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Jet quenching parameter in an expanding QCD plasma

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We present a new definition of the jet quenching parameter $\hat{q}$ in a weakly-coupled quark-gluon plasma undergoing boost-invariant longitudinal expansion. We propose a boost-invariant definition of $\hat{q}$, which is proportional to the broadening of the angular variables η (the pseudo-rapidity) and ϕ (the azimuthal angle). We furthermore consider radiative corrections to $\hat{q}$ and find potentially large corrections enhanced by a double logarithm like the case of a static medium. But unlike for the static medium, these corrections are now local in (proper) time.

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1. Introduction

The transport coefficient $\hat{q}$ is one of the most important parameters for studying jet quenching in relativistic heavy-ion collisions. $\hat{q}$ can be defined as the typical value of the transverse momentum broadening $p_\perp^2$ of a high-energy parton averaged over its path length in QCD matter. It has been shown that medium-induced radiative energy loss is also proportional to $\hat{q}$ [1].

Radiative corrections to $\hat{q}$ have been calculated in a static QCD medium with a fixed length L. The transverse momentum broadening is found to receive a double logarithmic correction [2], which can be resummed in perturbative QCD [3]. Such a double logarithmic enhancement has been shown to be universal both in $p_\perp^2-$broadening [2, 3] and radiative energy loss [4, 5]. And it can be formally absorbed into a renormalized $\hat{q}$ [4, 6].

It is important to measure $\hat{q}$ as transverse momentum broadening. By comparing with the measurement from parton energy loss [7, 8], one can hence test the fundamental relation between $p_\perp^2$ and parton energy loss. Such measurements have been proposed in [9, 10, 11, 12] (see also a recent measurement by the STAR collaboration at RHIC [13]).

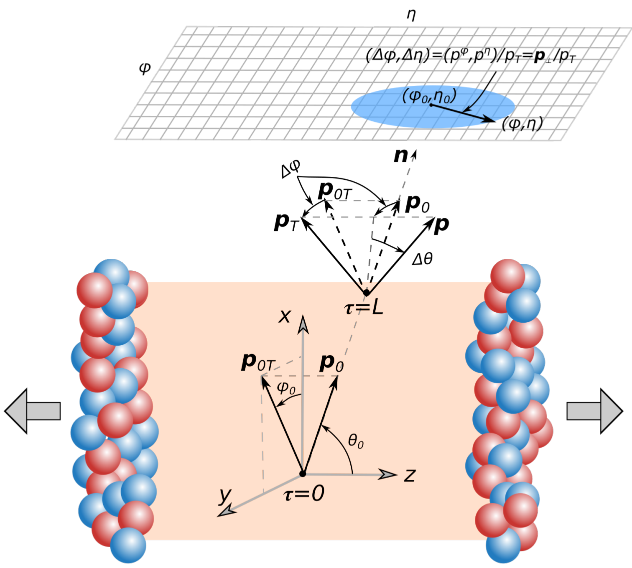

Figure 1: A pictorial representation of the definition for transverse momentum broadening in a nucleus-nucleus collision. An energetic parton is created at (proper) time τ=0 with the 4-momentum p_0^μ and decouples from the medium at $\tau = L$, with the 4-momentum p^μ. The broadening refers to a change $\Delta\phi$ in the azimuthal angle and a change $\Delta\eta$ in the pseudo-rapidity (corresponding to a change $\Delta\theta$ in the polar angle).These changes are summarized in a vector $\boldsymbol{p}_\perp$ in the 2-dimensional rapidity-azimuthal angle plane $(\Delta\eta, \Delta\phi)$, which is precisely the plane generally used to represent the kinematics of a high-energy collision.

As illustrated in Fig. 1, we found that it is convenient to choose a new coordinate system in which $\hat{q}$ can be defined as the broadening of the angular variables η (the pseudo-rapidity) and ϕ (the azimuthal angle) [14]. This was motivated by the proposal to measure $\hat{q}$ from the broadening of jets in $\Delta\phi$ [10, 11, 12]. We also found that using these new coordinates radiative corrections to $\hat{q}$ in a longitudinally boost-invariant plasma can be calculated in a way similar to the case in the static medium.

2. $\hat{q}$ as jet broadening in $\Delta\phi$ and $\Delta\eta$

Let us study the motion of an energetic parton which propagates through a longitudinally-expanding plasma (cf. Fig. 1). We choose the z-axis as the collision (or "longitudinal") axis and use the notation $\boldsymbol{x}_T = (x, y)$ for the coordinates of a point in the transverse plane. The collision starts at $z = t = 0$ and a high-p_T parton, initiating the jet, is produced almost instantaneously at $t = z = 0$. It is convenient to use the momentum rapidity[1] η and the space-time rapidity η_s, defined respectively as

$$\eta \equiv \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right), \qquad \eta_s \equiv \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right), \tag{2.1}$$

as well as the azimuthal angle ϕ, which is measured from the x-axis around the beam. In the situation at hand, one can identify the two rapidities, i.e., $\eta = \eta_s$.

In nuclear collisions a QCD jet is usually defined as a cluster of final-state particles in the ϕ-η space. It is hence important to find a convenient way to calculate the broadening of the energetic parton in (η, ϕ). Let us take $\phi = \phi_0$, $\eta = \eta_0$ and $p^\mu = p_0^\mu = p_T(\cosh\eta_0, \cos\phi_0, \sin\phi_0, \sinh\eta_0)$ at $t = 0$. At a later time $\tau \equiv \sqrt{t^2 - z^2} > \tau_0$, when bulk matter gets formed, this parton is subject to multiple scattering. Here, we shall focus on the calculation of jet broadening in $\Delta\phi = (\phi - \phi_0)$ and $\Delta\eta = (\eta - \eta_0)$ resulting from multiple scattering.

We shall perform our calculation in a new coordinate system by choosing four independent basis vectors. First, we choose the initial jet direction

$$\hat{n}^\mu = \frac{p_0^\mu}{p_T} = (\cosh\eta_0, \cos\phi_0, \sin\phi_0, \sinh\eta_0) \tag{2.2}$$

as one basis vector. Second, it is natural to choose two more basis vectors as follows

$$\hat{\phi}^\mu \equiv (0, \sin\phi_0, -\cos\phi_0, 0), \qquad \hat{\eta}^\mu \equiv (-\sinh\eta_0, 0, 0, -\cosh\eta_0), \tag{2.3}$$

which span the 2-dimensional vector space encoding the jet broadening in $\Delta\phi$ and $\Delta\eta$. These vectors are orthogonal in the 4-dimensional sense to the initial jet direction: $\hat{\eta}\cdot\hat{n} = \hat{\phi}\cdot\hat{n} = 0$. For that reason, the respective components $p^\eta \equiv \hat{\eta}\cdot p = p_T\Delta\eta$ and $p^\phi \equiv \hat{\phi}\cdot p = p_T\Delta\phi$ will be referred to as "transverse" and collectively denoted with the subscript $\perp$: $\boldsymbol{p}_\perp \equiv (p^\phi, p^\eta)$. This 2-dimensional vector should not be confused with the *other* transverse momentum in the problem, namely $\boldsymbol{p}_T = (p_x, p_y)$, which is orthogonal to the collision axis. The fourth basis vector can be chosen to be the norm to the hyper-surface of the constant proper time τ, which takes the form

$$\hat{\tau}^\mu = (\cosh\eta_0, 0, 0, \sinh\eta_0). \tag{2.4}$$

In this new coordinate system the "time" is the proper time τ, the "energy" is p_T and the "transverse" momentum broadening is the broadening in (η, ϕ). The reason is as follows. In terms of these four vectors, one can easily see that

$$g^{\mu\nu} = \hat{n}^\mu\hat{\tau}^\nu + \hat{\tau}^\mu\hat{n}^\nu - \hat{n}^\mu\hat{n}^\nu - \hat{\phi}^\mu\hat{\phi}^\nu - \hat{\eta}^\mu\hat{\eta}^\nu, \tag{2.5}$$

[1] We treat all particles as massless, so their momentum rapidity is the same as their pseudo-rapidity: $\eta = -\ln\tan(\theta/2)$, with θ their polar angle: $v_z = \cos\theta$.

and, accordingly,

$$p^\mu \simeq p_T \hat{n}^\mu - p_T \Delta\eta\, \hat{\eta}^\mu - p_T \Delta\phi\, \hat{\phi}^\mu \equiv (p_T, 0, \Delta\eta, \Delta\phi),$$
$$x^\mu \simeq \tau\hat{\tau}^\mu - x^\eta\, \hat{\eta}^\mu - x^\phi\, \hat{\phi}^\mu \equiv (0, \tau, x^\eta, x^\phi). \tag{2.6}$$

All the discussions in the static case can be straightforwardly generalized into the expanding case using such new coordinates. For example, $\hat{q}$ can be defined as

$$\frac{\partial}{\partial\tau} S(x, \boldsymbol{r}_\perp) = -\frac{1}{4}\hat{q}_0(x) r_\perp^2 S(x, \boldsymbol{r}_\perp), \tag{2.7}$$

which looks the same as the corresponding equation in the static medium with the replacement discussed above. The solution to this equation gives the distribution of the parton in the (η, ϕ) space at $\tau = L$

$$\frac{dN}{d^2\boldsymbol{p}_\perp} = \int \frac{d^2\boldsymbol{r}}{(2\pi)^2} e^{-i\boldsymbol{p}_\perp\cdot\boldsymbol{r}_\perp} S(\boldsymbol{r}_\perp) \simeq \frac{1}{\pi\mathscr{Q}_0^2(L)} e^{-\frac{\boldsymbol{p}_\perp^2}{\mathscr{Q}_0^2(L)}}$$
$$\Longleftrightarrow \quad \frac{dN}{d\Delta\phi\, d\Delta\eta} \simeq \frac{p_T^2}{\pi\mathscr{Q}_0^2(L)} e^{-\frac{p_T^2(\Delta\phi^2+\Delta\eta^2)}{\mathscr{Q}_0^2(L)}}. \tag{2.8}$$

In a longitudianlly boost-invariant plasma, one has [15]

$$\mathscr{Q}_0^2(L) = \int_{\tau_0}^{L} d\tau \hat{q}_0(\tau) \simeq \hat{q}_0(L) L \frac{1-(\tau_0/L)^{1-\beta}}{1-\beta} \tag{2.9}$$

with

$$\hat{q}_0(\tau) \simeq \hat{q}_0(\tau_0)\left(\frac{T(\tau)}{T_0}\right)^3 \simeq \hat{q}_0(\tau_0)\left(\frac{\tau_0}{\tau}\right)^\beta. \tag{2.10}$$

3. Radiative correction to $\hat{q}(\tau)$ in an expanding plasma

It is also straightforward to calculate the radiative corrections to $p_\perp^2$ and $\hat{q}$ using these coordinates in an expanding plasma. With $\hat{q}_0$ given by Eq. (2.10), the radiative correction to $p_\perp^2$ is given by

$$\delta\langle p_\perp^2\rangle = -\nabla_{\boldsymbol{r}}^2 \delta S(\boldsymbol{r})\Big|_{r_\perp=0}, \tag{3.1}$$

where the radiative correction from one gluon emission to $S(\boldsymbol{r}_\perp)$

$$\delta S(\boldsymbol{r}) = -\frac{\alpha_s N_c r^2}{2} \mathrm{Re} \int \frac{d\omega}{\omega^3} \int_{\tau_0}^{L} d\tau_2 \int_{\tau_0}^{\tau_2} d\tau_1$$
$$\times \left\{ e^{-\frac{r^2}{4}\int_{\tau_0}^{L} d\tau' \hat{q}_0(\tau')[\theta(\tau_1-\tau')+\theta(\tau'-\tau_2)]} \nabla_{\boldsymbol{B}_2}\cdot\nabla_{\boldsymbol{B}_1} G^{(3)}(\boldsymbol{B}_2, \tau_2; \boldsymbol{B}_1, \tau_1; \boldsymbol{r}) \right\} \Bigg|_{\boldsymbol{B}_2=0}^{\boldsymbol{B}_2=\boldsymbol{r}} \Bigg|_{\boldsymbol{B}_1=0}^{\boldsymbol{B}_1=\boldsymbol{r}}, \tag{3.2}$$

the propagator for a gluon and a color dipole in the plasma

$$G^{(3)}(\boldsymbol{B}_2, \tau_2; \boldsymbol{B}_1, \tau_1; \boldsymbol{r}) = G\left(\boldsymbol{B}_2 - \frac{\boldsymbol{r}}{2}, \tau_2; \boldsymbol{B}_1 - \frac{\boldsymbol{r}}{2}, \tau_1\right), \tag{3.3}$$

and

$$G(\boldsymbol{B}_2,\tau_2;\boldsymbol{B}_1,\tau_1)\equiv\frac{i\omega}{2\pi D(\tau_2,\tau_1)}\,e^{\frac{-i\omega}{2D(\tau_2,\tau_1)}\left[c_1\boldsymbol{B}_1^2+c_2\boldsymbol{B}_2^2-2\boldsymbol{B}_2\cdot\boldsymbol{B}_1\right]},\tag{3.4}$$

with $c_1\equiv c(\tau_2,\tau_1), c_2\equiv c(\tau_1,\tau_2)$ and the following definitions for the functions $D(\tau_2,\tau_1)$ and $c(\tau_2,\tau_1)$:

$$\begin{aligned}D(\tau_2,\tau_1)&=\pi\nu\sqrt{\tau_1\tau_2}\left[J_\nu\left(2\nu\Omega_1\tau_1\right)Y_\nu\left(2\nu\Omega_2\tau_2\right)-J_\nu\left(2\nu\Omega_2\tau_2\right)Y_\nu\left(2\nu\Omega_1\tau_1\right)\right],\\ c(\tau_2,\tau_1)&=\frac{\pi\nu\sqrt{\tau_1\tau_2}\Omega_2}{\sin(\pi\nu)}\left[J_{\nu-1}\left(2\nu\Omega_2\tau_2\right)J_{-\nu}\left(2\nu\Omega_1\tau_1\right)+J_{1-\nu}\left(2\nu\Omega_2\tau_2\right)J_\nu\left(2\nu\Omega_1\tau_1\right)\right].\end{aligned}\tag{3.5}$$

Here, we have used the shorthand notation $\Omega_{1,2}\equiv\Omega(\tau_{1,2})$ with $\Omega(\tau)\equiv\sqrt{i\hat{q}_0(\tau)/\omega}$ and $\nu\equiv 1/(2-\beta)$.

The extraction of the double logarithmic terms in the radiative correction to $p_\perp^2$ from the above equation involves exactly the same manipulations as in the case of the static medium in Ref. [3], except for the replacement $\hat{q}_0\to\hat{q}_0(\tau)$ and for the fact that the integral over the formation time $\tau_{\rm f}$ should now be restricted to $\tau_{\rm f}<\tau$. And we find

$$\begin{aligned}\delta\langle p_\perp^2(L)\rangle &= \bar{\alpha}\int_{\tau_0}^{L}d\tau\,\hat{q}_0(\tau)\int_{\lambda(\tau)}^{\tau}\frac{d\tau_{\rm f}}{\tau_{\rm f}}\int_{\hat{q}_0(\tau)\tau_{\rm f}^2}^{\mathscr{Q}_0^2(L)\tau_{\rm f}}\frac{d\omega}{\omega}\\ &=\int_{\tau_0}^{L}d\tau\,\delta\hat{q}(\tau),\qquad\text{with}\qquad \delta\hat{q}(\tau)\equiv\hat{q}_0(\tau)\,\frac{\bar{\alpha}}{2}\ln^2\frac{\tau}{\lambda(\tau)}\,.\end{aligned}\tag{3.6}$$

Plugging $\hat{q}_0(\tau)$ in (2.10) and the thermal wavelength $\lambda(\tau)=\lambda_0(\tau/\tau_0)^{\beta/3}$, with $\lambda_0\equiv 1/T_0$, into the above equation gives

$$\delta\langle p_\perp^2(L)\rangle\simeq\begin{cases}\mathscr{Q}_0^2(L)\,\dfrac{2\bar{\alpha}}{27}\ln^2\dfrac{L}{\tau_0} & \text{for } \beta=1\,,\\ \mathscr{Q}_0^2(L)\,\dfrac{\bar{\alpha}}{2}\left(1-\dfrac{\beta}{3}\right)^2\ln^2\dfrac{L}{\tau_0} & \text{for } \beta<1\,.\end{cases}\tag{3.7}$$

Here, we have neglected the difference between τ_0 and λ_0 within the arguments of the various logarithms.

It is also straightforward to carry out the all-order resummation of the leading double-logarithmic terms and we obtain

$$\hat{q}(\tau)=\hat{q}_0(\tau)\,\frac{I_1\left(2\sqrt{\bar{\alpha}}Y\right)}{\sqrt{\bar{\alpha}}Y}=\hat{q}_0(\tau)\,\frac{e^{2\sqrt{\bar{\alpha}}Y}}{\sqrt{4\pi}\,(\sqrt{\bar{\alpha}}Y)^{3/2}}\left[1+\mathscr{O}(1/\sqrt{\bar{\alpha}}Y)\right],\tag{3.8}$$

where $Y=\ln(\tau/\lambda(\tau))$ and the second equality holds when $Y\gg 1/\sqrt{\bar{\alpha}}$, i.e. for sufficiently large time. Eq. (3.8) is formally similar to the corresponding result for a static medium except that it depends on τ instead of L via the τ-dependence of the functions $\hat{q}_0(\tau)$ and $\lambda(\tau)$. Unlike the case in the static medium, this makes it possible to treat the renormalized $\hat{q}(\tau)$ as a *(quasi)local* transport coefficient, like its tree-level counterpart $\hat{q}_0(\tau)$.

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