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Femtoscopic correlations of two identical particles with nonzero spin in the model of one-particle multipole sources

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The process of emission of two identical particles with nonzero spin S and different helicities in relativistic heavy-ion collisions is theoretically investigated within the model of one-particle multipole sources. Taking into account the unitarity of the finite rotation matrix and the symmetry relations for d–functions, the general expression for the probability of emission of two identical particles by two multipole sources with angular momentum J, averaged over the angular momentum projections and over the space-time dimensions of the multiple particle generation region, has been obtained. For the case of unpolarized particles, the additional averaging over helicities is performed and the formula for two-particle correlation function at sufficiently large 4-momentum difference q is derived. For particles with nonzero mass, this formula is considerably simplified in the case when the angle β between the particle momenta equals zero, and also in the case when $J = S$.

In addition, the special cases of emission of two unpolarized photons by dipole and quadrupole sources, and emission of two "left" neutrinos ("right" antineutrinos) by sources with arbitrary J have been also considered, and the respective explicit expressions for the correlation function are obtained.

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1. Probability of emission of two identical particles with nonzero spin by two multipole sources

In the framework of the model of independent sources [1] with the angular momentum J and the projections of angular momentum onto the coordinate axis z, equaling M and M', the amplitude of emission of two identical particles with the momentum $\mathbf{p}_1$, helicity λ_1 and momentum $\mathbf{p}_2$, helicity λ_2 has the following structure :

$$A_{MM'}(\mathbf{p}_1,\lambda_1;\mathbf{p}_2,\lambda_2) =$$

$$= D^{(J)}_{\lambda_1 M}(\mathbf{n}_1) D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\, e^{ip_1x_1}\, e^{ip_2x_2} + D^{(J)}_{\lambda_2 M}(\mathbf{n}_2) D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)\, e^{ip_1x_2}\, e^{ip_2x_1}, \qquad (1.1)$$

where x_1 and x_2 are the 4-dimensional space-time coordinates of two multipole sources; in doing so, $p_1x_1 = E_1t_1 - \mathbf{p}_1\mathbf{x}_1$, $p_2x_2 = E_2t_2 - \mathbf{p}_2\mathbf{x}_2$; the functions

$$D^{(J)}_{\lambda_1 M}(\mathbf{n}_1) = D^{(J)}_{\lambda_1 M}(0,\theta_1,\phi_1) = \left(d_y(0,\theta_1,\phi_1)\, e^{iM\phi_1}\right)_{\lambda_1 M},$$

$$D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2) = D^{(J)}_{\lambda_2 M'}(0,\theta_2,\phi_2) = \left(d_y(0,\theta_2,\phi_2)\, e^{iM'\phi_2}\right)_{\lambda_2 M'}, \qquad (1.2)$$

are elements of the finite rotation matrix corresponding to the angular momentum J, $\mathbf{n}_1 = \mathbf{p}_1/|\mathbf{p}_1|$, $\mathbf{n}_2 = \mathbf{p}_2/|\mathbf{p}_2|$, θ_1,θ_2 and ϕ_1,ϕ_2 – polar and azimuthal angles of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively .

Thus, in accordance with Eq. (1.1), the probability of emission of two identical particles with spin S, respective 4-momenta p_1, p_2 and helicities λ_1,λ_2 by two multipole sources with the angular momentum J and projections M, M' of angular momentum onto the axis z is described by the following expression :

$$W_{MM'}(p_1,\lambda_1;p_2,\lambda_2) = |\,D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)\,|^2\,|\,D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\,|^2 + |\,D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)\,|^2\,|\,D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)\,|^2 +$$

$$+\, 2\,(-1)^{2S}\,\mathrm{Re}\left(D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)\, D^{*(J)}_{\lambda_2 M}(\mathbf{n}_2)\, D^{*(J)}_{\lambda_1 M'}(\mathbf{n}_1)\, D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\right)\cos(qx), \qquad (1.3)$$

where $q = p_1 - p_2$ is the difference of 4-momenta of two identical particles and $x = x_1 - x_2$ is the difference of 4-coordinates of two one-particle multipole sources.

Now let us average the expression (1.3) over the angular momentum projections M, M' and over the space-time dimensions of the emission region. In doing so, we take into account that, due to the unitarity of the finite rotation matrix, the following relations hold :

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$$\sum_{M=-J}^{J} |\, D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)|^2 = \sum_{M'=-J}^{J} |\, D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)|^2 =$$

$$= \sum_{M=-J}^{J} |\, D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)|^2 = \sum_{M'=-J}^{J} |\, D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)|^2 = 1\,. \tag{1.4}$$

Let us remark also that, without losing generality, we may choose the coordinate axis z as lying in the plane of the particle momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, with the axis y being perpendicular to this plane. Then the azimuthal angles of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ will be equal to zero: $\phi_1 = \phi_2 = 0$, and the angle $\beta = \theta_1 - \theta_2$ will have the meaning of angle between the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$.

In doing so, once again due to the unitarity of the finite rotation matrix, we obtain :

$$\sum_{M=-J}^{J} D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)\, D^{*(J)}_{M\lambda_2}(\mathbf{n}_2) = \sum_{M=-J}^{J} \left(e^{-i\,J_y\theta_1}\right)_{\lambda_1 M} \left(e^{i\,J_y\theta_2}\right)_{M\lambda_2} =$$

$$= \left(e^{-i\,J_y\,(\theta_1-\theta_2)}\right)_{\lambda_1\lambda_2} = \left(d^{(J)}_y(\beta)\right)_{\lambda_1\lambda_2}; \tag{1.5}$$

$$\sum_{M'=-J}^{J} D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\, D^{*(J)}_{M'\lambda_1}(\mathbf{n}_1) = \sum_{M'=-J}^{J} \left(e^{-i\,J_y\theta_2}\right)_{\lambda_2 M'} \left(e^{i\,J_y\theta_1}\right)_{M'\lambda_1} =$$

$$= \left(e^{i\,J_y(\theta_1-\theta_2)}\right)_{\lambda_2\lambda_1} = \left(d^{(J)}_y(-\beta)\right)_{\lambda_2\lambda_1}. \tag{1.6}$$

Using the well-known symmetry relation [2] :

$$(d^{(J)}_y(\beta))_{\lambda_1\lambda_2} = (d^{(J)}_y(-\beta))_{\lambda_2\lambda_1},$$

we come finally to the following result for the averaged emission probability $\overline{W_{MM'}}$ (see also [3]) :

$$\overline{W_{MM'}}(p_1,\lambda_1;p_2,\lambda_2) = \frac{1}{(2J+1)^2}\left(2+2\,(d^{(J)}_{\lambda_1\lambda_2}(\beta))^2\,(-1)^{2S}\,\langle\cos(qx)\,\rangle\right). \tag{1.7}$$

Let us emphasize that the quantity $r = (d^{(J)}_{\lambda_1\lambda_2}(\beta))^2$ in Eq. (1.7) has the meaning of the degree of non-orthogonality (non-distinguishability) of particle states with different helicities with respect to the momenta, the angle between which equals $\beta = \theta_1 - \theta_2$: $\langle\lambda_1|\lambda_2\rangle \neq 0$.

2. Correlation function for two unpolarized particles in the model of one-particle multipole sources

If the emitted identical particles with the momenta $\mathbf{p}_1, \mathbf{p}_2$ are unpolarized, then – after averaging Eq. (1.7) over all the $(2S+1)$ values of helicity allowed at spin S – we obtain (see also [3]):

$$\overline{W}(q) = \left(2\,(2S+1)^2 + (-1)^{2S}\, 2 \sum_{\lambda_1=-S}^{S} \sum_{\lambda_2=-S}^{S} |\, d^{(J)}_{\lambda_1\lambda_2}(\beta)\,|^2 \,\langle \cos(qx)\,\rangle \right) \frac{1}{(2J+1)^2}\, \frac{1}{(2S+1)^2}\,. \tag{2.1}$$

At sufficiently large momentum differences q the correlation function, normalized by unity, will take the form [3] :

$$R(q) = 1 + \frac{(-1)^{2S}}{(2S+1)^2} \sum_{\lambda_1=-S}^{S} \sum_{\lambda_2=-S}^{S} |\, d^{(J)}_{\lambda_1\lambda_2}(\beta)\,|^2 \,\langle \cos(qx)\,\rangle\,. \tag{2.2}$$

In particular, if $\beta = 0$, then we have $d^{(J)}_{\lambda_1\lambda_2}(0) = \delta_{\lambda_1\lambda_2}$, and formula (2.2) is considerably simplified:

$$R(q) = 1 + (-1)^{2S}\, \frac{1}{2S+1}\, \langle \cos(qx)\,\rangle\,. \tag{2.3}$$

Besides, taking into account the unitarity of the matrix $d^{(J)}_{\lambda_1\lambda_2}(\beta)$, it is easy to see from Eq. (2.2) that at $J = S$ expression (2.3) for the correlation function is valid for any angles between the particle momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. Let us stress that Eq. (2.3) is related to particles with nonzero mass .

3. Special cases of pair correlations of two unpolarized photons and two neutrinos

Now let us consider, within the model of one-particle multipole sources, the emission of two unpolarized photons – here the particle mass equals zero, spin $S = 1$ and each of the helicities λ_1, λ_2 takes only two $(2S)$ values: -1 and 1, irrespective of the momentum direction .

For the case of dipole sources, the two-photon correlation function has the form [4] :

$$R(q) = 1 + \frac{1}{4}\left[(d^{(1)}_{11}(\beta))^2 + (d^{(1)}_{-1,1}(\beta))^2 + (d^{(1)}_{-1,-1}(\beta))^2 + (d^{(1)}_{1,-1}(\beta))^2 \right] \langle \cos(qx)\,\rangle\,. \tag{3.1}$$

Taking into account the equalities :

$$d^{(1)}_{11}(\beta) = d^{(1)}_{-1,-1}(\beta) = \frac{1+\cos\beta}{2}, \quad d^{(1)}_{1,-1}(\beta) = d^{(1)}_{-1,1}(\beta) = \frac{1-\cos\beta}{2}, \tag{3.2}$$

we find :

$$R(q) = 1 + \frac{1}{4}\,(1+\cos^2\beta)\,\langle\cos(qx)\,\rangle. \tag{3.3}$$

At very small angles between the photon momenta ($\beta \ll 1$) we obtain the simple expression :

$$R(q) = 1 + \frac{1}{2}\,\langle\cos(qx)\,\rangle. \tag{3.4}$$

For the case of quadrupole sources , the general formula for the two-photon correlation function is as follows :

$$R(q) = 1 + \frac{1}{4}\left[(d^{(2)}_{11}(\beta))^2 + (d^{(2)}_{-1,1}(\beta))^2 + (d^{(2)}_{-1,-1}(\beta))^2 + (d^{(2)}_{1,-1}(\beta))^2\right]\langle\cos(qx)\,\rangle\,. \tag{3.5}$$

So, using the equalities :

$$d^{(2)}_{11}(\beta) = d^{(2)}_{-1,-1}(\beta) = \frac{1+\cos\beta}{2}\,(2\cos\beta - 1), \tag{3.6}$$

$$d^{(2)}_{1,-1}(\beta) = d^{(2)}_{-1,1}(\beta) = \frac{1-\cos\beta}{2}\,(2\cos\beta + 1), \tag{3.7}$$

we find the correlation function of two unpolarized photons emitted by the quadrupole sources :

$$R(q) = 1 + \frac{1}{4}\left(4\cos^4\beta - 3\cos^2\beta + 1\right)\langle\cos(qx)\,\rangle\,. \tag{3.8}$$

At $\beta \approx 0$ Eq. (3.8) gives : $R(q) = 1 + \frac{1}{2}\,\langle\cos(qx)\,\rangle$, i.e. here we also obtain – just as in the case of dipole sources – the standard formula (3.4) corresponding to two directions of polarization for each of the photons [4] .

Finally, let us consider also the case of emission of two "left" neutrinos (two "right" antineutrinos), with helicity taking only one value $\lambda_1 = \lambda_2 = +\frac{1}{2}$. Here, the two-neutrino correlation function in the model of multipole sources is as follows :

$$R(q) = 1 - (d^{(J)}_{\frac{1}{2}\frac{1}{2}}(\beta))^2\,\langle\cos(qx)\,\rangle\,. \tag{3.9}$$

In particular, at $J = S = \frac{1}{2}$ we obtain the expression :

$$R(q) = 1 - \cos^2 \frac{\beta}{2} \langle \cos(qx) \rangle \, ; \tag{3.10}$$

in the limit $\beta \to 0$ Eq. (3.10) gives

$$R(q) = 1 - \langle \cos(qx) \rangle \, . \tag{3.11}$$

References

[1]M. I. Podgoretsky, EChAYa, **20**, 266 (1989) .

[2]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics. Nonrelativistic theory* (Nauka, Moscow, 1989), § 58 .

[3]V. V. Lyuboshitz and V. L. Lyuboshitz, in Proceedings of the XV Advanced Research Workshop on High Energy Spin Physics – DSPIN-13 (Dubna, October 8 - 12, 2013) , JINR **E1,2-2014-12** , Dubna, 2014, p. 83 .

[4]V. L. Lyuboshitz and M. I. Podgoretsky, Yad. Fiz. **58**, 33 (1995) [Phys. At. Nucl. **58**, 30 (1995)] .