
</tr>
<tr>
<td>$W \rightarrow \tau \nu, \tau \rightarrow l \nu \bar{\nu}$</td>
<td>$t \rightarrow c l^+ l^-$</td>
</tr>
<tr>
<td>$Z \rightarrow b \bar{b} (l^+ l^-)$</td>
<td>$Z \rightarrow \mu \tau$</td>
</tr>
</tbody>
</table>

**Table 1:** Constraints from flavour variables and from lepton flavour violating processes.

The next step is to construct a model at TeV scale which can explain both B anomalies. The NP mediator can have either spin 0 or spin 1. Scalar leptoquarks are typical examples of these models (e.g. see [36, 37, 38, 20, 33, 39]). Spin 1 resonances are considered as a remnant of some techni-fermion models [40, 41] or as a gauge bosons [44, 43, 45, 46, 47].

3. Leptoquarks resolving B anomalies

The leptoquarks are particles interacting with quarks and leptons. One can denote the leptoquark (LQ) states according to their quantum numbers of $SU(3)_{c}$, $SU(2)_{w}$ and $U(1)_{y}$ representations of the SM [48]. In Table 2 leptoquark states which can resolve either $R_{D^{(*)}}$ or $R_{K^{(*)}}$ or both
Table 2: Single scalar and vector LQs explaining either one of B anomalies or both at tree level.

<table>
<thead>
<tr>
<th>(SU(3)_c, SU(2)_w, U(1)_Y)</th>
<th>Spin</th>
<th>R_{D(s)}</th>
<th>R_{K(s)}</th>
<th>R_{D(s)} and R_{K(s)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_3 \equiv (3, 3, 1/3)</td>
<td>0</td>
<td>x</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>R_2 \equiv (3, 2, 7/6)</td>
<td>0</td>
<td>✓</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>\tilde{R}_2 \equiv (3, 2, 1/6)</td>
<td>0</td>
<td>✓</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>S_1 \equiv (3, 1, 1/3)</td>
<td>0</td>
<td>✓</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>U_3 \equiv (3, 3, 2/3)</td>
<td>1</td>
<td>x</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>U_1 \equiv (3, 1, 2/3)</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

3.1 Pati-Salam unifying model

As presented in Table 2, the U_1 vector leptoquark can relatively well explain both B meson anomalies, but cannot explain the muon anomalous magnetic moments. The light vector leptoquark is most trivial to accommodate in the Pati-Salam-like model with a gauge group SU(4) \times SU(3) \times SU(2) \times U(1) which is then spontaneously broken [45, 46, 47]. The basic concept of this unification is to accommodate quarks and leptons in the fundamental representation of SU(4).

The approach of Refs. [44, 43] assumes the tri-site Pati-Salam model [PS]^3, which might lead to explanation of the "flavour puzzle" (understanding the mass pattern of the fundamental fermions). Naturally, in such model there are many new particles as new Z', new colorons with masses between 1.3, 1.9 TeV. A nice feature of that model is that the unification scale is rather low \sim 10^6 GeV and the proton does not decay.

3.2 Two scalar LQs solution of R_{D(s)} and R_{K(s)}

There are few appealing reasons to consider two scalar leptoquark solution of the B meson anomalies. First of all the unification of the three fundamental interactions within SU(5) grand unifying group is possible with 2 light leptoquarks as we showed in [48]. Two light scalar leptoquark might generate neutrino masses at loop level [51].

In [20] we constructed two scalar LQ extension of the SM that can accommodate all measured LFU ratios in B-meson decays and related flavour observables, while being compatible with direct search constraints at the LHC. The extension has an SU(5) origin that relates Yukawa couplings of the two LQs through a mixing angle and all Yukawas remain perturbative up to the unification scale. We provide prospects for future discoveries of the two light LQs at the LHC and spell out predictions for several yet-to-be-measured flavor observables. In particular, we predict and correlate \mathcal{B}(B \to K \mu \tau) with \mathcal{B}(B \to K^{(*)} \nu \nu). We also predict a lower bound for \mathcal{B}(\tau \to \mu \gamma) which is just below the current experimental limit.

4. Summary and outlook

The explanation of B physics anomalies itself is a huge task. One can immediately asks in
which processes the same NP can be observed. The correlations of B anomalies and $K \rightarrow \pi \nu \bar{\nu}$ were already studied, ($R_{D^{(*)}}$ [53] and $R_{K^{(*)}}$ [54]). Effects were found to be of the order $\sim 20\%$. Important results of all B anomalies studies are that NP seen in B decays should affect mostly $B \rightarrow K^{(*)}\nu \bar{\nu}$ and lepton flavour violating processes as $\tau \rightarrow \mu \gamma$, $\tau \rightarrow 3\pi$, $B \rightarrow K^{(*)}\tau\mu$, $B_s \rightarrow \tau\mu$. In our expectations of NP, a new UV complete theories based on B anomalies explanation might help in understanding of "Flavour puzzle".

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References


