Jet fragmentation in a QCD medium:
Universal quark/gluon ratio and Wave turbulence

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We calculate the evolution of a jet shower due to medium induced splittings in the deep LPM regime. Due to the characteristic energy dependence of the formation time $t_{\text{form}}(\omega) = \sqrt{\omega/\hat{q}}$, the radiative break-up process exhibits turbulent characteristics, allowing for analytic predictions of various inclusive properties of the medium induced cascade in an inertial range of energies $T \ll \omega \ll E$, where $E$ is the energy of the jet and $T$ is the temperature of the medium.

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1. Introduction

Highly energetic particles or jets produced in collider experiments provide a unique tool to study the dynamics of strong-interaction matter. The evolution of jets in vacuum is theoretically well established within pQCD and serves as a precision tool in pp collisions; conversely the evolution of jets in a de-confined Quark-Gluon Plasma (QGP), created in the collision of heavy nuclei, can serve as a probe of the QGP medium and remains an active area of research. While a full characterization of jets in a QCD medium represents a formidable task, involving many different scales from below the the temperature of the medium \( T \lesssim 1 \text{ GeV} \) to the energy of the jet \( E \gtrsim 100 \text{ GeV} \), we will not attempt to provide a complete description of the dynamics. Instead, we will focus on one particular aspect of jet evolution in a QCD medium, namely the radiative break-up or fragmentation of a jet due to medium induced radiation \[1\].

2. Evolution of q/g jets in the medium

We describe the jet fragments as a collection of highly energetic (\( \omega \gg T \)) on-shell quarks/gluons propagating in a thermal QGP. Various kinds of elastic (\( 2 \leftrightarrow 2 \)) and inelastic (\( 1 \leftrightarrow 2 \)) interactions with the constituents of the thermal QGP can modify the properties of the jet \[2, 3\]. However, for highly energetic fragments (\( \omega \gg T \)) the evolution is dominated by radiative branchings, where due to the interaction with the medium a hard parton of energy \( \omega \), splits into two fragments with energy fractions \( z \omega \) and \( (1-z)\omega \). In the regime where in-medium splittings are induced by multiple soft interactions, as characterized by a momentum diffusion constant \( \hat{q} \), the corresponding splitting rates are given by \[2, 3\]

\[
\Gamma^\text{split}_{f_i}(\omega | z \omega, (1-z)\omega) = \frac{\alpha}{\pi \tau_{\text{form}}(\omega)} K_{fi}(z)
\]

where \( i, f = q, g \) labels the initial and final states and \( K_{fi}(z) \) denote the in-medium splitting functions (see \[1\] for details). Due to the characteristic energy dependence of the formation time \( \tau_{\text{form}}(\omega) = \sqrt{\omega/\hat{q}} \) associated with the Landau-Pomeranchuk-Migdal (LPM) effect \[4, 5\], the splitting rates increase as \( \Gamma^\text{split}_{f_i}(\omega) = \sqrt{E/\omega} \Gamma^\text{split}_{f_i}(E) \) for less energetic particles, indicating the importance of successive splittings in the process. By treating successive splittings as quasi-instantaneous and independent of each other, the evolution of the in-medium fragmentation functions \( D_i(x) = \omega dN_i/d\omega \), which describe the inclusive distribution of \( i = q_f, \bar{q}_f, g \) fragments as function of the energy fractions \( x = \omega/E \), is then described by a coupled set of effective kinetic equations \[1\]

\[
\partial_t D_g(x) = \int_0^1 dz \left\{ \frac{K_{gg}(z)}{\sqrt{x/z}} D_g \left( \frac{x}{z} \right) + \frac{K_{gq}(z)}{\sqrt{x/z}} D_S \left( \frac{x}{z} \right) - \frac{K_{gg}(z) + K_{gq}(z)}{2\sqrt{x}} D_g(x) \right\}, \tag{2.2}
\]

\[
\partial_t D_S(x) = \int_0^1 dz \left\{ \frac{K_{gg}(z)}{\sqrt{x/z}} D_g \left( \frac{x}{z} \right) + \frac{K_{gq}(z)}{\sqrt{x/z}} D_S \left( \frac{x}{z} \right) - \frac{K_{gg}(z) + K_{gq}(z)}{2\sqrt{x}} D_S(x) \right\}, \tag{2.3}
\]

for the gluon distribution \( D_g \) and the flavor singlet quark distribution \( D_S = \sum_{f=1}^{N_f} (D_{q_f} + D_{\bar{q}_f}) \).\footnote{Note that color factors are removed from \( \hat{q} \) and absorbed into the splitting functions instead.} Note that in Eqns. (2.2,2.3), the dependence on the medium properties (\( \hat{q} \)) and the jet energy (\( E \)) have been adsorbed into the definition of the characteristic time scale \( \tau = \frac{\alpha}{\pi} \tau_{\text{form}}(E) \).

\footnote{We note that there is an additional set of kinetic equations for the evolution of the flavor non-singlet (valence)
Figure 1: Evolution of the in-medium fragmentation functions for gluons (left) and quarks (right) inside a gluon jet. Different curves in each panel show the results after evolution for different amounts of time $\tau$ inside the medium. [1]

2.1 Single vs. multiple splittings, wave turbulence & universal quark/gluon ratio

Numerical solutions of Eqns. (2.2,2.3) are presented in Fig. 1, which shows fragmentation functions $\sqrt{x} D_{g/S}(x, \tau)$ for a gluon jet ($D_g(x, \tau = 0) = \delta(1 - x)$, $D_S(x, \tau = 0) = 0$) after a short period of evolution $\tau = 0.002 - 0.12$ inside the medium. While at moderate values of $x$ the in-medium fragmentation functions are dominated by a single splitting, i.e. $D_i(x) \approx \tau x K_i(x)$, clear deviations from this behavior can be observed for smaller momentum fractions $x \lesssim 0.1$ already after very a short period of evolution inside the medium. Since the emission rates at small $x$ are enhanced by a factor $\Gamma_i^{\text{split}}(xE) = \frac{1}{\gamma_i} \Gamma_i^{\text{split}}(E)$ due to the shorter formation times for softer radiation, the fragmentation process for $x \lesssim 0.1$ is dominated by multiple quasi-democratic splittings and proceeds in analogy to the famous Richardson cascade. In particular one finds that the in-medium fragmentation functions approach turbulent spectra of the Kolmogorov-Zakharov (KZ) form [6, 7, 1]

$$D_g(x) = G/\sqrt{x}, \quad D_S(x) = 2N_f Q/\sqrt{x},$$

(2.4)

associated with a scale ($x$) independent energy flux

$$\frac{1}{E} \frac{dE}{d\tau} = -\gamma_i G - \gamma_i Q, \quad \gamma_i = \int_0^1 dz \sum_{j=q \bar{q}} K_{ji}(z) \log(1/z)$$

(2.5)

directed towards smaller energies. A careful analysis of the stationary turbulent solutions of the kinetic equations (2.2,2.3) reveals that the chemistry of small $x$ fragments is uniquely fixed by the balance of $g \rightarrow q\bar{q}$ and $q \rightarrow gq$ processes, yielding [1]

$$\frac{D_S(x)}{2N_f D_g(x)} \bigg|_{x \ll 1} = \frac{Q}{G} = \frac{\int_0^1 dz z K_{qg}(z)}{2N_f \int_0^1 dz z K_{gq}(z)} \approx 0.07.$$  

(2.6)

distributions $D_{NS}^{(f)} = D_{qf} - D_{\bar{q}f}$, which decouples from Eqns. (2.2,2.3), and refer to [1] for the evolution of the valence distribution.
Numerical solutions of the evolution equations shown in the left panel of Fig. 2 confirm that the universal ratio in Eq. (2.6), is indeed realized over a substantial range of momentum fractions $x$ and evolution times $\tau$ inside the medium, regardless of the nature (q/g) of the leading jet parton.

2.2 Energy loss of quark & gluon jets

Besides affecting the jet substructure, the medium induced cascade also leads to an efficient energy transfer to the thermal medium, as shown in the right panel of Fig. 2, where we present the rate $dE/d\tau$ of energy transferred to the thermal medium. Notably, for a short amount of evolution inside the medium ($\tau \ll 1$) the energy loss rate can be understood as a simple two-step process [6, 1], where initially a single emission off the original hard parton creates a soft gluon spectrum of the form $D_g(x, \tau) \sim x \Gamma_g(x) \tau C_A^{1/2} R / \sqrt{x}$. Subsequently, multiple splittings of the soft jet fragments create a turbulent energy flux

$$\frac{dE}{dt} = \frac{\alpha_s^2}{\pi^2} \hat{q} \gamma_{\hat{E}} C_A^{1/2} R \tau$$

transporting energy all the way to the scales of the medium $(x \sim T/E \ll 1)$. Since for $\tau \ll 1$, the turbulent cascade is initiated by the original hard parton, the energy loss in Eq. (2.7) features the familiar Casimir scaling, such that $dE/d\tau |_{q-\text{jet}} = C_F/C_A \ dE/d\tau |_{g-\text{jet}}$. However, beyond time scales $\tau \gtrsim 0.1$ the chemical composition of the hard components of the jet is strongly altered by medium induced branchings, leading to a break down of the naive Casimir scaling seen in Fig. 2.

3. Discussion

Based on an effective kinetic description of jet evolution in a thermal medium, we find that the in-medium fragmentation of jets is governed by a turbulent cascade associated with the transport of energy from the scale of jet $(\omega \sim E)$ all the way to the scale of the medium $(\omega \sim T)$.
Due to the turbulent nature of the process, important features of the medium induced cascade can be directly inferred from the stationary turbulent solutions of the underlying kinetic equations. Some of our findings, such as the modified chemical composition of jet fragments, could also have interesting consequences for various jet sub-structure observables, including e.g. modified strangeness/heavy-flavor production in heavy-ion jets [1]. However, to further explore phenomenological consequences it will be necessary to combine the results of our study of the in-medium evolution with a theoretical description of the initial jet production and the hadronization of jet fragments. While this work is in progress, it is also worth pointing out that these phenomena could also be studied within Monte-Carlo event generators such as MARTINI or JEWEL, which are based on the same underlying microscopic processes.

References


