Self-polarization in Storage Rings

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The possibility of electrons becoming polarized in an uniform field was predicted in the early 60s by Loskutov, Korovina, Sokolov and Ternov (Sokolov-Ternov effect of radiative polarization). The very first experimental observation of beam self-polarization followed in 1968 at ACO in Orsay and therefore this year marks the fiftieth anniversary of that event.

In particular at high energy, $e^{\pm}$ polarization is not really for free and in addition experiments are mainly interested in longitudinal polarization. This is obtained through spin rotators which may lead to depolarization. The $p/e^{\pm}$ high energy collider HERA in Hamburg was the first, and until now the only one, designed for delivering longitudinal $e^{\pm}$ polarization to the experiments.

In this paper, after a historical overview of the theory of radiative polarization in storage rings and its first observations, the unique case of HERAe will be highlighted. Measures needed for fostering high polarization will be illustrated with examples from HERAe operation.

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1. Sokolov-Ternov effect

The spin of an electron (or positron) in a homogeneous constant magnetic field has two stationary states, parallel or anti-parallel to the field direction. An electron (or positron) moving on a plane perpendicular to such a field describes circular orbits and emits synchrotron radiation. A part of the emitted radiation is accompanied by a flip of the particle spin direction from parallel to anti-parallel to the field and the other way round. The two processes having slightly different probability, there is an exponential build-up of polarization with rate

$$\tau_{\mathrm{ST}}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \hbar}{m_0} \frac{\gamma^5}{|\rho|^3}$$

and asymptotic value

$$|P_{\mathrm{ST}}| = \frac{|n^+ - n^-|}{n^+ + n^-} = \frac{8}{5\sqrt{3}} = 92.4\%$$

where n^+ and n^- is the number of electrons (positrons) with spin parallel and anti-parallel to the field respectively. The net polarization direction is anti-parallel to the field for electrons and parallel for positrons.

The very first published paper predicting the effect described above is by Ternov, Loskutov and Korovina [1] which appeared in russian in 1961 and in english one year later. However, it is in the 1963 paper (english translation in 1964), by Sokolov and Ternov [2] that the computation of asymptotic polarization and time constant were carried out by solving the exact Dirac equation.

Sokolov-Ternov formulas were generalized in 1970 by Baier, Katkov and Strakhovenko [3] to the case where spin and motion direction are not everywhere perpendicular, which leads to a reduction of the polarization rate and its asymptotic value.

The conditions described in these papers are somehow realized in particles storage rings. The possibility of polarizing $e^{\pm}$ beams for free was clearly appealing. Actually a $e^{\pm}$ beam in an ideally planar ring accelerator has finite radial and longitudinal dimensions. In addition the machine magnets may have mis-alignments which lead to a finite vertical size too. The effect of the fields seen by the particles in quadrupole and sextupole magnets, which are necessary to focus the beam and correct the natural chromaticity, must be therefore taken into account. The kinematics of the expectation value of the spin operator, $\vec{S}$, is described by the Thomas-BMT equation [4][5]

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$\vec{\Omega}$ depends in general upon machine azimuth and phase space position, $\vec{u}$. In the laboratory frame and MKS units it is given by

$$\vec{\Omega}(\vec{u};s) = -\frac{e}{m_0}\left[\left(a + \frac{1}{\gamma}\right)\vec{B} - \frac{a\gamma}{\gamma+1}\vec{\beta}\cdot\vec{B}\vec{\beta} - \left(a + \frac{1}{\gamma+1}\right)\vec{\beta}\times\frac{\vec{E}}{c}\right]$$

with $\vec{\beta} \equiv \vec{v}/c$ and $a = (g-2)/2$=0.0011597 (for $e^{\pm}$).

By applying Thomas-BMT equation to the spin of particles circulating in a storage ring, Baier and Orlov[6] first pointed out the existence of spin-orbit depolarizing resonances

$$\nu_{spin} = m \pm m_x Q_x \pm m_y Q_y \pm m_s Q_s$$

ν_{spin} being the spin tune, namely the number of precession per machine turn a spin performs around the periodic solution, $\hat{n}_0$, to the Thomas-BMT equation on the machine closed orbit. Those resonances are activated by the stochastic emissions of photons in presence of spin rotators and/or in a non-perfectly planar ring. The resulting spin diffusion is of course particularly important at high energy.

Finally, by using a semi-classical approximation, Derbenev and Kondratenko [7] gave an analytical expression for the asymptotic polarization and the build-up constant including spin diffusion. Following their results, outside from resonances, the asymptotic polarization is oriented along $\hat{n}_0$ and its value is

$$P_{\mathrm{DK}} = P_{\mathrm{ST}} \frac{\oint ds < \frac{1}{|\rho|^3} \hat{b} \cdot (\hat{n} - \frac{\partial \hat{n}}{\partial \delta}) >}{\oint ds < \frac{1}{|\rho|^3} \left[1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18}(\frac{\partial \hat{n}}{\partial \delta})^2 \right] >}$$

with $\hat{b} \equiv \hat{v} \times \dot{\hat{v}}/|\dot{\hat{v}}|$.

The corresponding polarization rate is

$$\tau_{\mathrm{DK}}^{-1} = P_{\mathrm{ST}} \frac{r_e \gamma^5 \hbar}{m_0 C} \oint < \frac{1}{|\rho|^3} \left[1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left(\frac{\partial \hat{n}}{\partial \delta} \right)^2 \right] >$$

The average is meant over the 6D phase space. The quantity $\partial \hat{n}/\partial \delta$, with δ relative energy deviation from the nominal machine energy, describes the effect on the particle spin direction of the trajectory perturbation after photon emission.

Following the Derbenev and Kondratenko papers appearance in the western countries, there were some disputes about the meaning of $\hat{n}(\vec{u};s)$ and $\partial \hat{n}/\partial \delta$. Many authors have contributed to settle the question (see for instance [8] [9] [10]) and nowadays $\hat{n}(\vec{u};s)$ is understood as the solution to the Thomas-BMT equation satisfying the periodicity condition $\hat{n}(\vec{u};s+C) = \hat{n}(\vec{u};s)$, C being the machine circumference.

1.1 Polarization computation

While the correct definition of $\hat{n}(\vec{u};s)$ is of course essential for implementing Derbenev-Kondratenko formulas in a computer code able of evaluating polarization in a realistic machine, implementations attempted so far by perturbation expansion of $\hat{n}(\vec{u};s)$ [11], Lie algebra [12] and Fourier series [13] have shown all some limitations.

By linearizing orbit and spin motion it is possible to calculate polarization "analytically" in terms of one turn maps. However, this approach, developed by K. Yokoya and A. Chao and implemented in SLIM [14], can detect only linear resonances.

A statistical approach has been used by J. Kewisch in SITROS [15]. The initially fully polarized beam is tracked and stochastic photon emission is simulated by random emission of "big photons" at user selected machine dipoles. Polarization evolves as

$$P(t) = P_\infty (1 - e^{-t/\tau_p}) + P(0)e^{-t/\tau_p}$$

with

$$\frac{1}{\tau_p} \simeq \frac{1}{\tau_{\mathrm{BKS}}} + \frac{1}{\tau_d} \qquad \text{and} \qquad P_\infty \simeq \frac{\tau_p}{\tau_{\mathrm{BKS}}} P_{\mathrm{BKS}}$$

where P_{BKS} and τ_{BKS} are the Baier, Katkov and Strakhovenko generalizations of Sokolov-Ternov quantities, namely

$$P_{\text{BKS}} = P_{\text{ST}} \frac{\oint ds \frac{1}{|\rho|^3} \hat{b} \cdot \hat{n}_0}{\oint ds \frac{1}{|\rho|^3} \left[1 - \frac{2}{9} (\hat{n}_0 \cdot \hat{v})^2 \right]}$$

and

$$\tau_{\text{BKS}}^{-1} = P_{\text{ST}} \frac{r_e \gamma^5 \hbar}{m_0 C} \oint < \frac{1}{|\rho|^3} \left[1 - \frac{2}{9} (\hat{n}_0 \cdot \hat{v})^2 \right]$$

SITROS evaluates τ_p from the fit of $P(t)$ and thus P_∞ from $\frac{\tau_p}{\tau_{\text{BKS}}} P_{\text{BKS}}$.

More recently E. Forest Polymorphic Tracking Code[16] has been used for a modern implementation of this approach [17].

2. Measurements of radiative polarization

The very first observation of radiative polarization was in 1968 [18] at ACO (Anneau de Collisions d'Orsay) which was in operation as e^+/e^- collider from 1965 to 1973. Polarization was measured by exploiting the spin dependent cross section of Coulomb scattering. Figure 1 [19] shows a schematic view of the experimental set-up and the measured loss rate which exhibits the expected behavior, namely losses decrease when polarization increases as expected from the Coulomb scattering cross section.
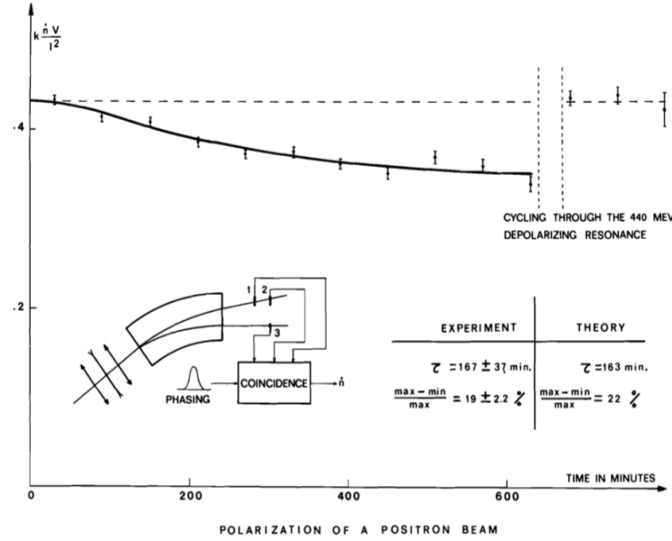

Figure 1: Experimental set-up for the ACO polarization measurement at 536 MeV (from [19]). Positrons exchanging upstreams 7% energy are separated by the following dipole and intercepted by 2 set of counters. Losses decrease when polarization increases.

Polarization was observed in 1972 also at Novosibirsk storage ring VEPP-2 [20]. Polarization close to 92% was later measured again at ACO [21] and at VEPP-2M [22] at 500 MeV. Figure 2

(from [23]) shows one polarization build-up measurement at VEPP-2M. The measurement set-up was similar to the one used at Orsay.
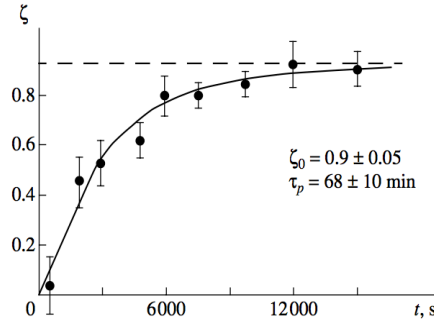

Figure 2: Polarization buildup in VEPP-2M (from [23]).

At VEPP-2M the method of beam energy measurement by resonant depolarization was invented [24]. A time varying longitudinal magnetic field was used for depolarizing the beam. Depolarization occurs when spin precession and field frequency are in resonance

$$\nu_{spin} = \frac{f_{exc}}{f_{rev}} + k$$

Owing to $\nu_{spin} = a\gamma$ (in a planar ring, w/o solenoids) the beam energy is given by $E_{beam} = \left[k \pm \frac{f_{exc}}{f_{rev}} \right] \frac{E_0}{a}$.

Large transverse beam polarization was measured later at 3.7 GeV in SPEAR (SLAC) and at 5 GeV in VEPP-4 (Novosibirsk) and DORIS (DESY).

At SLAC the first Compton polarimeter was build for SPEAR2 [25] which allowed more precise and faster polarization measurement wrt the loss rate method. It was based on the spin dependent cross section of Compton scattering of circularly polarized photons from a laser on transversely polarized electrons and was first suggested by Baier and Khoze in 1969.

Figure 3 shows a comparison between measured and computed (by SLIM) polarization for SPEAR [26].

At low energy spin diffusion is small and high polarization could be achieved without having to resort to special corrections. At PETRA (DESY) at 15 GeV a dedicated correction [27] of $\delta\hat{n}_0$, the deviation of the periodic solution to the Thomas-BMT equation from the ideal one, was invented and successfully applied to improve polarization from 40% to 80%. Variations of the same technique, sometime called "harmonic spin matching", were applied later at LEP (CERN), TRISTAN (KEK) and HERAe for improving polarization. Figure 4 [28] summarizes the achieved transverse polarization in various storage rings and shows that spin diffusion becomes stronger at high energies.

At LEP polarization was used solely for precise energy calibrations, but it gave the opportunity for some very interesting observations like the effect of the terrestrial tides on the machine length and hence on the beam energy [29].
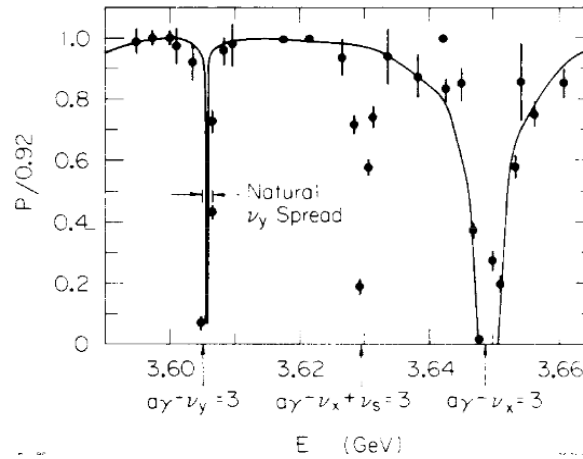
Figure 3: Polarization vs. beam energy at SPEAR (from [26]). Solid line: computation by SLIM; dots: measurements . The second order resonance is missed by the computation with linearized spin motion.
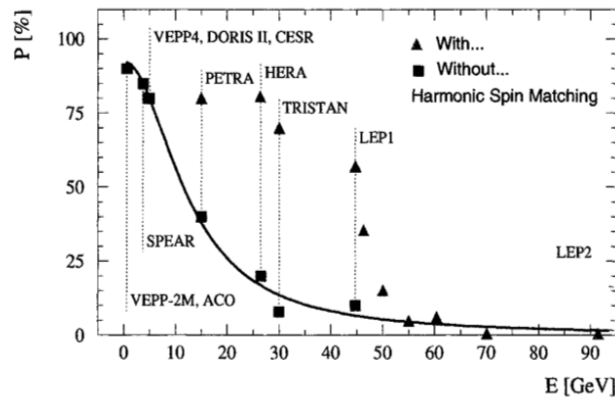

Figure 4: Achieved transverse radiative polarization in various storage rings (from [28]). At high energy $\delta \hat{n}_0$ correction is needed to achieve high polarization. At the very large energy achieved at LEP2 the correction was no more sufficient to get high polarization.

3. Radiative polarization at HERAe

HERA was a 6.3 km long $p/e^{\pm}$ collider operating at DESY from 1992 to 2007. Protons were accelerated in a superconducting ring from 40 GeV to 820 GeV and later to 920 GeV, while the $e^{\pm}$ ring was designed for accelerating leptons from 12 GeV to 30 GeV. In practice, however, the $e^{\pm}$ top energy was limited to about 27 GeV. The wish for a $p/e^{\pm}$ collider arose in the early 70s. Ideas based on CERN SPS and PETRA were abandoned in favor of a new facility to be build at DESY [30]. The $e^{\pm}$ ring was conceived from the beginning on for delivering longitudinal beam polarization which is an essential feature of an $p/e^{\pm}$ collider. For this reason spin rotators were integrated in the design and a large number of correctors, beam position monitors (BPMs) and

independently powered quadrupoles for "spin matching" (see later), was foreseen.

3.1 HERAe Mini-rotator and spin matching

For bringing polarization in the longitudinal direction at the Interaction Point (IP), while keeping it vertical in the machine arcs, a pair of 90^0 spin rotators is needed before and after the IP. The finally chosen scheme [31] consisted of a sequence of vertical and horizontal bending magnets and fulfilled the requirements of being easily integrable in the machine design, short enough not to require quadrupoles between rotator dipoles and flexible to work between 27 and 35 GeV. Spin helicity at the IP was flipped by reversing the sign of the vertical bending angles. The maximum vertical excursion being ± 20 cm (at 27 GeV), the magnets had to be mounted on remotely controlled jacks for adjusting their elevation. Energy changes larger than about ± 100 MeV were possible by manual machine re-alignment.

The presence of rotators breaks the spin transparency of an otherwise planar machine: spin diffusion originates in the quadrupoles between the rotator pair where $\hat{n}_0 \neq \hat{y}$; vertical dispersion, D_y, introduced by vertical bending magnets (or by solenoids, through coupling) leads to a finite vertical beam dimension which leads to spin diffusion in the quadrupoles of the whole ring.

Spin transparency for (linear) spin/orbit motion may be recovered by spin matching, which consists in designing the focusing structure so that spin and orbit motion are decoupled [32]. For the HERAe mini-rotator, assuming symmetric focusing around IP and around arc center, this adds 5 optics conditions which in terms of Twiss functions write

$$\int_{IP}^{s_R} ds\, K\sqrt{\beta_x}\cos\mu_x = 0 \qquad \int_{IP}^{s_R} ds\, \hat{n}_0 \cdot \hat{s}\, K\sqrt{\beta_y}\cos\mu_y = 0$$

$$\int_{ARCS} ds\, e^{i(\psi \pm \mu_y)} K\sqrt{\beta_y} = 0 \qquad \int_{IP}^{s_R} ds\, D_x K = 0$$

with s_R= rotator entrance and $\psi = \nu_s \times \phi_b$, $\phi_b = \phi_b(s)$ being the accumulated bending angle. After the HERA luminosity upgrade in 2000-2001, the interaction regions lost their symmetry wrt the IP and the number of spin matching constraints increased to 6.

3.2 Effect of misalignments and counter-measures

In a real machine spin transparency is also randomly broken by magnet misalignments. Usually the most dangerous for beam polarization is the vertical displacement of the quadrupoles which lead to a non-vanishing vertical dispersion and to $\delta\hat{n}_0 \neq 0$ all along the ring. At HERAe usual closed orbit correction was sufficient to correct also the spurious vertical dispersion. At LEP it was used a "dispersion free" correction aiming to correct closed orbit and spurious dispersion simultaneously.

The correction of $\hat{n}_0$ wrt the nominal direction required special care. The expected rms value of the quadrupole vertical displacement was 300 μm. Simulations showed that after correcting the vertical closed orbit down to about 0.7 mm it was $|\delta\hat{n}_0|_{rms} \approx$30 mrad and polarization very low. By introducing extra radial fields through the vertical correctors it is possible to compensate the effect of the displaced quadrupoles. In particular the harmonic expansion of $\delta\hat{n}_0$ reveals that the most

important harmonics of the "spin-orbit" function f [33] are those close to the spin tune

$$\delta\hat{n}_0(s) = -i\frac{L}{2\pi}\sum_k \frac{f_k}{k-\nu_s}e^{i2\pi ks/C}$$

and therefore $\delta\hat{n}_0(s)$ can be effectively corrected by minimizing those few harmonics. In HERAe up to 8 harmonics were corrected by using 8 closed orbit bumps, each exciting one harmonic at the time while keeping the other 7 unchanged. Figure 5 shows the results of a simulation of the effect of the "harmonic bumps" on HERAe polarization.
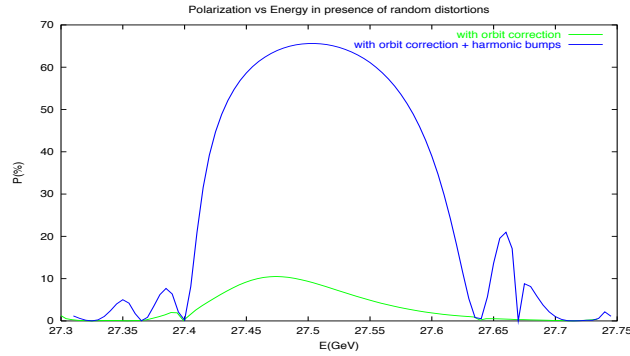

Figure 5: HERAe expected polarization before and after $\delta\hat{n}_0$ correction for a typical error realization.
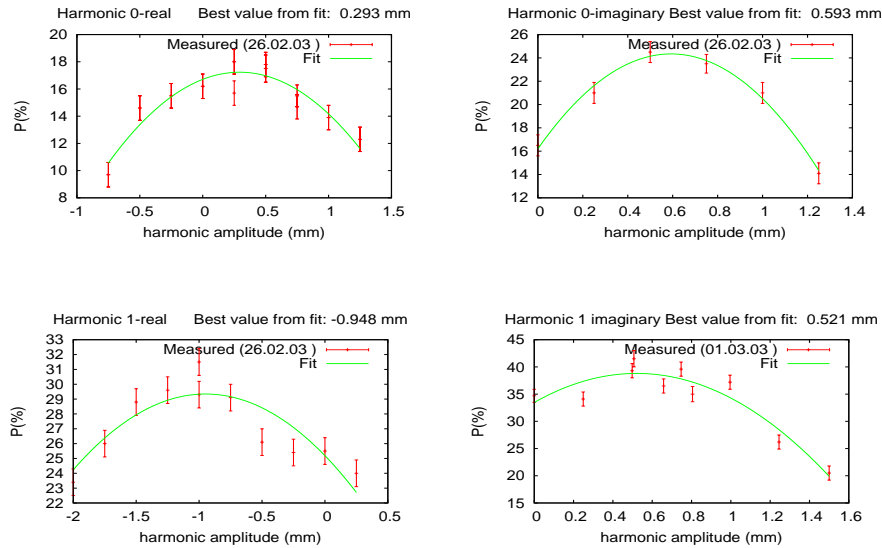

Figure 6: HERAe harmonics optimization in 2003 with 3 pairs of spin rotators.

In HERAe it was not possible to extract the harmonic content of $\delta\hat{n}_0$ from the BPMs reading. However, owing to the orthogonality of the components, an empirical optimization, by observing polarization rise/fall, was possible. Figure 6 shows polarization optimization through scanning of the $\delta\hat{n}_0$ 4 most important harmonics (2003 data).

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3.3 Beam parameters optimization

As spin diffusion is larger on the spin-orbit resonances, a key parameter for fostering high polarization is the choice of the working point. In general small fractional parts of the betatron tunes are convenient. HERAe operated with $q_x \simeq 0.12$ and $q_y \simeq 0.20$. The role of synchrotron tune is more intriguing (see [34], [35], [36], [37]), the spin tune being modulated by the the synchrotron motion.

Another important parameter is the betatron coupling which must be minimized (see for instance [38]). In HERAe there were no skew quadrupoles for a global betatron coupling correction and the observed beam ellipse tilt was corrected through vertical bumps in the arcs sextupoles. Although those bumps alone perturbed $\hat{n}_0$ and introduced some spurious vertical dispersion, polarization was improved.

As the spin tune may be not exactly known and resonances are not equally strong, a machine energy scan allows finding the best spot for polarization.

Once high polarization is established a meticulous book keeping may be needed to preserve it, especially if the machine reproducibility is poor: recording of golden orbits; tracking of beam energy changes due to different horizontal corrector settings; run-to-run tracking of harmonic bump settings; analysis of harmonic content of changed vertical corrector kicks.

3.4 HERAe polarization milestones

In November 1991 the Compton polarimeter in HERA West was brought into operation and $\simeq 10\%$ transverse beam polarization was observed at 26.66 GeV w/o dedicated optimizations [39]. In 1992 the collider experiments H1 and ZEUS started data taking; after re-alignment of some machine magnets and correction of the betatron coupling (by vertical bumps through the sextupoles), polarization increased to $\simeq 18\%$. In June 1992, after dedicated machine tuning (energy and harmonic bumps scans, tunes optimization) it was possible to establish 60% polarization on routine basis [40]. This success demonstrated the reliability of the polarization simulations and paved the way to the approval of the HERMES experiment to be hosted in the East straight section and to the installation of a pair of Buon-Steffens spin rotators during the 1993/1994 shut down for the experiment.

At the beginning of May 1994 the machine energy was increased to 27.5 GeV for operating the newly installed rotators. Dedicated polarization optimization with rotators still off brought 65% polarization. However, there was still some nervousness for eventual detrimental effects of the rotators on polarization. On May 4 the rotators were brought into operation and polarization reached 56% and, after some re-tuning, 65% [41]. This was the very first time that longitudinal polarization was achieved in a high energy storage ring.

Hermes started data taking in 1995 and, in 1996, the collaboration brought into operation a polarimeter measuring the longitudinal polarization [42]. The longitudinal polarimeter multi-photon mode allowed to measure single bunch polarization with high accuracy in real time providing valuable informations about the effect of the beam-beam interaction on polarization by comparing the polarization of colliding and non-colliding bunches [1].

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[1] a small number of non-colliding ("pilot") bunches was kept in each store to correct the luminosity monitor measurement for the bremsstrahlung of $e^{\pm}$ on the residual gas.

Average beam polarization vs. run number from 1996 to 2000 is shown in Figure 7. Despite the increasing beam-beam forces, it was possible to deliver high longitudinal polarization to HERMES. The bad performance in the first half of 1998 was due to lack of time for systematic optimizations.
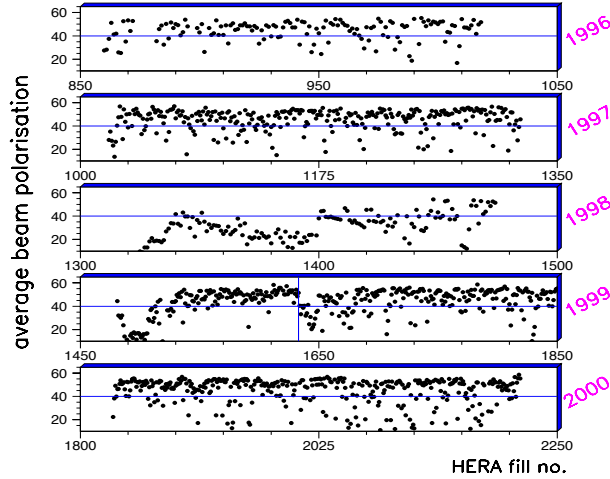

Figure 7: Average longitudinal polarization delivered to HERMES until 2000 (E.C. Aschenauer courtesy).

In 2000/2001 the machine underwent an important luminosity upgrade which involved a re-building of the Interaction Regions (IRs). The luminosity upgrade design was of course conceived to maximize the luminosity yield, but wasn't easy on polarization. The compensating solenoids were removed for lack of space. Two more pairs of spin rotators were installed for the collider experiments. The residual $\delta\hat{n}_0$ due to the H1 solenoid overlapping with a machine dipole, was corrected with special settings of the H1 spin rotators. The betatron coupling due to the solenoids was locally corrected by newly installed skew quadrupoles. When in 2001 the machine was brought back into operation, large closed orbit drifts were observed from run to run as well as within the same run. For compensating those drifts, imputable to the weaker magnet supports of the new IR magnets, an orbit feed-back was needed, which effect on polarization was unclear. In February 2003 all 3 rotators pairs were brought into operation and the e^+ beam parameters optimized, first with experiment solenoids turned off and later with them on. Polarization reached 54% to be compared with a level of 54-60% expected from non-linear tracking w/o beam-beam forces. Test luminosity was established and polarization reached 50% w/o further corrections. Details are found in [43],[44].

The main concern of the luminosity upgrade for polarization was the increased beam-beam force on the $e^\pm$ beam. A proton bunch colliding with a $e^\pm$ bunch acts as a non-linear lenses introducing a shift and spread of the tunes, the effect being larger when the proton bunch intensity increases. The extra lenses is missing for the pilot bunches which therefore have different betatron tunes. As a consequence the colliding bunches may lay on a spin-orbit resonance while the non-colliding are not and the other way around. In addition there is a difference between the effect on e^- and on e^+ because the proton lenses is focusing for the former and defocusing for the latter. The focusing beam-beam force pushed the e^- tunes upwards, while for polarization value close to

an integer are more convenient. Compensating for this effect on the bunch core would had lead to a poor lifetime of the pilot bunches and of the large amplitude particles. The situation was better with e^+ for which the proton lenses was defocusing. In June 2005 "mirror tunes" (i.e. just below the integer) were tried with e^-. They indeed improved polarization, but at expenses of luminosity and therefore became never operational[45].

4. Summary

We have reviewed the milestones of Radiative Polarization in storage rings from the theoretical predictions (by Korovina, Sokolov, Ternov et al.) and the very first observations at ACO (1968), to the unique experience of the HERA $p/e^\pm$ collider where radiative longitudinal $e^\pm$ beam polarization was for the first time achieved and delivered to the experiments on a routine basis.

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