I begin by briefly reviewing the evidence for the existence of Dark Matter (DM), emphasizing that many observations, at length scales between kpc (the size of the smallest galaxies) and Gpc (the Hubble radius) can be described by the same simple model, ΛCDM. I will then argue that primordial black holes, the only DM candidates that can be realized within the Standard Model (SM) of particle physics, are very unlikely to provide all of DM. After giving a (probably incomplete) list of possible DM candidates, I end by mentioning some recent developments in the theory of Weakly Interacting Massive Particles (WIMPs).
1. Evidence for Dark Matter

For a cosmologist, “matter” is stuff whose pressure $p$ is much less than its energy density $\rho$ (in natural units where $\hbar = c = 1$). In contrast, “radiation” is stuff with $p \simeq \rho/3$, and “dark energy” is stuff with negative pressure, $p < -\rho/3$ (the simplest example is vacuum energy with $p = -\rho$, which is indistinguishable from Einstein’s cosmological constant). In the matter category, we need to further distinguish between “baryonic” matter, which consists of protons, neutrons and electrons (yes, electrons are baryons for cosmological purposes – but neutrinos aren’t), and Dark Matter, which doesn’t. (This is the modern definition of DM. It differs somewhat from the older definition as “matter which cannot be detected optically”. Most, but not all, DM by the old definition is also DM by the new definition but by the old definition there’s also baryonic DM; more on that below.) If we assume that gravitational interactions at all length scales $\gtrsim 1$ kpc (about 54 orders of magnitude above the Planck length, so most likely we need not worry about quantum gravity) is described by Einstein’s general theory of relativity, a great many independent observations require the existence of DM [1]:

- **Galactic rotation curves**: Assuming (approximately) spherical symmetry, bodies on stable Keplerian orbits with radius $r$ around a galaxy have rotational (tangential) velocity $v(r) \propto M(r)/r$, where $M(r)$ is the mass within the orbit. Observationally, $v(r)$ remains rather flat (although not exactly constant) even at radius $r$ well outside of nearly the entire visible mass of the galaxy. This can be explained by the presence of a dark component, such that $M(r) \propto r$ (approximately).

- **Clusters of galaxies**: The mass of a cluster of galaxies can be estimated in different ways: via the relative velocities of the galaxies in the cluster (assuming the system is virialized); via the temperature of the hot, X–ray emitting gas that contains most of the baryons in the cluster$^1$; and, most directly, via gravitational lensing of background galaxies. In all cases one finds that the total mass of the cluster exceeds its baryonic mass by about a factor of 5.

- **Old galaxies**: We know from the cosmic microwave background (CMB) that the universe was very homogeneous at temperature $T \simeq 0.3$ eV. Small density perturbations can grow (by overdensities gravitationally attracting more matter) only in a matter–dominated epoch. Large overdensities, as required for galaxy formation, can therefore occur earlier for larger matter content. In particular, given the size of fluctuations in the CMB, the observations of galaxies at $z > 10$ can only be understood if a sizable amount of DM exists.

- **Bullet cluster**: This is actually a system of two clusters, which “recently” passed through each other with rather large relative velocity. The hot plasma in the two clusters interacted strongly, heating up and slowing down. As a result, the plasma clouds are now well separated from the bulk of the galaxies in the cluster. At the same time gravitational lensing shows that the total mass of the clusters centers at the same location as the galaxies do – not where most

$^1$ Zwicky used the first method to estimate the mass of the Coma cluster. Back in the 1930’s X–ray emitting matter was invisible to astronomers, so this hot gas was counted as part of Zwicky’s DM. This illustrates a problem with baryonic DM: the observational definition of “dark” depends on technology, and is hence time–dependent; the distinction between baryonic and non–baryonic matter isn’t.
baryonic matter, stored in the hot X–ray emitting plasma, is located. In the context of DM, this can be used to derive an upper bound on the DM–DM scattering cross section [2],

$$\sigma_{\chi \chi} \leq (\text{few}) \, \text{b} \cdot \frac{m_\chi}{1 \text{ GeV}}, \tag{1.1}$$

where $\chi$ stands for a DM particle.\(^2\) Note that the $pp$ and $p\bar{p}$ scattering cross sections are well below this bound; pions come closer to saturating it.

- Acoustic oscillations: In addition to fixing the normalization of density perturbations at $T \simeq 0.3$ eV, the analysis of CMB anisotropies also allows to measure the “sound horizon”, showing that the visible universe is essentially spatially flat, and to determine the free parameters of the $\Lambda$CDM model with percent–level precision. In particular, the overall relic density of DM is given by [5]

$$\Omega_\chi h^2 = 0.1198 \pm 0.0012. \tag{1.2}$$

Here $\Omega_\chi$ is the $\chi$ mass density in units of the critical density, and $h \simeq 0.7$ is today’s Hubble constant in units of $100 \text{ km/}(\text{s} \cdot \text{Mpc})$. It should be noted that the most prominent of these acoustic oscillations have also been detected in galaxy correlation functions at low redshift.

While $\Lambda$CDM can account for the large–scale structure of the visible Universe very well, there are some challenges at smaller scales [6], often involving dwarf galaxies. All these problems occur in systems with very high density contrast (so that density fluctuations can no longer be treated perturbatively) and where baryonic physics (sometimes called “baryochemistry” by cosmologists) is likely to play an important role. The latter is currently not sufficiently well understood to decide whether all observations can be reproduced by the $\Lambda$CDM model.

However, the “Lyman $\alpha$ forest” – the spatial distribution of hydrogen clouds, detected through the absorption of photons at the Ly$\alpha$ wave length – is very well described by $\Lambda$CDM [6]; these are much simpler systems, at similar length scales as small galaxies. This success strongly constrains extensions of the $\Lambda$CDM model (e.g., interacting DM) that is meant to improve the description of small galaxies. Therefore currently there is no compelling need to go beyond $\Lambda$CDM.

So far we have assumed that GR describes gravitational interactions at the relevant length scales. Many of the observations alluded to above (e.g., of galactic rotations curves and peculiar velocities in clusters) primarily indicate a missing force, i.e. the force on the observed objects is larger than predicted by GR if all the mass is in the visible baryons. Within the framework of GR this can be fixed by adding mass, i.e. DM. Alternatively, one can contemplate changing the law of gravity. In fact, MOdified Newtonian Dynamics (MOND) [7] as a purely phenomenological modification of Newton’s law of gravity at small accelerations can account quite well for galactic rotation curves.

\(^2\)Soon after the discovery of this system it was claimed [3] that it argues against the standard $\Lambda$CDM cosmology, because simulations did not produce encounters of similarly large clusters with similarly high relative velocity. Such systems are indeed rare in the observable universe: no comparable system has yet been found. It seems that simulations available at that time did not encompass a sufficiently large volume to include similar systems. More recent simulations conclude [4] that $\Lambda$CDM has no problem generating such a system.
However, embedding this into a consistent covariant (classical) field theory is quite difficult: it is not easy to devise an interaction whose potential falls off more slowly than $1/r$ at large distances, as required if MOND is to explain the observations. Existing attempts include either an additional vector and an additional scalar field (“TeV eS”) [8], or a second tensor (“bi–metric gravity”) [9]. Moreover, observations of the bullet cluster, and cosmological observations, seem to require some sort of DM even in MOND [10]. Hence, MOND is certainly no more economical than GR plus DM; besides, GR can explain a large body of data, including some of the most precise measurements ever performed (on binary neutron stars) [11] and the recent observation of gravitational waves [12]. In contrast to these theoretical difficulties with MOND, DM can be introduced using the well–established formalism of quantum field theory; it thus seems the far more plausible explanation to me.

2. Properties of Dark Matter

Many of the observations requiring the existence of DM are made in the nearby universe. This implies that DM particles must be extremely long–lived, $\tau_\chi > 10^{10}$ years. For DM particles heavier than a few GeV the bound on the lifetime is even stronger, typically $\tau_\chi > 10^{25}$ seconds [13]; this comes from unsuccessful searches for the energetic $\chi$ decay products. Most (but not all) DM models therefore assume $\chi$ to be absolutely stable.

By definition DM should be “dark”, meaning it should not interact (much) with photons. The simplest choice is to consider neutral particles, of course, but “millicharged” particles with electric charge $|q_\chi| \lesssim 10^{-3} e$ may also be acceptable [14]. The strongest bound on charged stable particles comes from the search for exotic isotopes [15]. A massive particle with charge $-e$ could bind to some nucleus $Z^X$. For $m_\chi > 10$ GeV or so the binding energy would exceed 10 MeV even for quite light nuclei (e.g. carbon or oxygen). The bound state would thus chemically look like an isotope with charge $Z-1$ but with mass $\simeq A m_p + m_\chi$. Large–scale searches for such exotic isotopes have failed to find anything. For example at most one out of $5 \cdot 10^{15}$ carbon nuclei can be bound to a charged particle of mass 10 TeV. For yet heavier particles, the best limit comes from looking for superheavy water, made from hydrogen–like $\chi^+ e^-$ “atoms”. It seems plausible that similar limits apply to stable massive particles that carry color by no electric charge, since they should bind to ordinary nuclei as well.

Of course, in order to count as matter, $\chi$ must have non–vanishing mass. Moreover, simulations of galaxy formation strongly favor “cold” dark matter [6], i.e. $\chi$ must have been non–relativistic at the latest when one Hubble volume contained a galactic mass. For particles that have been produced more or less thermally, this implies a lower bound on the mass in the keV range. Similarly, fermionic DM, which is subject to the Pauli exclusion principle, must have a mass at least in the keV range in order to be able to accommodate a sufficient amount of mass in the limited phase space of a dwarf galaxy [16]. On the other hand, bosonic DM that never was in thermal equilibrium might be much lighter than this. The lower bound than comes from the requirement that the de Broglie wavelength $1/(v_\chi m_\chi)$ (with $v_\chi \lesssim 10^{-3}$ in small galaxies) should not be larger than the scale length of the smallest “known” DM halos, those of dwarf galaxies [17]; this gives a lower bound of $10^{-23}$ eV or so. To summarize:
\[ 10^{-23} \text{eV} \lesssim m_\chi \lesssim M_{\text{Pl}} \text{ (non-thermal, bosonic DM)}; \]
\[ 1 \text{keV} \lesssim m_\chi \lesssim M_{\text{Pl}} \text{ (thermal or fermionic DM)}. \]

Here \( M_{\text{Pl}} \approx 2.4 \times 10^{18} \text{GeV} \) is the reduced Planck mass; I used it as (approximate) upper bound since I find it hard to envision elementary particles with even larger mass. Even if we accept this as upper bound, the range (2.1) spans 50 orders of magnitude for DM bosons that were not produced thermally, and about 24 orders of magnitude for other candidates.

Finally, we have the (quite weak) upper bound (1.1) on the \( \chi \chi \rightarrow \chi \chi \) scattering cross section, which is actually not easy to violate in concrete models. Of course, the DM density is by now known quite accurately, at least within \( \Lambda \)CDM, see eq.(1.2). (In simple extensions of \( \Lambda \)CDM the errors become a bit larger, but the central value doesn’t change much.) However, in general this does not translate directly into a bound on a property, or a combination of properties, of \( \chi \).

We are thus forced to conclude that, while we have very solid evidence for the existence of DM, we know very little about its properties. Remarkably enough, the above list of known properties suffices to exclude all elementary particles described by the SM as primary component of DM. In particular, the only known massive, non–baryonic particles – the (active) neutrinos – are way too light.

There is one possible loophole in this argument. The earliest determination of the baryon content of the Universe comes from analyses of Big Bang Nucleosynthesis (BBN). While less accurate than the constraint from the CMB data, it also implies that the baryonic density cannot be much more than 5% of critical density, which is well below what is needed for the DM density. However, baryons that were bound in black holes at the time of BBN do not count as baryons any more. Such “primordial” black holes (PBHs) [18] even have some independent motivation: as early seeds of the superheavy black holes seen in AGNs (formerly known as quasars) already at quite high redshift; and as explanation of the fact that all black holes mergers that have been observed via their gravitational wave signature so far have quite large masses, perhaps larger than what prior modeling of “population III” stars indicated. (Somewhat counter-intuitively, “population I” refers to the youngest stars. “Population III” stars would have had to form from primordial matter, basically only consisting of hydrogen and helium, i.e. with essentially vanishing “metallicity” in astronomers’ language. Such stars are thought to have been very massive, and hence short-lived; this explains why no such stars can be found in our “neighborhood”.) This triggered quite a lot of work on constraining PBHs as possible DM candidates using a multitude of astronomical observations and astrophysical considerations. By now it seems clear that only PBHs with mass near \( 10^{16} \) to \( 10^{17} \text{g} \) can form a sizable fraction of DM [19]. The density of PBHs with slightly lower mass is very strongly constrained because such objects would evaporate via Hawking radiation just about now, which would give very spectacular signals. PBHs can form only if the primordial density perturbations at the relevant (very small) length scales are at least 7 orders of magnitude larger than those probed by the CMB; the deviation of scale invariance observed in the CMB goes in the wrong direction. This raises two problems: one has to find a mechanism that generates very strong primordial density perturbations at length scales corresponding to \( 10^{17} \text{g} \) PBHs; and the density perturbations at slightly smaller length scales, which would produce PBHs decaying now,
has to be considerably smaller again. While this is technically possible, it seems very contrived. Very recently it has also been pointed out that models that are engineered accordingly will generate a detectable amount of gravitational waves in the LISA band [20]. In summary, severe finetuning is required for PBHs to form a significant fraction of DM; and models of this kind can soon be tested.

3. Particle Physics Candidates

Given how little we know about the properties of DM (see the previous Section) it is not surprising that a large number of different particle physics candidates have been suggested. These cover the entire allowed range (2.1) of masses, from “fuzzy” [17] to “Planckian” [21] DM; their self interactions range from essentially negligible (in the large majority of models) to saturating the bound (1.1). However, I personally still prefer candidates that have some independent motivation, not related to DM, or even to cosmology. This is true in particular for axions [22], which have been introduced so solve the strong CP problem but can also make good CDM if the “Peccei–Quinn” scale where the global $U(1)$ symmetry of the axions is broken is around $10^{10}$ GeV. It is also true for some Weakly Interacting Massive Particles (WIMPs) [1] that emerge in attempts to solve, or at least alleviate, the hierarchy problem of the Standard Model, the prime (but not quite only) example here being supersymmetric WIMPs [23]. To the best of my knowledge axions are still perfectly viable CDM candidates; but since I have never worked on them, for the rest of this write–up I will focus on WIMPs.

4. News on WIMPs

In the last couple of decades, WIMPs have been the by far most popular DM candidates, to the extent that sometimes WIMP and DM are taken to be synonyms. They aren’t – as I just mentioned, axions are perfectly fine CDM candidates, and they are certainly not WIMPs – but the popularity of WIMPs is not without reason.

In fact, there are two reasons, a more theoretical and a phenomenological one. The theoretical argument in favor of WIMPs rests on the fact that the simplest production mechanism for massive relics from the very early universe, freeze–out from the thermal plasma [24], in minimal cosmology automatically points towards the weak scale. The main assumption here is that the post–inflationary universe once was sufficiently hot and dense that reactions of the kind $\chi\chi \rightarrow \text{SM}$ were in equilibrium. Note that in a static universe all reactions eventually reach equilibrium. However, in an expanding universe equilibrium can be maintained only if the reaction rate, given by $n_\chi \langle \sigma_\chi \chi v \rangle$, is larger than the expansion rate, given by the Hubble parameter, $H \propto T^2/M_{Pl}$ in the radiation–dominated era. Here $n_\chi$ is the $\chi$ number density, $\sigma_\chi \chi$ is the total cross section for the annihilation of two DM particles into an SM final state, $v$ is the relative velocity between the annihilating DM particles, and $M_{Pl}$ is the Planck mass. Due to the $1/M_{Pl}$ factor in $H$, if the couplings of $\chi$ are not very tiny the reaction rate will be larger than the expansion rate for temperature $T \sim m_\chi$. However, if $\chi$ is in equilibrium, $n_\chi \propto e^{-m_\chi/T}$ for $T < m_\chi$ is exponentially suppressed, and will thus become smaller than $H$ eventually; for typical WIMPs this “freeze–out” occurs at $T \sim m_\chi/20$ (with logarithmic dependence on $\langle \sigma_\chi \chi v \rangle$). After freeze–out the number of $\chi$ particles per co–moving volume
is basically constant. This gives the final result

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\chi \chi} v \rangle}.$$  \hspace{1cm} (4.1)\]

The observational value (1.2) is saturated for weak–scale (i.e., picobarnish) cross section. This is sometimes\(^3\) called the “WIMP miracle”. While there is nothing miraculous (meaning supernatural) about this result, it is at the very least a coincidence linking weak–scale physics with DM.\(^4\) The fact that naturalness arguments favor the existence of new weak–scale particles makes this coincidence all the more intriguing. This is the theoretical argument in favor of WIMPs.

WIMPs are interesting phenomenologically because a roughly weak–scale annihilation cross section indicates that both the annihilation of WIMPs in the current universe and the elastic scattering of WIMPs on target nuclei might be detectable; the former goes under indirect detection, while the latter constitutes direct detection [1]. In today’s universe WIMP annihilation can only be detectable in regions with enhanced WIMP density, e.g. near the centers of galaxies or within the Sun (which can capture WIMPs that have lost energy by elastically scattering off a nucleus in the Sun). However, this argument is not airtight. For example, if at low \(v \chi \chi\) annihilation from an \(S\)–wave initial state is suppressed, then \(\langle \sigma_{\chi \chi} v \rangle \propto v^{2n}\) with \(n \geq 1\) \((n = 1\) for annihilation from a \(P\)–wave\), and the average \(v^2\) in our galaxy is about 5 orders of magnitude smaller than that during WIMP decoupling. Moreover, the relic density might actually be determined by co–annihilation of \(\chi\) with another slightly heavier partner particle (e.g. a squark if \(\chi\) is the LSP in the MSSM) [26]; by now these partner particles will all have decayed, and the true \(\chi \chi\) annihilation cross section can be very small in such a scenario. On the other hand, direct WIMP detection can also be suppressed, e.g. if \(\chi\) predominantly couples to top quarks, in which case \(\chi\) couples to nucleons only at one–loop level.

Nevertheless the fact that even the recent ton–scale direct detection experiments, e.g. Xenon–1t [27], have not found a signal puts severe strain on many well–motivated WIMP candidates. This has led to a lot of activity in model–building, which in turn is starting to spur new experimental developments.

There are basically three ways to evade the current direct detection bounds. One is to go for very light WIMPs. They cannot be detected with current methods since a WIMP that is much lighter than the target nucleus cannot transfer a detectable amount of energy to it (just like a table tennis ball hitting a basketball will just bounce back, leaving the basket ball at rest). Such models typically need new “mediator” particles to facility WIMP annihilation into SM particles with a sufficiently large cross section. Both the light DM particles (which at some point become too light to be properly called WIMPs) and the mediators are being searched for in a large variety of (relatively low–energy) experiments.

The second option is to go for very heavy WIMPs. Here the direct detection bound on the scattering cross section weakens because the bound is really on the event rate, which is proportional

\(^3\)Pompously or derisively, I’m not sure.

\(^4\)This argument assumes that the main \(\chi\) number changing reactions are \(\chi \chi\) annihilations into SM particles. In some models the main number changing reactions are of the form \(3\chi \leftrightarrow 2\chi\), i.e. occur purely within the dark sector. The desired relic density (1.2) is then obtained for \(\chi\) mass and interaction strength very roughly comparable to those of ordinary pions, with \(2 \rightarrow 2\) scattering cross sections not far below the bound (1.1) [25].
to the product of WIMP flux and scattering cross section, and for given WIMP mass density in the solar neighborhood the WIMP flux drops \( \propto 1/m_\chi \). Hence for \( m_\chi > 100 \) GeV or so the bound on the scattering cross section scales basically linearly with \( m_\chi \). Of course, for dimensional reasons the annihilation cross section will scale like \( 1/m_\chi^2 \). For very large \( m_\chi \) it thus becomes difficult to design the model such that the \( \chi \) annihilation cross section is sufficiently large (in minimal cosmology). In fact, unitarity arguments [28] imply the upper limit \( m_\chi \leq 120 \) TeV. The largest mass of a thermal WIMP in minimal cosmology that has been found [29] in a reasonably well motivated particle physics model (the MSSM extended with right–handed sneutrino superfields and an extra \( U(1) \) gauge factor) is about 40 TeV, which is already within a factor of 3 of the unitarity bound.

The final option is to consider WIMPs with very small scattering cross section on nuclei. This is e.g. true for Bino–like neutralinos (i.e. superpartners of the hypercharge gauge boson) in the MSSM. These are gauge singlets, which couple to Higgs bosons only via mixing with higgsinos; this mixing is small if the higgsinos are heavy. Binos can scatter also via squark exchange, but LHC experiments tell us that squark masses are in the TeV range at least [30], so these contributions are also small.

However, because Binos have such small couplings, their relic density in minimal cosmology typically comes out (much) too large. One way to fix this is to consider co–annihilation with colored superparticles, the most widely candidate being the lighter stop squark [31]. Recent calculations show that this can give the correct relic density for Binos with mass up to 6 TeV [32]. However, co–annihilation requires that the stop and the Bino have similar masses, the mass difference being below 10%; Bino masses near 6 TeV even require a mass splitting \( \lesssim 0.5\% \), which appears rather finetuned.

Another possibility is to consider deviations from minimal cosmology. In particular, if for some range of temperatures above MeV (i.e., safely above the range relevant for Big Bang nucleosynthesis) the energy density of the universe was dominated by some heavy but long–lived field many possibilities open up [33]. “Moduli” fields with these properties in fact occur quite naturally in many string theory constructions [34]. Detailed calculations show that in this case Binos can indeed be good DM candidates in generic parts of MSSM parameter space if the branching ratio for moduli decays into superparticles is very small, below \( 10^{-4} \) or so [35].

5. Take–Home Messages

- Dark Matter exists!
- Most likely it consists of new kind(s) of particle.
- We don’t know much about these particles.
- Several decades of searches for Dark Matter have not yielded a convincing signal.
- Some of the oldest/simplest/best motivated candidates (axion, WIMPs, gravitinos) are not excluded.
• Since WIMPs are getting squeezed, theorists are extending WIMP parameter space, and are suggesting entirely new kinds of DM candidates, with masses between $10^{-22}$ eV and $10^{18}$ GeV.

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References

[15] See the review on Other Particle Searches in the Particle Data Book [1].
[22] See e.g. the review on Axions in the Particle Data Book [1].
[23] See e.g. the review on Supersymmetry in the Particle Data Book [1].


[30] See e.g. “Supersymmetric Particle Searches” in the Particle Data book [1].


