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$\mathbf{O}(D,D)$ completion of the Einstein Field Equations

Jeong-Hyuck Park*

Department of Physics, Sogang University, 35 Baekbeom-ro, Mapo-gu, Seoul 04107, Korea
E-mail: park@sogang.ac.kr

Upon treating the whole closed-string massless NS-NS sector as stringy graviton fields, Double Field Theory may evolve into 'Stringy Gravity'. In terms of an $\mathbf{O}(D,D)$ covariant differential geometry beyond Riemann, we present the definitions of the off-shell conserved stringy Einstein curvature tensor and the on-shell conserved stringy Energy-Momentum tensor. Equating them, all the equations of motion of the massless sector are unified into a single expression, $G_{AB} = 8\pi G T_{AB}$, carrying $\mathbf{O}(D,D)$ vector indices, which we dub *the Einstein Double Field Equations.*

Proceeding based on a work [1] with Stephen Angus and Kyoungho Cho

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To the memory of Cornelius Sochichiu

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*Speaker.

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1. Core Idea

String theory may predict its own gravity rather than General Relativity. In GR, the metric is the only geometric and gravitational field, whereas in string theory the closed-string massless NS-NS sector comprises a skew-symmetric B-field and the string dilaton in addition to the Riemannian metric. $\mathbf{O}(D,D)$ T-duality rotations transform them into each other [2, 3]. This hints at a natural augmentation of GR: upon treating the whole closed-string massless NS-NS sector as stringy graviton fields, Double Field Theory (DFT) [4–8] may evolve into Stringy Gravity. In terms of an $\mathbf{O}(D,D)$ covariant stringy differential geometry beyond Riemann, or the so-called semi-covariant formalism [9, 10], we present the definitions of the off-shell conserved stringy Einstein curvature tensor [11] and the on-shell conserved stringy Energy-Momentum tensor [1]. Equating them as prescribed by the action principle of DFT coupled to generic matter, all the equations of motion of the closed string massless NS-NS sector are unified into a single expression,

$$G_{AB} = 8\pi G T_{AB}\,, \tag{1.1}$$

which carry $\mathbf{O}(D,D)$ vector indices. As they correspond to the $\mathbf{O}(D,D)$ completion of the (undoubled) Einstein Field Equations, we dub them *the Einstein Double Field Equations* [1].

2. DFT as Stringy Gravity – *Essential Constituents*

- **Built-in symmetries & Notation:**

– $\mathbf{O}(D,D)$ T-duality

– DFT diffeomorphisms (ordinary diffeomorphisms plus B-field gauge symmetry)

– Twofold local Lorentz symmetries, $\mathbf{Spin}(1,D{-}1)\times\mathbf{Spin}(D{-}1,1)$

$\Rightarrow$ Two locally inertial frames exist separately for the left and the right modes.

Index	Representation	Metric (raising/lowering indices)
$A,B,\cdots,M,N,\cdots$	$\mathbf{O}(D,D)$ vector	$\mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$p,q,\cdots$	$\mathbf{Spin}(1,D{-}1)$ vector	$\eta_{pq} = \mathrm{diag}(-++\cdots+)$
$\alpha,\beta,\cdots$	$\mathbf{Spin}(1,D{-}1)$ spinor	$C_{\alpha\beta}, \quad (\gamma^p)^T = C\gamma^p C^{-1}$
$\bar{p},\bar{q},\cdots$	$\mathbf{Spin}(D{-}1,1)$ vector	$\bar{\eta}_{\bar{p}\bar{q}} = \mathrm{diag}(+--\cdots-)$
$\bar{\alpha},\bar{\beta},\cdots$	$\mathbf{Spin}(D{-}1,1)$ spinor	$\bar{C}_{\bar{\alpha}\bar{\beta}}, \quad (\bar{\gamma}^{\bar{p}})^T = \bar{C}\bar{\gamma}^{\bar{p}}\bar{C}^{-1}$

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The $\mathbf{O}(D,D)$ metric $\mathcal{J}_{AB}$ divides doubled coordinates into two: $x^A = (\tilde{x}_\mu, x^\nu), \partial_A = (\tilde{\partial}^\mu, \partial_\nu)$.

• Doubled-yet-gauged spacetime:
The doubled coordinates are *gauged* through a certain equivalence relation [12],

$$x^A \sim x^A + \Delta^A\,, \qquad\qquad \Delta^A = \Phi\partial^A\Psi\,,$$

where, with $\partial^A = \mathcal{J}^{AB}\partial_B$, Δ^A is derivative-index-valued for arbitrary functions, Φ, Ψ, appearing in DFT. Each equivalence class, or gauge orbit in $\mathbb{R}^{D+D}$, then corresponds to a single physical point in $\mathbb{R}^D$. This implies, and also is implied by, a section condition,

$$\partial_A\partial^A = 0\,,$$

which can be conveniently solved by switching off the tilde-coordinate dependence, *i.e.* $\tilde{\partial}^\mu \equiv 0$.

In fact, if we gauge the infinitesimal coordinate one-form, $\mathrm{d}x^A$, explicitly introducing a derivative-index-valued auxiliary gauge potential,

$$\mathrm{d}x^A \quad\longrightarrow\quad \mathrm{D}x^A = \mathrm{d}x^A - \mathcal{A}^A\,, \qquad\qquad \mathcal{A}^A\partial_A = 0\,,$$

it is possible to define an $\mathbf{O}(D,D)$ and DFT-diffeomorphism covariant 'proper length' in the doubled space through a path integral [13], and accordingly string worldsheet actions which are fully covariant with respect to symmetries like $\mathbf{O}(D,D)$ T-duality, Weyl symmetry, target as well as worldsheet diffeomorphisms [14–16] (*c.f.* [17–22]), and κ-symmetry [23] (3.2).

• Stringy graviton fields (closed-string massless NS-NS sector) as represented by $\{d, V_{Mp}, \bar{V}_{N\bar{q}}\}$:
The defining properties of the DFT metric are

$$\mathcal{H}_{MN} = \mathcal{H}_{NM}\,, \qquad\qquad \mathcal{H}_K{}^L\mathcal{H}_M{}^N\mathcal{J}_{LN} = \mathcal{J}_{KM}\,, \tag{2.1}$$

from which one can set a pair of symmetric and orthogonal projectors,

$$\begin{array}{lll} P_{MN} = P_{NM} = \frac{1}{2}(\mathcal{J}_{MN} + \mathcal{H}_{MN})\,, & P_L{}^MP_M{}^N = P_L{}^N\,, & \\ \bar{P}_{MN} = \bar{P}_{NM} = \frac{1}{2}(\mathcal{J}_{MN} - \mathcal{H}_{MN})\,, & \bar{P}_L{}^M\bar{P}_M{}^N = \bar{P}_L{}^N\,, & P_L{}^M\bar{P}_M{}^N = 0\,. \end{array}$$

Taking the "square roots" of the projectors, we acquire a pair of DFT vielbeins,

$$P_{MN} = V_M{}^pV_N{}^q\eta_{pq}\,, \qquad \bar{P}_{MN} = \bar{V}_M{}^{\bar{p}}\bar{V}_N{}^{\bar{q}}\bar{\eta}_{\bar{p}\bar{q}}\,,$$

satisfying their own defining properties,

$$V_{Mp}V^M{}_q = \eta_{pq}\,, \qquad\qquad \bar{V}_{M\bar{p}}\bar{V}^M{}_{\bar{q}} = \bar{\eta}_{\bar{p}\bar{q}}\,, \qquad\qquad V_{Mp}\bar{V}^M{}_{\bar{q}} = 0\,,$$

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which are — as the left inverse of a matrix coincides with the right inverse — equivalent to

$$V_M{}^p V_{Np} + \bar{V}_M{}^{\bar{p}} \bar{V}_{N\bar{p}} = \mathcal{J}_{MN}\,.$$

The most general solutions to (2.1) can be classified by two non-negative integers $(n,\bar{n})$ [24],

$$\mathcal{H}_{MN} = \begin{pmatrix} H^{\mu\nu} & -H^{\mu\sigma}B_{\sigma\lambda} + Y_i^\mu X_\lambda^i - \bar{Y}_{\bar{\imath}}^\mu \bar{X}_\lambda^{\bar{\imath}} \\ B_{\kappa\rho}H^{\rho\nu} + X_\kappa^i Y_i^\nu - \bar{X}_\kappa^{\bar{\imath}} \bar{Y}_{\bar{\imath}}^\nu & K_{\kappa\lambda} - B_{\kappa\rho}H^{\rho\sigma}B_{\sigma\lambda} + 2X_{(\kappa}^i B_{\lambda)\rho} Y_i^\rho - 2\bar{X}_{(\kappa}^{\bar{\imath}} B_{\lambda)\rho} \bar{Y}_{\bar{\imath}}^\rho \end{pmatrix}, \tag{2.2}$$

where $1 \le i \le n$, $1 \le \bar{\imath} \le \bar{n}$ and

$$H^{\mu\nu}X_\nu^i = 0\,,\quad H^{\mu\nu}\bar{X}_\nu^{\bar{\imath}} = 0\,,\quad K_{\mu\nu}Y_i^\nu = 0\,,\quad K_{\mu\nu}\bar{Y}_{\bar{\imath}}^\nu = 0\,,\quad H^{\mu\rho}K_{\rho\nu} + Y_i^\mu X_\nu^i + \bar{Y}_{\bar{\imath}}^\mu \bar{X}_\nu^{\bar{\imath}} = \delta^\mu{}_\nu\,.$$

The corresponding coset is, with $D = t+s+n+\bar{n}$,

$$\frac{\mathbf{O}(D,D)}{\mathbf{O}(t+n,s+n)\times\mathbf{O}(s+\bar{n},t+\bar{n})}\,,$$

which has the dimension, $D^2 - (n-\bar{n})^2$ [25], while $\mathcal{H}_M{}^M = 2(n-\bar{n})$ is $\mathbf{O}(D,D)$ invariant.

Upon the generic $(n,\bar{n})$ background, strings become chiral and anti-chiral over the n and $\bar{n}$ directions:

$$X_\mu^i \partial_+ x^\mu = 0\,, \qquad \bar{X}_\mu^{\bar{\imath}} \partial_- x^\mu = 0\,.$$

Examples include Riemannian geometry as $(0,0)$ where $K_{\mu\nu} = g_{\mu\nu}$, $H^{\mu\nu} = g^{\mu\nu}$, Newton–Cartan gravity as $(1,0)$, Gomis–Ooguri or Newton–Cartan non-relativistic strings as $(1,1)$ [26–29], Carroll gravity as $(D-1,0)$, and Poisson–Lie dual $(1,1)$ backgrounds [30]. In particular, the extreme case of $(D,0)$ corresponds to the maximally non-Riemannian, perfectly $\mathbf{O}(D,D)$ symmetric, vacuum geometry of DFT, where the DFT metric coincides with the $\mathbf{O}(D,D)$ metric, $\mathcal{H}_{AB} = \mathcal{J}_{AB}$. Intriguingly then, the Riemannian as well as partially non-Riemannian, $n+\bar{n} < D$, spacetimes 'emerge' after spontaneously breaking the $\mathbf{O}(D,D)$ symmetry with the component fields in (2.2) interpreted as Goldstone bosons [27]. Furtheremore, the maximally non-Riemannian $(D,0)$ background does not allow any linear fluctuation: from the defining property (2.1), any linear fluctuation of the DFT metric must satisfy $\delta\mathcal{H}_A{}^B\mathcal{H}_B{}^C + \mathcal{H}_A{}^B\delta\mathcal{H}_B{}^C = 0$, and thus if $\mathcal{H}_{AB} = \mathcal{J}_{AB}$, we have $\delta\mathcal{H}_{AB} = 0$. Thus, taken as an internal space, it realizes a graviscalar-moduli-free Scherk–Schwarz twistable Kaluza–Klein reduction of DFT, in fact, to heterotic supergravity [31] .

• **Covariant derivative:**
The 'master' covariant derivative,

$$\mathcal{D}_A = \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A\,,$$

is characterized by compatibilities with the whole NS-NS sector,

$$\mathcal{D}_A d = 0\,, \qquad \mathcal{D}_A V_{Bp} = 0\,, \qquad \mathcal{D}_A \bar{V}_{B\bar{p}} = 0\,,$$

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as well as with the kinematical constant metrics,

$$\mathcal{D}_A \mathcal{J}_{BC} = 0\,, \qquad \mathcal{D}_A \eta_{pq} = 0\,, \qquad \mathcal{D}_A \bar{\eta}_{\bar{p}\bar{q}} = 0\,, \qquad \mathcal{D}_A C_{\alpha\beta} = 0\,, \qquad \mathcal{D}_A \bar{C}_{\bar{\alpha}\bar{\beta}} = 0\,.$$

The DFT-Christoffel symbols are [10]

$$\begin{array}{ll}\Gamma_{CAB} = & 2\left(P\partial_C P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^D\bar{P}_{B]}{}^E - P_{[A}{}^D P_{B]}{}^E\right)\partial_D P_{EC} \\ & -4\left(\frac{1}{P_M{}^M-1}P_{C[A}P_{B]}{}^D + \frac{1}{\bar{P}_M{}^M-1}\bar{P}_{C[A}\bar{P}_{B]}{}^D\right)\left(\partial_D d + (P\partial^E P\bar{P})_{[ED]}\right)\,,\end{array}$$

and the spin connections are

$$\Phi_{Apq} = V^B{}_p(\partial_A V_{Bq} + \Gamma_{AB}{}^C V_{Cq})\,, \qquad \bar{\Phi}_{A\bar{p}\bar{q}} = \bar{V}^B{}_{\bar{p}}(\partial_A \bar{V}_{B\bar{q}} + \Gamma_{AB}{}^C \bar{V}_{C\bar{q}})\,.$$

In Stringy Gravity there are no normal coordinates where Γ_{CAB} would vanish point-wise: the Equivalence Principle does not hold for strings, or extended objects. However, when the formalism is applied and restricted to the case of point particles, Γ_{CAB} reduces to the ordinary Christoffel symbols and the Equivalence Principle is restored.

• Scalar and 'Ricci' curvatures:
The semi-covariant Riemann curvature in Stringy Gravity is defined by

$$S_{ABCD} := \tfrac{1}{2}\left(R_{ABCD} + R_{CDAB} - \Gamma^E{}_{AB}\Gamma_{ECD}\right)\,,$$

where $R_{CDAB} = \partial_A\Gamma_{BCD} - \partial_B\Gamma_{ACD} + \Gamma_{ACE}\Gamma_B{}^E{}_D - \Gamma_{BCE}\Gamma_A{}^E{}_D$ (the "field strength" of Γ_{CAB}). The completely covariant 'Ricci' and scalar curvatures are, with $S_{AB} = S_{ACB}{}^C$,

$$S_{p\bar{q}} := V^A{}_p\bar{V}^B{}_{\bar{q}}S_{AB}\,, \qquad S_{(0)} := \left(P^{AC}P^{BD} - \bar{P}^{AC}\bar{P}^{CD}\right)S_{ABCD}\,.$$

• DFT minimally coupled to matter:
While $e^{-2d}S_{(0)}$ corresponds to the original DFT Lagrangian density [4, 6], or the 'pure' Stringy Gravity, the master covariant derivative fixes its minimal coupling to extra matter fields, *e.g.* type II $D = 10$ maximally supersymmetric DFT [32],

$$\mathcal{L}_{\mathrm{type\,II}} = e^{-2d}\left[\begin{array}{l}\frac{1}{8}S_{(0)} + \frac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}}) + i\bar{\rho}\mathcal{F}\rho' + i\bar{\psi}_{\bar{p}}\gamma_q\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^q + i\frac{1}{2}\bar{\rho}\gamma^p\mathcal{D}_p\rho - i\frac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho' \\ -i\bar{\psi}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho - i\frac{1}{2}\bar{\psi}^{\bar{p}}\gamma^q\mathcal{D}_q\psi_{\bar{p}} + i\bar{\psi}'^p\mathcal{D}_p\rho' + i\frac{1}{2}\bar{\psi}'^p\bar{\gamma}^{\bar{q}}\mathcal{D}_{\bar{q}}\psi'_p\end{array}\right], \tag{2.3}$$

or the Standard Model coupled to DFT [33],

$$\mathcal{L}_{\mathrm{SM}} = e^{-2d}\left[\begin{array}{l}\frac{1}{16\pi G_N}S_{(0)} + \sum_{\mathcal{V}}\mathrm{Tr}(\mathcal{F}_{p\bar{q}}\mathcal{F}^{p\bar{q}}) + \sum_{\psi}\bar{\psi}\gamma^a\mathcal{D}_a\psi + \sum_{\psi'}\bar{\psi}'\bar{\gamma}^{\bar{a}}\mathcal{D}_{\bar{a}}\psi' \\ -\mathcal{H}^{AB}(\mathcal{D}_A\phi)^\dagger\mathcal{D}_B\phi - V(\phi) + y_d\,\bar{q}\cdot\phi\,d + y_u\,\bar{q}\cdot\tilde{\phi}\,u + y_e\,\bar{l}'\cdot\phi\,e'\end{array}\right]. \tag{2.4}$$

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The former Lagrangian (2.3) was constructed to the full *i.e.* quartic order in fermions. It unifies IIA and IIB supergravities as well as “Gomis–Ooguri supergravity" as different solution sectors. The latter Lagrangian (2.4) may put quarks and leptons in two distinct spin group sectors, *i.e.* $\mathbf{Spin}(1,3)$ *vs.* $\mathbf{Spin}(3,1)$. Every single term in the above two Lagrangians is completely invariant with respect to the diffeomorphisms, twofold local Lorentz symmetries, and $\mathbf{O}(D,D)$ T-duality.

3. Derivation of the Einstein Double Field Equations

We consider a general action for Stringy Gravity (*i.e.* DFT) coupled to generic matter fields, Υ_a, for example (2.3), (2.4). The variation of the action gives

$$\begin{aligned}
&\delta\int e^{-2d}\left[\tfrac{1}{16\pi G}S_{(0)} + L_{\mathrm{matter}}\right]\\
=&\int e^{-2d}\left[\tfrac{1}{4\pi G}\bar{V}^{A\bar{q}}\delta V_A{}^p(S_{p\bar{q}} - 8\pi G K_{p\bar{q}}) - \tfrac{1}{8\pi G}\delta d(S_{(0)} - 8\pi G T_{(0)}) + \delta\Upsilon_a\frac{\delta L_{\mathrm{matter}}}{\delta\Upsilon_a}\right]\\
=&\int e^{-2d}\left[\tfrac{1}{8\pi G}\xi^B\mathscr{D}^A\{G_{AB} - 8\pi G T_{AB}\} + (\hat{\mathscr{L}}_\xi\Upsilon_a)\frac{\delta L_{\mathrm{matter}}}{\delta\Upsilon_a}\right],
\end{aligned}$$

where the second line is for generic variation and the third line is specifically for diffeomorphic transformation. While deriving the above, one is naturally led to define

$$K_{p\bar{q}} := \frac{1}{2}\left(V_{Ap}\frac{\delta L_{\mathrm{matter}}}{\delta\bar{V}_A{}^{\bar{q}}} - \bar{V}_{A\bar{q}}\frac{\delta L_{\mathrm{matter}}}{\delta V_A{}^p}\right), \qquad T_{(0)} := e^{2d}\times\frac{\delta\left(e^{-2d}L_{\mathrm{matter}}\right)}{\delta d},$$

and subsequently also the stringy *Einstein curvature*, G_{AB}, and *Energy-Momentum tensor*, T_{AB},

$$\begin{aligned}
G_{AB} &= 4V_{[A}{}^p\bar{V}_{B]}{}^{\bar{q}}S_{p\bar{q}} - \tfrac{1}{2}\mathscr{J}_{AB}S_{(0)}, && \mathscr{D}_A G^{AB} = 0 && (\text{off-shell}),\\
T_{AB} &:= 4V_{[A}{}^p\bar{V}_{B]}{}^{\bar{q}}K_{p\bar{q}} - \tfrac{1}{2}\mathscr{J}_{AB}T_{(0)}, && \mathscr{D}_A T^{AB} = 0 && (\text{on-shell}),
\end{aligned}$$

which satisfy $G_A{}^A = -DS_{(0)}$, $T_A{}^A = -DT_{(0)}$. Therefore, the equations of motion of the stringy graviton fields are unified into a single expression, *the Einstein Double Field Equations* (1.1).

Restricting to the $(0,0)$ Riemannian background, the Einstein Double Field Equations reduce to

$$\begin{aligned}
R_{\mu\nu} + 2\bigtriangledown_\mu(\partial_\nu\phi) - \tfrac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} &= 8\pi G K_{(\mu\nu)},\\
\bigtriangledown^\rho\left(e^{-2\phi}H_{\rho\mu\nu}\right) &= 16\pi G e^{-2\phi}K_{[\mu\nu]},\\
R + 4\Box\phi - 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} &= 8\pi G T_{(0)},
\end{aligned}$$

which imply the conservation law, $\mathscr{D}_A T^{AB} = 0$, now given explicitly by

$$\nabla^\mu K_{(\mu\nu)} - 2\partial^\mu\phi\, K_{(\mu\nu)} + \tfrac{1}{2}H_\nu{}^{\lambda\mu}K_{[\lambda\mu]} - \tfrac{1}{2}\partial_\nu T_{(0)} = 0, \qquad \nabla^\mu\left(e^{-2\phi}K_{[\mu\nu]}\right) = 0.$$

The Einstein Double Field Equations also govern the dynamics of other non-Riemannian cases, $(n,\bar{n}) \neq (0,0)$, where the invertible Riemannian metric, $g_{\mu\nu}$, cannot be defined.

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Examples

– *Pure Stringy Gravity with the* $\mathbf{O}(D,D)$ *invariant cosmological constant:*

$$\tfrac{1}{16\pi G}e^{-2d}\left(S_{(0)} - 2\Lambda_{\mathrm{DFT}}\right), \qquad K_{p\bar{q}} = 0\,, \qquad T_{(0)} = \tfrac{1}{4\pi G}\Lambda_{\mathrm{DFT}}\,.$$

– *RR sector, represented by a* $\mathbf{Spin}(1,9)\times\mathbf{Spin}(9,1)$ *bi-spinorial potential,* $\mathscr{C}^{\alpha}{}_{\bar{\alpha}}$ *:*

$$L_{\mathrm{RR}} = \tfrac{1}{2}\mathrm{Tr}\left(\mathscr{F}\bar{\mathscr{F}}\right), \qquad K_{p\bar{q}} = -\tfrac{1}{4}\mathrm{Tr}\left(\gamma_p\mathscr{F}\bar{\gamma}_{\bar{q}}\bar{\mathscr{F}}\right), \qquad T_{(0)} = 0\,,$$

where

$$\mathscr{F} = \mathscr{D}_+\mathscr{C} = \gamma^p\mathscr{D}_p\mathscr{C} + \gamma^{(11)}\mathscr{D}_{\bar{p}}\mathscr{C}\bar{\gamma}^{\bar{p}}\,,$$

which is the RR flux set by an $\mathbf{O}(D,D)$ covariant 'H-twisted' cohomology, $(\mathscr{D}_+)^2 = 0$, and $\bar{\mathscr{F}} = \bar{C}^{-1}\mathscr{F}^T C$ is its charge conjugate [32].

– *Scalar field :*

$$L_\Phi = -\tfrac{1}{2}\mathscr{H}^{MN}\partial_M\Phi\partial_N\Phi - \tfrac{1}{2}m_\Phi^2\Phi^2\,, \qquad K_{p\bar{q}} = \partial_p\Phi\partial_{\bar{q}}\Phi\,, \qquad T_{(0)} = -2L_\Phi\,.$$

– *Spinor field:*

$$L_\psi = \bar{\psi}\gamma^p\mathscr{D}_p\psi + m_\psi\bar{\psi}\psi\,, \qquad K_{p\bar{q}} = -\tfrac{1}{4}\left(\bar{\psi}\gamma_p\mathscr{D}_{\bar{q}}\psi - \mathscr{D}_{\bar{q}}\bar{\psi}\gamma_p\psi\right), \qquad T_{(0)} = 0\,.$$

– *Point particle:*

$$e^{-2d}L_{\mathrm{particle}} = \int \mathrm{d}\tau\,\left[e^{-1}D_\tau y^A D_\tau y^B\mathscr{H}_{AB}(x) - \tfrac{1}{4}m^2 e\right]\delta^D\big(x - y(\tau)\big)\,, \tag{3.1}$$

$$K_{p\bar{q}} = -\int \mathrm{d}\tau\; 2e^{-1}\left(D_\tau y^A V_{Ap}\right)\left(D_\tau y^B\bar{V}_{B\bar{q}}\right)e^{2d(x)}\,\delta^D\big(x - y(\tau)\big)\,, \qquad T_{(0)} = 0\,.$$

– *Green-Schwarz superstring (κ-symmetric) :*

$$e^{-2d}L_{\mathrm{string}} = \tfrac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\left[-\tfrac{1}{2}\sqrt{-h}h^{ij}\Pi_i^M\Pi_j^N\mathscr{H}_{MN} - \varepsilon^{ij}D_i y^M\left(\mathscr{A}_{jM} - i\Sigma_{jM}\right)\right]\delta^D\big(x - y(\sigma)\big)\,, \tag{3.2}$$

$$K_{p\bar{q}}(x) = \tfrac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{-h}h^{ij}\left(\Pi_i^M V_{Mp}\right)\left(\Pi_j^N\bar{V}_{N\bar{q}}\right)e^{2d}\,\delta^D\big(x - y(\sigma)\big)\,, \qquad T_{(0)} = 0\,,$$

where $\Sigma_i^M = \bar{\theta}\gamma^M\partial_i\theta + \bar{\theta}'\bar{\gamma}^M\partial_i\theta'$ and $\Pi_i^M = \partial_i y^M - \mathscr{A}_i^M - i\Sigma_i^M$ [23].

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4. DFT as Modified Gravity

As DFT evolves into Stringy Gravity, which appears to, at least conceptually, differ from General Relativity, it should be of interest, if not the duty of a physicist, to investigate how DFT as a graviational theory modifies GR. Since the stringy Energy-Momentum tensor has D^2+1 components, and this is certainly larger than $\frac{1}{2}D(D+1)$ which is the number of components in GR, it is natural to expect that the gravitational phenomena are richer in Stringy Gravity than in General Relativity. As a first step to verify this, henceforth we focus on the most general, static, spherically symmeric, asymtotically flat, Riemannian, regular 'star-like' solution to the $D=4$ Einstein Double Field Equations,

$$G_{AB} = \begin{cases} 8\pi G T_{AB} & \text{for} \quad r \le r_{\mathrm{c}} \quad \text{(inside the stringy star)} \\ 0 & \text{for} \quad r > r_{\mathrm{c}} \quad \text{(ourside)}\,. \end{cases} \tag{4.1}$$

Outside the stringy star, we have the spherical 'vacuum' geometry [34, 35],

$$\begin{array}{ll} e^{2\phi} = \gamma_{+}\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{b}{\sqrt{a^2+b^2}}} + \gamma_{-}\left(\frac{r+\beta}{r-\alpha}\right)^{\frac{b}{\sqrt{a^2+b^2}}}\,, & H_{(3)} = h \sin\vartheta\, \mathrm{d}t\wedge \mathrm{d}\vartheta \wedge \mathrm{d}\varphi\,, \\ \mathrm{d}s^2 = e^{2\phi}\left[-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}}\mathrm{d}t^2 + \left(\frac{r+\beta}{r-\alpha}\right)^{\frac{a}{\sqrt{a^2+b^2}}}\left\{\mathrm{d}r^2 + (r-\alpha)(r+\beta)\mathrm{d}\Omega^2\right\}\right]\,, & \end{array} \tag{4.2}$$

where a,b,h,α,β are constant parameters satisfying the constraint, $a^2+b^2 = (\alpha+\beta)^2$; we let $\gamma_{\pm} := \frac{1}{2}(1 \pm \sqrt{1-h^2/b^2})$; and $\mathrm{d}s^2$ is given in string frame. Thus there are four independent free parameters in the spherical vacuum geometry, in contrast to the Schwarzschild geometry which possesses only one free parameter, *i.e.* mass. The Einstein Double Field Equations (4.1) then determine — and hence reveal the physical meaning of — all of these "free" parameters in terms of the Energy-Momentum tensor inside the stringy star, for example,

$$a = \int_0^{r_{\mathrm{c}}} \mathrm{d}r \int_0^{\pi} \mathrm{d}\vartheta \int_0^{2\pi} \mathrm{d}\varphi\; e^{-2d}\left[\tfrac{1}{4\pi} H_{r\vartheta\varphi}H^{r\vartheta\varphi} + 2G\left(K_r{}^r + K_\vartheta{}^\vartheta + K_\varphi{}^\varphi - K_t{}^t - T_{(0)}\right)\right].$$

The $\mathbf{O}(D,D)$ symmetric doubled-yet-gauged particle action (3.1) implies that a point-like particle should follow a geodesic defined in string frame [35], rather than in Einstein frame.[1] In terms of the areal radius, R, which normalizes the angular part of the metric, $\mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + g_{RR}\mathrm{d}R^2 + R^2\mathrm{d}\Omega^2$, the orbital velocity of a point particle probe can be computed from

$$V_{\mathrm{orbit}} = \sqrt{R\,\frac{\mathrm{d}\Phi_{\mathrm{Newton}}}{\mathrm{d}R}}\,,$$

$$\Phi_{\mathrm{Newton}} = -\tfrac{1}{2}(1+g_{tt}) = -\frac{MG}{R} + \left(\frac{2b^2-h^2+2ab\sqrt{1-h^2/b^2}}{a^2+b^2-h^2+2ab\sqrt{1-h^2/b^2}}\right)\left(\frac{MG}{R}\right)^2 + \cdots\,,$$

[1]However, this is not an S-duality invariant statement. The author would like to thank Chris Hull for this remark. Our discussion is thus restricted to the implications of $\mathbf{O}(D,D)$ T-duality rather than S- or U-dualities.

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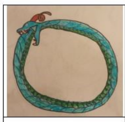
where the ellipses in (4) denote higher order terms in $\frac{MG}{R}$, and the mass is given by

$$MG = \tfrac{1}{2}\left(a + b\sqrt{1 - h^2/b^2}\right) = \int_0^\infty \mathrm{d}r \int_0^\pi \mathrm{d}\vartheta \int_0^{2\pi} \mathrm{d}\varphi \, e^{-2d}\left(-2GK_t{}^t + \tfrac{1}{8\pi}\left|H_{t\vartheta\varphi}H^{t\vartheta\varphi}\right|\right). \qquad (4.3)$$

Thus, in terms of the dimensionless radius, $R/(MG)$, normalized by the mass times the Newton constant, the orbital motion becomes Keplerian, *i.e.* $V_{\text{orbit}} \simeq \sqrt{\frac{MG}{R}}$, for large $R/(MG)$, while it is non-Keplerian for small $R/(MG)$. That is to say, Stringy Gravity modifies General Relativity at "short" dimensionless scales. In fact, depending on the parameters, the gravitational force can even be repulsive at "short" scale. This might shed new light upon the dark matter/energy problems, as they arise essentially from "short" dimensionless scale observations:

	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_\odot G$)	Milky Way (visible)	Galaxy Cluster	Universe ($M \propto R^3$)
$R/(MG)$	0^+	7.1×10^{38}	2.0×10^{43}	2.4×10^{26}	1.4×10^{9}	1.0×10^{8}	1.5×10^{6}	$\sim 10^{5}$	0^+

'Uroboros' spectrum of the dimensionless radial variable normazized by mass in natural units [13, 35]. The observations of stars and galaxies far away may reveal the short-distance nature of gravity. Repulsive gravitational force at short scale may explain the acceleration of the Universe.

Finally, we speculate that electric H-flux may be *dark matter*, since it contributes to the mass formula (4.3) while it decouples from point particles (3.1). We call for verification.

Dedication

I would like to dedicate this humble writing to the memory of Cornelius Sochichiu who has taught me how to balance life and physics until his last moment.

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