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Anomaly and Polarisation in Heavy-Ion Collisions

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The role of axial anomaly in the generation of hyperons polarisation is discussed. The appearance of vorticit6y as gauge field strength and chemical potential as a coupling constant provide the robust exlanation of the energy dependence of polarisation related to induced quark axial current. The transition from quarks to hadrons is controlled by quark-hadfron duality, quantified in terms of axial charge or quantized vortices in pionic superfluid. The polarisation of antibaryons is expected to be of the same size and larger magnitude than that of baryons. The interplay with the thermodynamoc approach to polarisation is studied by calculating the induced axial current in Wigner function approach and is attributed to depndence on fermion mass.

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1. Introduction

The pioneering results on global polarization of Λ and $\bar{\Lambda}$ hyperons in $Au-Au$ collisions in Beam Energy Scan at RHIC reveal qualitative tendency of polarization decrease with energy in agreement with the prediction [1] based on axial anomaly/ Here we address this issue and explore the relevant details of theoretical description.

2. Axial anomaly and hyperon polarization

We explore the mechanism of generation of polarisation [1, 2] related to axial current and famous axial anomaly. In the medium described by chemical potential $\mu(x)$ there is a contribution to the interaction lagrangian [3] proportional to the appropriate conserved charge density in the medium rest frame $\rho(x) = j_0(x)$:

$$\Delta L(x) = \mu(x)\rho(x).$$

The Lorentz covariance allows one to transform this expression using the hydrodynamical four-velocity $u_\alpha = \gamma(1, \vec{v})$ where γ is the Lorentz factor:

$$\Delta L(x) = \mu(x)u^\alpha(x)j_\alpha(x).$$

Here, the velocity $u_\alpha(x)$ and the chemical potential $\mu(x)$ play the role of the gauge field $A(x)$ and the corresponding coupling g, respectively:

$$gA^\beta(x)j_\beta(x) \to \mu(x)u^\alpha(x)j_\alpha(x) \tag{2.1}$$

This substitution can be applied to any diagram with the lines of external (classical) gauge fields leading to various medium effects. In the case of famous anomalous triangle diagram it leads to the induced (classical) axial current.

As soon as in the $SU(3)$ wave function of u and d quarks form the spin singlet, we assume that Λ spin is carried predominantly by the strange quark. Let us therefore consider the classical strange axial charge [4] being the way to implement the quark-hadron duality in this particular case

$$Q_5^s = \frac{N_c}{2\pi^2}\int d^3x\,\mu_s^2(x)\gamma^2\varepsilon^{ijk}v_i\partial_j v_k, \tag{2.2}$$

iwhere N_c is the number of colors.

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c\int d^3x\,c_V\gamma^2\varepsilon^{ijk}v_i\partial_j v_k \tag{2.3}$$

since we assume that the chiral chemical potential μ_A is much smaller than the strange one μ_s. As to the temperature dependent term in eq. (2.3) related to gravitational anomaly [5], naively it can be quite substantial. However, lattice simulations [6] lead to the zero result in the confined phase and to the suppression by one order of magnitude at high temperatures. As soon as for free fermion gas the $T^2/6$ term is recovered [7] for large lattice volume at fixed temperature, the above-mentioned suppression should be attributed to the correlation effects.

In order to relate the strange axial charge Q_5^s (2.2) to the hydrodynamical quantities one can use the mean-value theorem to evaluate it [4]:

$$Q_5^s = \frac{< \mu^2 \gamma^2 > N_c H}{2\pi^2}, \tag{2.4}$$

where the hydrodynamical helicity

$$H \equiv \int d^3x (\vec{v} \cdot \vec{w})$$

is the integrated projection of the velocity $\vec{v}$ to the vorticity $\vec{w} = curl\vec{v}$.

Another, complementary, description of quark-hadron duality for axial anomaly may be achieved in terms of quantized vortices in pionic superfluid [9]:

$$j_5^\alpha \;=\; \frac{1}{4\pi^2} \varepsilon^{\alpha\beta\gamma\delta} \partial_\beta \phi \partial_\gamma \partial_\delta \phi \quad , \tag{2.5}$$

where the phase $\phi \;\equiv\; \pi^0/f_\pi$. Then the current induced by a given vortex, with the axis of rotation looking in the z-direction is given by:

$$\left(j_5^z \right)_{vortex} \;=\; \frac{\mu_5^I \kappa}{2\pi}\, \delta(x,y), \tag{2.6}$$

where κ is integer.

The appearance of μ in the second power, related to the positive C-parity of axial current, immediately leads to the same expressions for axial charge of strange quarks and antiquarks. As soon as there are the less number of $\bar{\Lambda}$s than the number of Λs, so that the same axial charge should be distributed among smaller number of antiquarks comparing with the number of quarks, which results in increase of the effect for the latter. Thus, one could expect that the polarization of $\bar{\Lambda}$ has to be of the same sign but a larger magnitude than the polarization of Λ, which is compatible with the quite recent STAR data. This effect is partly compensated by the fact, that a larger amount of axial charge in the case of strange antiquarks might be carried by more numerous K^*-mesons [10].

As soon as the strange chemical potential is rapidly decreasing with energy, this provides a robust explanation of the observed decrease of polarization with energy. More accurate measurements of Λ and $\bar{\Lambda}$ polarization at RHIC, and at NICA and FAIR in future, might allow one to test the suppression of T^2 term and, at best, even to check experimentally the magnitude of its theoretically predicted coefficient and search for possible or complementary [8] contributions which will be discussed in the next section.

3. Interplay between anomalous and thermodynamical approaches to baryon polarisation

The interplay with the anomalous and statistical approaches may be studied by calculation of axial current using the statistical methods [11, 12], based on the Wigner function approach pioneered by our Chairman [13] which is expressed in terms of the distribution function $X(x,p)$ in the form of a modified Fermi-Dirac distribution

$$X(x,p) = \Big(\exp[\beta_\mu p^\mu - \zeta] \exp\Big[-\frac{1}{2} \varpi_{\mu\nu} \Sigma^{\mu\nu} \Big] + I \Big)^{-1}, \tag{3.1}$$

where $\zeta = \frac{\mu}{T}$, $\varpi_{\mu\nu}$ is the thermal vorticity tensor, and $\Sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$. The mean value of various physical quantities can be found by integrating the trace of the operator of the considered quantity with the function $X(x, p)$ over the momentum space. Thus, for the axial current we have the following formula [13]

$$\langle j_\mu^5 \rangle = -\frac{1}{16\pi^3}\varepsilon_{\mu\alpha\nu\beta}\int \frac{d^3p}{\varepsilon} p^\alpha \Big\{ \mathrm{tr}\big(X\Sigma^{\nu\beta}\big) - \mathrm{tr}\big(\bar{X}\Sigma^{\nu\beta}\big) \Big\}, \tag{3.2}$$

where $\langle\cdot\rangle$ gives a statistical averaging with normal ordering, $\bar{X}$ describes the contribution of the antiparticles and differs from (3.1) in sign of ζ and ϖ. The matrix traces in (3.2) were exactly found in [11] in formula (4.3)

$$\mathrm{tr}\big(X\Sigma^{\nu\beta}\big) = \Big\{ \big(\exp\big[(\beta p) - \zeta - \frac{g_\omega}{2T} + i\frac{g_a}{2T}\big] + 1\big)^{-1} - \big(\exp\big[(\beta p) - \zeta + \frac{g_\omega}{2T} - i\frac{g_a}{2T}\big] + 1\big)^{-1} \Big\}$$
$$\frac{T}{2(g_\omega - ig_a)}[\varpi^{\nu\beta} - i\,\mathrm{sgn}(\varpi_{\mu\alpha}\widetilde{\varpi}^{\mu\alpha})\widetilde{\varpi}^{\nu\beta}] + c.c., \tag{3.3}$$

where $\widetilde{\varpi}^{\nu\beta}$ is the tensor dual to $\varpi^{\nu\beta}$, while g_ω and g_a are scalar quantities that depend on acceleration $a^\mu = u^\nu\partial_\nu u^\mu$ and vorticity $\omega_\mu = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha u^\beta$

$$g_\omega = \frac{1}{\sqrt{2}}\Big(\sqrt{(a^2-\omega^2)^2 + 4(\omega a)^2} + a^2 - \omega^2\Big)^{1/2},$$
$$g_a = \frac{1}{\sqrt{2}}\Big(\sqrt{(a^2-\omega^2)^2 + 4(\omega a)^2} - a^2 + \omega^2\Big)^{1/2}. \tag{3.4}$$

Substituting (3.3) into (3.2), we obtain

$$\langle j_\mu^5 \rangle = \frac{\omega_\mu + i\,\mathrm{sgn}(\omega a)a_\mu}{2(g_\omega - ig_a)}\int \frac{d^3p}{(2\pi)^3}\Big\{ n_F(E_p - \mu - g_\omega/2 + ig_a/2) -$$
$$n_F(E_p - \mu + g_\omega/2 - ig_a/2) + n_F(E_p + \mu - g_\omega/2 + ig_a/2) -$$
$$n_F(E_p + \mu + g_\omega/2 - ig_a/2)\Big\} + c.c., \tag{3.5}$$

which is another form of formula (4.6) from [11]. Here $n_F(E) = (e^{E/T}+1)^{-1}$ is the Fermi distribution, $a^\mu = u^\nu\partial_\nu u^\mu$ and $\omega_\mu = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha u^\beta$ - 4-acceleration and vorticity.

It is useful to consider a particular case by going into the comoving reference system and assuming that $\vec{\Omega}\,||\,\vec{a}$, that is, the acceleration occurs along the rotation axis. Then $g_\omega = \Omega$, $g_a = a$, where Ω and a are the modulus of three dimensional rotational speed and acceleration in the comoving frame, and (3.5) leads to

$$\langle \mathbf{j}^5 \rangle = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\Big\{ n_F(E_p - \mu - \frac{\Omega}{2} + i\frac{a}{2}) - n_F(E_p - \mu + \frac{\Omega}{2} + i\frac{a}{2}) +$$
$$n_F(E_p + \mu - \frac{\Omega}{2} + i\frac{a}{2}) - n_F(E_p + \mu + \frac{\Omega}{2} + i\frac{a}{2}) + c.c.\Big\}\,\mathbf{e}_\Omega, \tag{3.6}$$

where $\mathbf{e}_\Omega = \frac{\blacksquare}{\Omega}$ is the unit vector in the direction of the rotation speed. Formula (3.6) is noteworthy in that in it Ω and a come in combination with the chemical potential. The coefficient $1/2$ may be interpreted due to equivalence principle providing the same angular velocity of spin and orbital

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momentum precession [14], so that spin precession is twice slower than in the case of usual magnetic field, leading, in particular, to the disappearance of zero mode in th gravitomagnetic field , so that the axial anomaly in gravitational field is proportional to the curvature rather than connection.

In the limit $m = 0$ for $T > \frac{g_a}{2\pi}$, one get

$$\langle j_\mu^5 \rangle = \left(\frac{1}{6} \left[T^2 + \frac{a^2 - \omega^2}{4\pi^2} \right] + \frac{\mu^2}{2\pi^2} \right) \omega_\mu + \frac{1}{12\pi^2} (\omega a)\, a_\mu \,. \tag{3.7}$$

The angular velocity and acceleration can be therefore interpreted as a special kind of real and imaginary chemical potentials, respectively. We note that the possibility of considering the rotational speed as a chemical potential was previously shown in the literature in [15].

The apperance of such combination can be also understood from the point of view of interplay of angular momentum J and boost K generators

$$N = J + iK$$

$$J\omega + aK = \Re((J + iK)(\omega - ia))$$

The imaginary unit in front of a leads to appearance of only its even powers in the expression for axial current Note, however that thermal vortical effect [16] combined with the equivalcence principle argument leading to relation $\nabla T/T \sim \mathbf{a}$ [3] leads to the appearance of the term linear term $\mathbf{a} \cdot \mathbf{r}$. One may expect that such a violation of the translation invariance would mean the appearance of the distance from the center of mass (charge) in the axial current. If so, the polarization for different regions of phase space might change the sign, which might be similar to the recently discovered quadrupole structure [17, 10].

The appearance of the expression for anomalous current in the chiral limit may signal that the diffetrence between anomalous and statistical approaches is actually the mass effect. For massive particles the statistical approach is applicable leading to the "flavour-blind" expression of universal sign and weak (for baryons, having the masses not too different from each other) dependence on mass only. At the same time, in the chiral limit (after averaging over momenta, leading to the expression for induced axial current) the anomalous result strongly dependent on quark structure is recovered.

4. Conclusions

The anomalous approach to baryon polarisation provides the robust description of its key features. Its interplay with statistical and hydrodynamical approaches still remains to be studied.

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