PROCEEDINGS OF SCIENCE

Soft corrections to inclusive DIS at four loops and beyond

Goutam Das*
Theory Group, Deutsches Elektronen-Synchrotron (DESY), D-22607 Hamburg, Germany
E-mail: goutam.das@desy.de

Sven-Olaf Moch
II. Institute for Theoretical Physics, Hamburg University, D-22761 Hamburg, Germany
E-mail: sven-olaf.moch@desy.de

Andreas Vogt
Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, United Kingdom
E-mail: andreas.vogt@liverpool.ac.uk

We study the threshold corrections to the structure functions in deep-inelastic scattering (DIS) at the fifth logarithmic (N^4LL) order of the soft-gluon exponentiation in massless perturbative QCD. Using recent results for the splitting functions and the quark form factor, we derive the fourth-order contribution to the coefficient $f^{\,q}$ of the form factor and from it the N^4LL part of the exponentiation coefficient $B^{\rm DIS}$ in the limit of a large number of colours. An approximation scheme is shown that leads to sufficiently accurate N^4LL results for full QCD. The N^4LL corrections are small and lead to a further stabilization of the perturbative expansion for the soft-gluon exponent.

XXVII International Workshop on Deep-Inelastic Scattering and Related Subjects - DIS2019
8-12 April, 2019, Torino, Italy

*Speaker.

PoS(DIS2019)010

1. Introduction

The Wilson coefficients (coefficient functions) for the structure functions of inclusive DIS have been a subject of research since the early days of QCD. These quantities are not only relevant for determining the parton distribution functions (PDFs) and the strong coupling constant α_s using structure function data, see, e.g., [1], but also to other processes and less inclusive observables in DIS, see, e.g., [2]. The main Wilson coefficients for DIS are presently known up to the third order in α_s in massless perturbative QCD [3]. Their perturbative expansion is well-behaved except close to the kinematic endpoints $x = 0$ and $x = 1$ of the Bjorken variable. The dominant terms $\ln^{\ell}(1-x)/(1-x)_+$ in the latter (threshold) limit are resummed by the soft-gluon exponentiation, see, e.g., [4–6], which is best formulated in Mellin N-space [4]. So far this resummation has been performed up to the next-to-next-to-next-to-leading logarithmic (N^{3}LL) accuracy [7].

The threshold resummation coefficients are closely related to the large-x limit of the quark-quark splitting functions for the PDFs and to the quark form factor [8,9] which are both fully known to order α_s^3 [10–13]. Recently the computations of these quantities have been extended to order α_s^4 in the (Ln_c) limit of a large number of colours [14, 15]. Together with approximate results for the n_f-independent [16] and exact expression for the n_f-dependent contributions to the cusp anomalous dimension in full QCD [17] these results facilitate the effective extension of the threshold resummation for the DIS Wilson coefficients to the next (N^{4}LL) logarithmic order. In the following we recall the theoretical framework, present the N^{4}LL resummation coefficient and briefly address the numerical implications of this result for the resummation of DIS in QCD.

2. Theoretical framework and new fourth-order coefficients

The all-order large-N behaviour of the DIS Wilson coefficients for F_1, F_2 and F_3 can be written as

$$C^N(Q^2) \;=\; g_0(Q^2)\cdot\exp\left[G^N(Q^2)\right] \;+\; \mathscr{O}(N^{-1}\ln^n N)\;, \tag{2.1}$$

where the resummation exponent G^N of the dominant $N^0\ln^n N$ contributions is given by [18]

$$G^N = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\,\left[\int_{\mu_f^2}^{(1-z)Q^2}\frac{dq^2}{q^2}\,A^{\mathrm{q}}\big(\alpha_s(q^2)\big) + B^{\mathrm{DIS}}\big(\alpha_s((1-z)q^2)\big)\right]. \tag{2.2}$$

Here A^{q} is the (light-like) quark cusp anomalous dimension and B^{DIS} is the resummation coefficient for DIS. Both have perturbative series, $A^{\mathrm{q}} = \sum_i a_s^i A_i^{\mathrm{q}}$ etc, in terms of the strong coupling which we normalize as $a_s \equiv \alpha_s/4\pi$. Performing the integrations one can organize the exponent as

$$G^N = \ln N\, g^{(1)}(\lambda) + g^{(2)}(\lambda) + a_s g^{(3)}(\lambda) + a_s^2 g^{(4)}(\lambda) + a_s^3 g^{(5)}(\lambda) + \ldots\,, \tag{2.3}$$

where $\lambda = \beta_0 a_s \ln N$ or $\lambda = \beta_0 a_s \ln \widetilde{N}$ with $\ln\widetilde{N} = \ln N + \gamma_e$. The first $n{+}1$ terms in (2.3) are required for the resummation at N^{n}LL accuracy. The N^{2}LL and N^{3}LL contributions to G^N have been derived in [7, 19]; explicit expressions can be found in (3.3) – (3.6) of [7]. The lengthy new function $g^{(5)}(\lambda)$ entering at N^{4}LL will be presented in [20]. The N-independent prefactor g_0 is presently known to order α_s^3 from the all-N calculation in [3], see (4.6) – (4.8) of [7],

$$g_0 = 1 + a_s g_{01} + a_s^2 g_{02} + a_s^3 g_{03} + \mathscr{O}(a_s^4)\,. \tag{2.4}$$

The resummation to N^{4}LL requires the terms up to $A_5^{\rm q}$ and $B_4^{\rm DIS}$ in their corresponding expansions. The impact of the former quantity, for which a first estimate has been obtained in [21], is very small. $B^{\rm DIS}$ can be calculated from knowledge of the quark form factor or the DIS Wilson coefficients. The form factor satisfies a differential equation which follows from the renormalization group and gauge invariance. Its solution can be found in terms of the cusp anomalous dimension $A^{\rm q}$ and the function $G^{\rm q}$ containing the quantity $f^{\rm q}$ related to a universal eikonal anomalous dimension and the coefficient $B^{\rm q}$ of $\delta(1-x)$ in the quark-quark splitting function. The four-loop coefficient of $G^{\rm q}$ (which appears in the $1/\varepsilon$ coefficient in the solution of the form factor) can be written as

$$G_4^{\rm q} = 2B_4^{\rm q} + f_4^{\rm q} + \beta_2 f_{01}^{\rm q} + \beta_1 f_{02}^{\rm q} + \beta_0 f_{03}^{\rm q} + \mathscr{O}(\varepsilon)\,, \tag{2.5}$$

where the quantities $f_{0\rm n}^{\rm q}$ are (combinations of) known lower-order coefficients of $G^{\rm q}$, see [12] and (20) of [9]. Hence $f_4^{\rm q}$ can be determined in the large-n_c limit from the results of [14, 15]. We find

$$\begin{aligned} f_4^{\rm q}\Big|_{{\rm L}n_c} = {}& C_F n_c^3 \left(\frac{9364079}{6561} - \frac{1186735}{729}\zeta_2 - \frac{837988}{243}\zeta_3 + \frac{115801}{27}\zeta_4 + \frac{11896}{9}\zeta_2\zeta_3 + 3952\,\zeta_5 \right. \\ & \left. - \frac{4796}{9}\zeta_3^2 - \frac{129547}{54}\zeta_6 - 416\,\zeta_2\zeta_5 - 720\,\zeta_3\zeta_4 - 1700\,\zeta_7 \right) + C_F n_c^2 n_f \left(-\frac{247315}{432} \right. \\ & \left. + \frac{412232}{729}\zeta_2 + \frac{102205}{243}\zeta_3 - \frac{7589}{6}\zeta_4 - \frac{824}{9}\zeta_2\zeta_3 - \frac{740}{9}\zeta_5 + \frac{2816}{9}\zeta_3^2 + \frac{15611}{27}\zeta_6 \right) \\ & + C_F n_c n_f^2 \left(\frac{329069}{17496} - \frac{22447}{729}\zeta_2 + \frac{25300}{243}\zeta_3 + \frac{140}{3}\zeta_4 - \frac{176}{9}\zeta_2\zeta_3 - \frac{856}{9}\zeta_5 \right) \\ & + C_F n_f^3 \left(-\frac{16160}{6561} - \frac{16}{81}\zeta_2 - \frac{400}{243}\zeta_3 + \frac{128}{27}\zeta_4 \right). \end{aligned} \tag{2.6}$$

The $a_{\rm s}^4$ contribution to resummation $B^{\rm DIS}$ reads, in terms of the genuine four-loop contributions $f_4^{\rm q}$ and $B_4^{\rm q}$, which are exactly known only in the Ln_c limit for now, and lower-order coefficients,

$$\begin{aligned} B_4^{\rm DIS} = {}& -f_4^{\rm q} - B_4^{\rm q} - \beta_2 \left(f_{01}^{\rm q} + g_{01} - \frac{1}{2}\zeta_2 A_1^{\rm q} \right) + \beta_0^3 \left(3\zeta_2 f_{01}^{\rm q} + 3\zeta_2 g_{01} + 2\zeta_3 f_1^{\rm q} + 2\zeta_3 B_1^{\rm q} \right. \\ & \left. + \frac{3}{2}\zeta_4 A_1^{\rm q} - \frac{3}{4}\zeta_2^2 A_1^{\rm q} \right) + \beta_0\beta_1 \left(\frac{5}{2}\zeta_2 f_1^{\rm q} + \frac{5}{2}\zeta_2 B_1^{\rm q} + \frac{5}{3}\zeta_3 A_1^{\rm q} \right) + \beta_0^2 \left(3\zeta_2 f_2^{\rm q} + 3\zeta_2 B_2^{\rm q} + 2\zeta_3 A_2^{\rm q} \right) \\ & - \beta_1 \left(f_{02}^{\rm q} + 2g_{02} - (g_{01})^2 - \zeta_2 A_2^{\rm q} \right) - \beta_0 \left(f_{03}^{\rm q} + 3g_{03} - 3g_{02}g_{01} - (g_{01})^3 - \frac{3}{2}\zeta_2 A_3^{\rm q} \right), \end{aligned} \tag{2.7}$$

where g_{0i} are to be taken without the $\gamma_{\rm e}$ terms in (4.6) – (4.8) of [7]. Its explicit form is given by

$$\begin{aligned} B_4^{\rm DIS}\Big|_{{\rm L}n_c} = {}& C_F n_c^3 \left(-\frac{2040092429}{139968} + \frac{23011973}{1944}\zeta_2 + \frac{517537}{36}\zeta_3 - \frac{312481}{36}\zeta_4 - \frac{39838}{9}\zeta_2\zeta_3 \right. \\ & \left. - \frac{50680}{9}\zeta_5 - 988\,\zeta_3^2 + \frac{12467}{6}\zeta_6 + 496\,\zeta_2\zeta_5 + 688\,\zeta_3\zeta_4 + 2260\,\zeta_7 \right) \\ & + C_F n_c^2 n_f \left(\frac{83655179}{11664} - \frac{5160215}{972}\zeta_2 - \frac{639191}{162}\zeta_3 + \frac{24856}{9}\zeta_4 + \frac{8624}{9}\zeta_2\zeta_3 \right. \\ & \left. + 200\,\zeta_5 - 32\,\zeta_3^2 - \frac{1201}{3}\zeta_6 \right) + C_F n_f^3 \left(\frac{50558}{2187} + \frac{80}{81}\zeta_3 - \frac{1880}{81}\zeta_2 + \frac{40}{9}\zeta_4 \right) \\ & + C_F n_c n_f^2 \left(-\frac{5070943}{5832} + \frac{160903}{243}\zeta_2 + \frac{14618}{81}\zeta_3 - \frac{2110}{9}\zeta_4 - \frac{400}{9}\zeta_2\zeta_3 + \frac{904}{9}\zeta_5 \right). \end{aligned} \tag{2.8}$$

3. Numerical implications

The lower-order coefficients $B_l^{\rm DIS}$ have the same structure as (2.7), i.e., they contain $-f_l^{\rm q} - B_l^{\rm q}$ and lower-order coefficients. Therefore, by comparing the exact results to an approximation at N^{l}LL in which the Ln_c expression for $-f_l^{\rm q} - B_l^{\rm q}$ is used together with the exact lower-order coefficients, we can check whether (2.7) with the Ln_c results for $f_4^{\rm q}$ and $B_4^{\rm q}$ can be expected to provide a good approximation for $B_4^{\rm DIS}$ and hence G^N at the N^{4}LL accuracy of full QCD.

This comparison is carried out in Fig. 1 for $l = 2$ and $l = 3$ (at $l = 1$ there is no difference between the Ln_c limit and full QCD). The Ln_c curves are off by less than 0.5% at N^{2}LL and 0.25% at N^{3}LL for $G^{\rm DIS}$ in the N-range shown and, at $x \leq 0.9$, for the convolution of its exponential with a schematic but sufficiently realistic form for a quark PDF. Therefore we can safely expect that the Ln_c numbers will deviate from (presumably exceed) the exact QCD results by well below 1%.

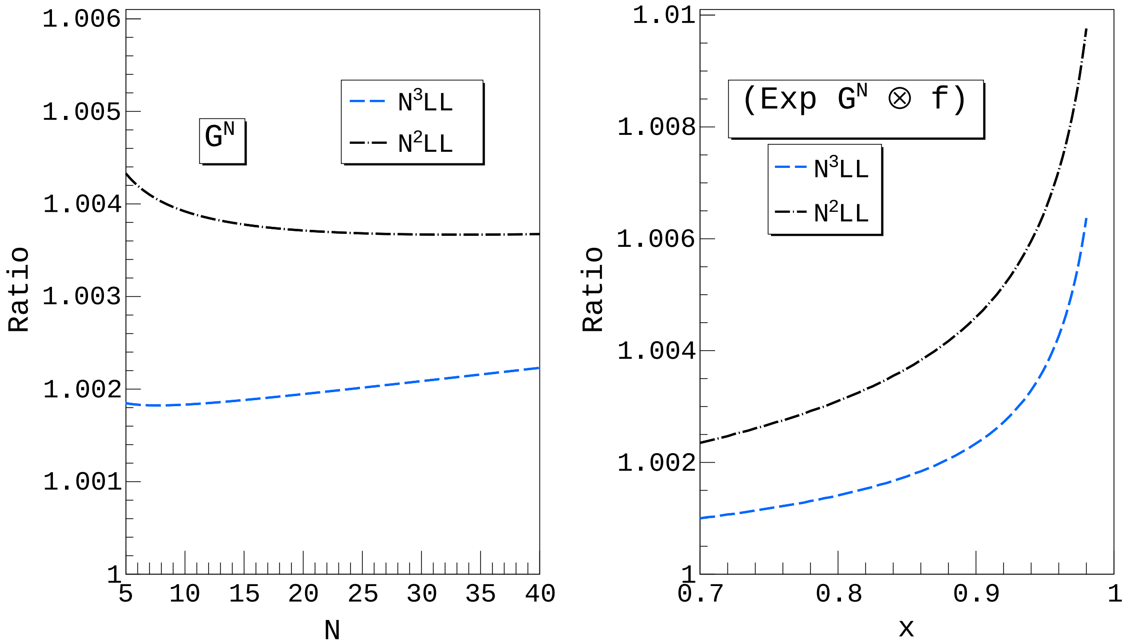

Figure 1: The ratio of the large-n_c approximation, defined as above, and the exact results at N^{2}LL and N^{3}LL for the DIS resummation exponent G^N (left) and for the convolution of the exponential with the schematic quark PDF shape $xf = x^{0.5}(1-x)^3$ (right) for $\alpha_s = 0.2$ and $n_f = 3$ flavours.

The cumulative effect, relative to the NLL results, of the exact N^{2}LL and N^{3}LL contributions and our new N^{4}LL corrections, as above determined using the Ln_c limit of $-f_l^{\rm q} - B_l^{\rm q}$ in (2.7), is illustrated in Fig. 2. Unlike the N^{3}LL contribution, the N^{4}LL correction is almost negligible at $N \leq 15$ and $x \leq 0.9$. Even at $N = 40$, the functions $g^{(n)}(\lambda)$ add only 6%, 1.6% and 1% to the NLL result, respectively, for $n = 2$, $n = 3$ and $n = 4$, where the latter Ln_c result is presumably a slight overestimate. The corresponding N^{2}LL, N^{3}LL and N^{4}LL percentages for the convolution of $\exp G^N$ with $xf = x^{0.5}(1-x)^3$ at $x = 0.95$ read 9.5%, 1.5% and 0.5%, where we have performed the Mellin inversion using a standard contour, see, e.g., [22], which constitutes a 'minimal prescription' contour [4] in the context of the present exponentiation. It appears that the expansion of G^N to N^{4}LL for the structure functions in inclusive DIS is sufficient for all practical purposes.

PoS(DIS2019)010

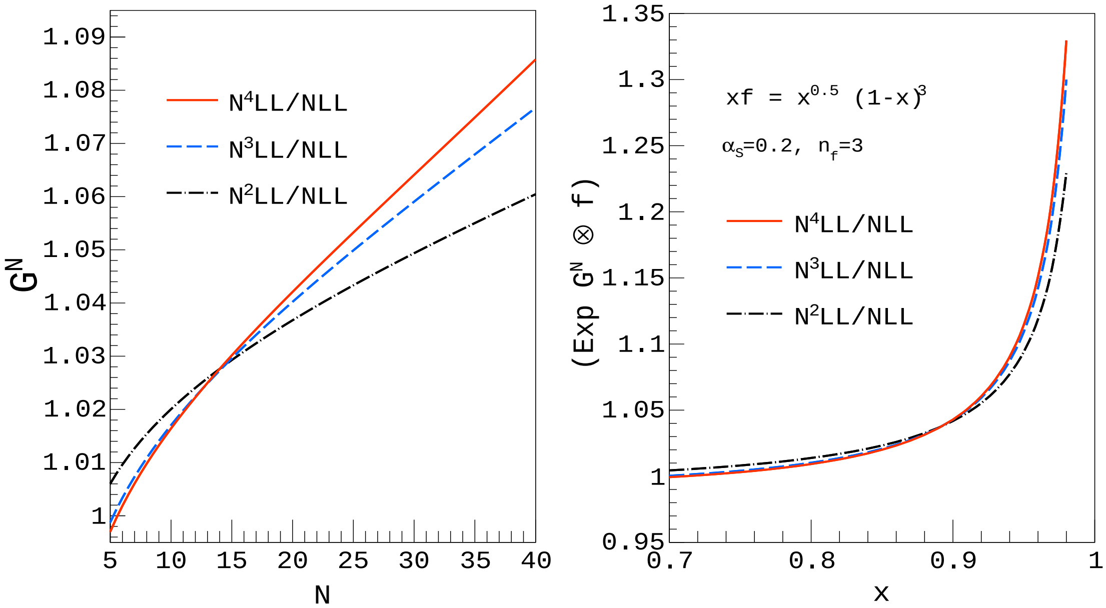

Figure 2: Left: The DIS resummation exponent G^N in (2.3) up to N^{4}LL accuracy, normalized to the NLL result at the standard reference point $\alpha_s = 0.2$ for $n_f = 3$. Right: corresponding x-space results for $\exp G^N$ after convoluted with a schematic form of a quark PDF of the proton.

4. Summary and outlook

We have studied the soft-gluon exponentiation (SGE) of inclusive DIS at the fifth logarithmic (N^{4}LL) order. Recent four-loop results on splitting functions and the quark form factor [14, 15] facilitate the exact determination of the form-factor coefficient $f^{\rm q}$ and the SGE coefficient $B^{\rm DIS}$ at order α_s^4 in the large-n_c (Ln_c) limit. Both coefficients are relevant beyond the context of DIS: Like the lightlike quark and gluon cusp anomalous dimensions $A^{\rm q,g}$ [10], the quantities $f^{\rm q,g}$ are maximally non-Abelian and related by a simple Casimir scaling up to three loops. We expect that the generalized Casimir scaling of [16] also applies to $f^{\rm q,g}$, hence our result (2.6) fixes also $f^{\rm g}$ at large n_c. The coefficient here called $B^{\rm DIS}$ is due to the outgoing unobserved quark; hence it contributes to the SGE for many other processes including, e.g., direct photon production [6].

The Ln_c approximation to the N^{4}LL resummation exponent G^N for inclusive DIS, defined as discussed above, is sufficiently accurate to demonstrate that the N^{4}LL corrections are small: they contribute well below 1% over a wide range in N and x. As shown in [23], the $1/N \ln^{\ell} N$ non-SGE contributions are larger; the highest four of these logarithms are currently known to all orders [23,24] – recall the parameter $\xi_{\rm DIS_4}$ unspecified in [23] was fixed in [24]. We have considered the case of $n_f = 3$ light flavours. In electromagnetic and neutral-current DIS, also charm production close to threshold needs to be taken into account beyond the threshold for $c\bar{c}$ production, see [25].

References

[1] A. Accardi et al., Eur. Phys. J. C76 (2016) 471, arXiv:1603.08906.

[2] P. Bolzoni, F. Maltoni, S. Moch and M. Zaro, Phys. Rev. Lett. 105 (2010) 011801, arXiv:1003.4451, M. Cacciari et al., Phys. Rev. Lett. 115 (2015) 082002, arXiv:1506.02660,

F.A. Dreyer and A. Karlberg, Phys. Rev. Lett. 117 (2016) 072001, arXiv:1606.00840,
J. Currie et al., JHEP 05 (2018) 209, 1803.09973.

[3] J.A.M. Vermaseren, A. Vogt and S. Moch, Nucl. Phys. B724 (2005) 3, hep-ph/0504242,
S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B813 (2009) 220, arXiv:0812.4168.

[4] S. Catani, M.L. Mangano, P. Nason and L. Trentadue, Nucl. Phys. B478 (1996) 273, hep-ph/9604351.

[5] H. Contopanagos, E. Laenen and G.F. Sterman, Nucl. Phys. B484 (1997) 303, hep-ph/9604313.

[6] S. Catani, M.L. Mangano and P. Nason, JHEP 07 (1998) 024, hep-ph/9806484.

[7] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B726 (2005) 317, hep-ph/0506288.

[8] S. Moch and A. Vogt, Phys. Lett. B631 (2005) 48, hep-ph/0508265,
E. Laenen and L. Magnea, Phys. Lett. B632 (2006) 270, hep-ph/0508284.

[9] V. Ravindran, Nucl. Phys. B752 (2006) 173, hep-ph/0603041.

[10] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B688 (2004) 101, hep-ph/0403192,
A. Vogt, S. Moch and J.A.M. Vermaseren, Nucl. Phys. B691 (2004) 129, hep-ph/0404111

[11] S. Moch, J.A.M. Vermaseren and A. Vogt, JHEP 08 (2005) 049, hep-ph/0507039.

[12] S. Moch, J.A.M. Vermaseren and A. Vogt, Phys. Lett. B625 (2005) 245, hep-ph/0508055.

[13] P.A. Baikov et al., Phys. Rev. Lett. 102 (2009) 212002, arXiv:0902.3519,
R.N. Lee, A.V. Smirnov and V.A. Smirnov, JHEP 04 (2010) 020, arXiv:1001.2887,
T. Gehrmann et al., JHEP 06 (2010) 094, arXiv:1004.3653,
T. Gehrmann et al., JHEP 11 (2010) 102, arXiv:1010.4478.

[14] J. Henn et al., JHEP 03 (2017) 139, arXiv:1612.04389.

[15] S. Moch et al., JHEP 10 (2017) 041, arXiv:1707.08315.

[16] S. Moch et al., Phys. Lett. B782 (2018) 627, arXiv:1805.09638.

[17] J.M. Henn, A.V. Smirnov, V.A. Smirnov and M. Steinhauser, JHEP 05 (2016) 066,
arXiv:1604.03126, A. Grozin, PoS LL2016 (2016) 053, arXiv:1605.03886,
J. Davies et al., Nucl. Phys. B915 (2017) 335, arXiv:1610.07477,
R.N. Lee, A.V. Smirnov, V.A. Smirnov and M. Steinhauser, Phys. Rev. D96 (2017) 014008,
arXiv:1705.06862, A. Grozin, JHEP 06 (2018) 073, arXiv:1805.05050,
R.N. Lee, A.V. Smirnov, V.A. Smirnov and M. Steinhauser, JHEP 02 (2019) 172, arXiv:1901.02898,
J.M. Henn, T. Peraro, M. Stahlhofen and P. Wasser, (2019), arXiv:1901.03693.

[18] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B646 (2002) 181, hep-ph/0209100,
S. Forte and G. Ridolfi, Nucl. Phys. B650 (2003) 229, hep-ph/0209154,
E. Gardi and R.G. Roberts, Nucl. Phys. B653 (2003) 227, hep-ph/0210429.

[19] A. Vogt, Phys. Lett. B497 (2001) 228, hep-ph/0010146,
S. Catani, D. de Florian, M. Grazzini and P. Nason, JHEP 07 (2003) 028, hep-ph/0306211.

[20] G. Das, S. Moch and A. Vogt, DESY 19-088, LTH 1205, to appear.

[21] F. Herzog et al., Phys. Lett. B790 (2019) 436, 1812.11818.

[22] A. Vogt, Comput. Phys. Commun. 170 (2005) 65, hep-ph/0408244.

[23] S. Moch and A. Vogt, JHEP 11 (2009) 099, arXiv:0909.2124.

[24] G.Grunberg, Phys. Lett. B687 (2010) 405, arXiv:0911.4471v5,
A.A. Almasy, G. Soar and A. Vogt, JHEP 03 (2011) 030, arXiv:1012.3352.

[25] H. Kawamura, N.A. Lo Presti, S. Moch and A. Vogt, Nucl. Phys. B864 (2012) 399, arXiv:1205.5727

PoS(DIS2019)010