PROCEEDINGS OF SCIENCE

Toward a unified picture of particle production in high energy collisions at all $p_\perp$

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We describe a new formalism which aims to unify particle production in high energy hadronic/nuclear collisions in the Color Glass Condensate formalism at small x (low p_t) with that of collinear factorization of perturbative QCD at high p_t (large x). To do so, in addition to the classical color field representing small x gluons of a target hadron or nucleus we also include large x gluons of the target. We then calculate the scattering amplitude for a projectile quark scattering from both large and small x gluons of the target. This allows us to generalize the Color Glass Condensate formalism to include high p_t deflection of the quark projectile. We discuss the remaining steps needed to derive expressions for particle production cross sections in high energy collisions, in both small and large x (low and high p_t) kinematics , including quantum effects which would contain both large $1/x$ and Q^2 logarithms.

International Conference on Hard and Electromagnetic Probes of High-Energy Nuclear Collisions
30 September - 5 October 2018
Aix-Les-Bains, Savoie, France

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PoS(HardProbes2018)120

1. Introduction

High p_t particle production in high energy hadronic (proton-proton, $\cdots$) collisions is usually described in the collinear factorization framework of QCD. Here one makes the approximation that only one parton (quark or gluon) in each of the incoming protons undergoes a hard scattering, after which the protons break up and their remnants form the soft underlying event. The hard scattering is calculable in perturbative QCD to any order in the coupling constant α_s while the parton distribution functions are truly non-perturbative. The predictive power of the formalism lies in the fact that the parton distribution functions are universal, i.e., they can be measured in one process and used in any other unlike the hard scattering part which is process dependent. In case of single inclusive hadron production the invariant cross section can be symbolically written as

$$E\,\frac{d\sigma^{pp\to\pi^0 X}}{d^3p} = f_1(x_1,Q^2)\otimes f_2(x_2,Q^2)\otimes\frac{d\sigma}{dt}\otimes D(z,Q^2) \tag{1.1}$$

where $\frac{d\sigma}{dt}$ is the hard scattering part with $t=p_t^2$ while $f_1(x_1,Q^2), f_2(x_2,Q^2)$ describe the distribution of partons carrying fractions x_1,x_2 of the energy of the incoming protons. The parton distribution functions depend on the hard scale Q^2 which is usually taken to be the p_t^2 of the produced hadron. The fragmentation function $D(z,Q^2)$ summarizes our ignorance of the details of hadronization in QCD, with z being the energy fraction of a parton carried by a hadron. Both fragmentation and distribution functions satisfy the DGLAP evolution equation which determines their Q^2 evolution arising from singular logarithms ever present in quantum (α_s) corrections to tree level diagrams. The DGLAP evolution equation for one quark flavor can be written as

$$\frac{d}{d\log Q^2}\begin{pmatrix} q(x,Q^2)\\ g(x,Q^2)\end{pmatrix} = \int_x^1 \frac{dz}{z}\begin{pmatrix} P_{qq}(z,\alpha_s) & P_{qg}(z,\alpha_s)\\ P_{gq}(z,\alpha_s) & P_{gg}(z,\alpha_s)\end{pmatrix}\begin{pmatrix} q(x/z,Q^2)\\ g(x/z,Q^2)\end{pmatrix}. \tag{1.2}$$

The splitting functions $P_{ab}(z,\alpha_s)$ are calculated in perturbation theory with the most important one (kinematically) being P_{gg} which describes radiation of a gluon by another gluon and is singular as $\frac{1}{z}$ as $z\to 0$. This leads to a fast rise of gluon (and sea quark) distribution function with energy fraction x. This fast growth of parton distribution functions is observed experimentally in Deep Inelastic Scattering (DIS) at HERA. The fast rise of gluon (and sea quark) distribution function with inverse of energy fraction x naturally leads to a breakdown of collinear factorization; as number of gluons in the wave function of a proton grows there must be an eventual breakdown of the parton model due high gluon densities, called gluon saturation. A back of the envelope estimate shows that when the probability of gluon-gluon interactions $\frac{\alpha_s\, xG(x,Q^2)}{S_\perp Q^2}\sim 1$ one can not treat partons as quasi-free anymore. This relation defines a saturation scale $Q_s^2(x,b_\perp,A)$, which roughly corresponds to the scale where *high parton density* effects become dominant. This leads to a breakdown of collinear factorization formalism and a new approach is needed.

1.1 QCD in the small x limit and the Color Glass Condensate formalism

The Color Glass Condensate (CGC) formalism [1] goes beyond the QCD parton model by treating small x gluons of a proton or nucleus coherently as a classical color field. This field is assumed to be radiated by the large x partons (quarks and gluons) jointly referred to as color charge density ρ [2]. Due to the high energy kinematics the color current has only one large component, along the direction of motion. One then solves the classical Yang-Mills equations in the presence of this color charge density. The solution in the light cone gauge is

$$\partial_i\, \tilde{\alpha}_i^a(x_t) = g\, \tilde{\rho}^a(x_t)\,, \tag{1.3}$$

where $\tilde{\alpha}_i^a(x_t)$ is the classical solution with $i = 1,2$. This relation however can not be inverted and it is much easier to work in the covariant gauge where the solution has a particularly simple form

$$\partial_t^2\, A_a^+(x_t) = g\, \rho_a(x_t)\,, \tag{1.4}$$

which is very useful for analytic calculations. Armed with this classical solution for the color field of a proton or nucleus one can use it to calculate parton production cross sections in the small x limit of QCD. For example, this approach is used to calculate the energy and number density of gluons produced in a high energy heavy ion collision, which can then be used as initial conditions for further evolution of the produced system using other approaches, for example, hydrodynamics. Nevertheless due to the complexities of the problem, analytic solutions are not known and one has to use numerical methods to solve these equations.

A much simpler process which allows an analytic approach is the so-called dilute-dense collisions [3] where one projectile is treated as a collection of quasi-free partons while the other (the target) is treated as a classical color field given by (1.4). Examples are proton-proton and proton-nucleus collisions in the forward rapidity region where partons of the proton are at moderate to large x while the target gluons are at small x. This is the essence of the hybrid formulation in contrast to the "k_t" factorization approach which treats both the projectile and target as classical color fields with the projectile field being weak and therefore expanded in powers of the color charge density. Here we will focus on the hybrid formulation. In this approach the basic building block is the Wilson line, a path-ordered exponential which sums multiple scattering of a quark (or gluon) from the projectile proton on the classical color field of the target proton or nucleus. The Wilson line is given by (for a left moving target and a right moving quark)

$$V(x_t) \equiv \hat{P} \exp\left\{ ig \int_{-\infty}^{+\infty} dx^+\, A_a^-(x^+, x_t)\, t_a \right\} \tag{1.5}$$

so that the elastic scattering amplitude is

$$i\mathscr{M}_{eikonal}(p,q) = 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\, [V(x_t) - 1]\, u(p)\,. \tag{1.6}$$

It is then clear that the scattering cross section for elastic scattering of a quark on the target is given in terms of (Fourier transform of) correlators of this Wilson line, the so-called dipole,

$$T(x_t, y_t) \equiv \frac{1}{N_c} < Tr\left[1 - V(x_t)\, V^\dagger(y_t)\right] >_\rho \tag{1.7}$$

where x_t, y_t are the transverse positions of the quark (anti-quark) in the amplitude (complex conjugate amplitude) and $< \cdots >_\rho$ denotes an averaging over all color charges. All the observables in dilute-dense collisions (as well as in DIS) in the hybrid formalism are given in terms of multi-point correlators of Wilson lines [4]. The energy (or equivalently x) dependence of observables results from one-loop ($O(\alpha_s)$) corrections which contain singular logarithms of $1/x$ resummed by JIMWLK evolution equation (or its large-N_c counterpart BK equation) [5] . It is important to realize that scattering is done in the eikonal approximation where the projectile quark (or gluon) does not get a sizable deflection after scattering from the target. Therefore one expects that this approximation will not be valid when considering particle production at high p_t. Furthermore since transverse momentum and rapidity of the produced parton are kinematically related to the energy fraction x of the target proton or nucleus via $x \sim \frac{p_t}{\sqrt{s}} e^y$, high p_t particle production probes intermediate to large x modes of the target and small x (eikonal) approximation is not applicable.

2. Toward a unified formalism for particle production for all transverse momentum

It should be clear that due to eikonal approximation and the kinematic relation between transverse momentum and rapidity of the produced parton with energy fraction x the CGC formalism is limited to particle production at low $p_t \sim Q_s$ where $Q_s \sim 2-3\,GeV$ at RHIC and the LHC. Keeping in mind that the target energy fraction x increases as the p_t of the produced parton increases it is more appropriate to treat the large x modes of the target as the standard gluons rather than a classical field. Therefore we reconsider the dilute-dense scattering and include scattering of a projectile quark not only from the small x modes of the target represented as a classical color field but also from the large x gluons [6]. The simplest process is $q\,T \to q\,X$ where T stands for a proton or nucleus. Since we are going beyond the eikonal approximation the scattered quark can now get deflected by a large angle, i.e. have a large transverse momentum. Furthermore, the large x gluon is now a dynamical degree of freedom and can itself rescatter from the classical color field representing the small x degrees of freedom. The scattering amplitude can be written as

$$i\mathcal{M} = i\mathcal{M}_{eikonal} + i\mathcal{M}_1 + i\mathcal{M}_2 + i\mathcal{M}_3 \tag{2.1}$$

where $i\mathcal{M}_{eikonal}$, $i\mathcal{M}_1$, $i\mathcal{M}_2$ and $i\mathcal{M}_3$ are given by eqs. (1.6,2.2,2.3,2.4) respectively,

$$\begin{aligned} i\mathcal{M}_1 \;=\; & \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\, \frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t} \\ & \bar{u}(\bar{q}) \left[\bar{V}_{AP}(x^+,\bar{z}_t)\, \not{n}\, \frac{\not{\bar{k}}}{2\bar{k}^+}\, \left[ig\not{A}(x)\right]\, \frac{\not{k}}{2k^+}\, \not{n}\, V_{AP}(z_t,x^+)\right] u(p) \end{aligned} \tag{2.2}$$

with $k^+ = p^+, k^- = \frac{k_t^2}{2k^+}, \bar{k}^+ = \bar{q}^+, \bar{k}^- = \frac{\bar{k}_t^2}{2\bar{k}^+}$,

$$\begin{aligned} i\mathcal{M}_2 \;=\; & \frac{2i}{(p-\bar{q})^2} \int d^4x\, e^{i(\bar{q}-p)x}\, \bar{u}(\bar{q}) \left[(ig\,t^a)\, \left[\partial_{x^+} U^\dagger_{AP}(x_t,x^+)\right]^{ab} \right. \\ & \left. \left[n\cdot(p-\bar{q})\not{A}_b(x) - (p-\bar{q})\cdot A_b(x)\,\not{n}\right]\right] u(p) \end{aligned} \tag{2.3}$$

$$\begin{aligned} i\mathcal{M}_3 = -2i & \int d^4x\, d^2\bar{x}_t\, d\bar{x}^+\, \frac{d^2\bar{p}_{1t}}{(2\pi)^2}\, e^{i(\bar{q}^+-p^+)x^-}\, e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\, e^{-i(\bar{q}_t-\bar{p}_{1t})\cdot\bar{x}_t} \\ & \bar{u}(\bar{q}) \left[\left[\partial_{\bar{x}^+} \bar{V}_{AP}(\bar{x}^+,\bar{x}_t)\right] \not{n}\, \not{\bar{p}}_1\, (ig\,t^a)\, \left[\partial_{x^+} U^\dagger_{AP}(x_t,x^+)\right]^{ab} \right. \\ & \left. \frac{\left[n\cdot(p-\bar{q})\not{A}^b(x) - (p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right]}{\left[2n\cdot\bar{q}\, 2n\cdot(p-\bar{q})\, p^- - 2n\cdot(p-\bar{q})\, \bar{p}_{1t}^2 - 2n\cdot\bar{q}\, (\bar{p}_{1t}-p_t)^2\right]}\right] u(p)\,, \end{aligned} \tag{2.4}$$

the semi-infinite, anti path-ordered Wilson lines in the fundamental representation are defined as

$$\overline{V}_{AP}(x^+, \bar{z}_t) \equiv \hat{P} \exp\left\{ig \int_{x^+}^{+\infty} d\bar{z}^+ \, \bar{S}_a^-(\bar{z}_t, \bar{z}^+) \, t_a\right\} \tag{2.5}$$

$$V_{AP}(z_t, x^+) \equiv \hat{P} \exp\left\{ig \int_{-\infty}^{x^+} dz^+ \, S_a^-(z_t, z^+) \, t_a\right\}. \tag{2.6}$$

The first term in (2.1) is just the standard small x result (1.6) while the second term (2.2) corresponds to the case when the projectile quark scatters at large angle but the large x gluon does not interact with the small x gluon modes. The third and fourth terms (2.3,2.4) correspond to the case when the large x and both large x and final state quark interact with the small x gluon modes. We note that in the small x limit all the non-eikonal terms vanish and one recovers the usual eikonal result of CGC formalism. To proceed further one would need to calculate the gluon propagator [7] in this approach first and then the one-loop corrections to this result which would then bring about the x and Q^2 dependence of the cross section and would hence unify the DGLAP and CGC description of particle production at the Leading Order in α_s. Work in this direction is in progress and will be reported elsewhere.

References

[1] F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, Ann. Rev. Nucl. Part. Sci. **60**, 463 (2010) doi:10.1146/annurev.nucl.010909.083629 [arXiv:1002.0333 [hep-ph]]; J. Jalilian-Marian and Y. V. Kovchegov, Prog. Part. Nucl. Phys. **56**, 104 (2006) doi:10.1016/j.ppnp.2005.07.002 [hep-ph/0505052].

[2] L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994) doi:10.1103/PhysRevD.49.2233 [hep-ph/9309289]; A. Dumitru, J. Jalilian-Marian and E. Petreska, Phys. Rev. D **84**, 014018 (2011) doi:10.1103/PhysRevD.84.014018 [arXiv:1105.4155 [hep-ph]].

[3] A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett. **89**, 022301 (2002) doi:10.1103/PhysRevLett.89.022301 [hep-ph/0204028]; A. Dumitru, A. Hayashigaki and J. Jalilian-Marian, Nucl. Phys. A **765**, 464 (2006) doi:10.1016/j.nuclphysa.2005.11.014 [hep-ph/0506308].

[4] A. Dumitru, J. Jalilian-Marian, T. Lappi, B. Schenke and R. Venugopalan, Phys. Lett. B **706**, 219 (2011) doi:10.1016/j.physletb.2011.11.002 [arXiv:1108.4764 [hep-ph]]; J. Jalilian-Marian, Y. Nara and R. Venugopalan, Phys. Lett. B **577**, 54 (2003) doi:10.1016/j.physletb.2003.09.097 [nucl-th/0307022].

[5] J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **55**, 5414 (1997); J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Nucl. Phys. B **504**, 415 (1997), Phys. Rev. D **59**, 014014 (1999), Phys. Rev. D **59**, 014015 (1999), Phys. Rev. D **59**, 034007 (1999); I. Balitsky, Nucl. Phys. B **463**, 99 (1996) doi:10.1016/0550-3213(95)00638-9 [hep-ph/9509348]; Y. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999) doi:10.1103/PhysRevD.60.034008 [hep-ph/9901281].

[6] J. Jalilian-Marian, arXiv:1809.04625 [hep-ph], Phys. Rev. D **96**, no. 7, 074020 (2017) doi:10.1103/PhysRevD.96.074020 [arXiv:1708.07533 [hep-ph]].

[7] A. Ayala, J. Jalilian-Marian, L. D. McLerran and R. Venugopalan, Phys. Rev. D **52**, 2935 (1995) doi:10.1103/PhysRevD.52.2935 [hep-ph/9501324].