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Non-eikonal corrections to multi-particle production in the CGC

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We discuss the non-eikonal effects due to a finite longitudinal width of the target on single and double inclusive gluon production in proton-proton collisions in the framework of the Color Glass Condensate. We show that these non-eikonal corrections can be summed to all orders in proton-proton collisions and can be identified as corrections to the Lipatov vertex. By using this non-eikonal Lipatov vertex, we calculate single and double inclusive gluon production in proton-proton collisions and argue that the non-eikonal corrections that we treat in this work, break the accidental symmetry of the CGC and lead to non-vanishing odd azimuthal harmonics.

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1. Introduction

One of the most commonly used approximation in the Color Glass Condensate (CGC) framework to study high energy collisions is the eikonal approximation. In dilute-dense systems, this corresponds to the adopting the following three approximations in the background field, A^μ, of the target: (i) considering only the largest component of the background field ($-$ component), (ii) neglecting the dynamics of the target (assuming that the target field does not depend on the z^- coordinate) and (iii) assuming that the target field is localized around $z^+ = 0$ (the shockwave approximation). Under these assumptions, the form of the background field that defines the target simply reads

$$A^\mu = \delta^{\mu -}\delta(z^+)A^-(z) \tag{1.1}$$

In order to go beyond the eikonal accuracy one should relax all these three approximations simultaneously. However, the leading contribution beyond eikonal accuracy arises from relaxing the shockwave approximation and considering a finite longitudinal width target. This is due to the fact that the finite width of the target gives a $A^{1/3}$ nuclear enhancement factor. This step has been taken in [1, 2] and the subeikonal corrections to the gluon production amplitude has been calculated to second order for proton-nucleus (pA) collisions. In [3], the dilute target limit of this result has been studied and corrections to the eikonal Lipatov vertex have been calculated for proton-proton (pp) collisions.

2. Non-eikonal Lipatov vertex

In [3], the non-eikonal corrections to the Lipatov vertex that are due to the finite longitudinal width of the target have been calculated to the next-to-next-to eikonal (NNEik) accuracy and read:

$$L_{\text{NNEik}}(k^+,k,q;x^+) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right]\left\{1 + i\frac{k^2}{2k^+}x^+ - \frac{1}{2}\left(\frac{k^2}{2k^+}x^+\right)^2\right\}, \tag{2.1}$$

where $(k-q)$ is the transverse momenta of the gluon in the projectile, and k is the transverse momenta, k^+ the longitudinal momenta and x^+ the longitudinal position of the produced gluon. The eikonal Lipatov vertex $L(k,q)$ is the $\mathcal{O}(1)$ term on the right hand side of Eq. (2.1). The derivation of the NNEik Lipatov vertex given in Eq. (2.1) relies on the results of [1, 2] where the background gluon propagator was calculated to NNEik order in powers of the finite width of the target, valid for pA collisions. In order to extract the Lipatov vertex at NNEik order, the target have been expanded in powers of the background field [3], which effectively corresponds to going from pA to pp collisions in the CGC framework. The form of the Lipatov vertex given in Eq. (2.1) suggests an exponentiation. However, this was only shown to second order in the expansion.

On the other hand, the Lipatov vertex can be computed to all orders in powers of the finite longitudinal width of the target in the dilute target limit, i.e., for pp collisions, for particle production processes by considering the three diagrams shown in Fig.(1) and keeping the phases in the calculation which are set to unity in the eikonal calculations [4]. The calculation yields the following non-eikonal Lipatov vertex that resums all order eikonal corrections that originate from the finite longitudinal width of the target (see [4] for details of the derivation):

$$L_{\text{NonEik}}(k^+,k,q:x^+) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right]e^{i\frac{k^2}{2k^+}x^+}. \tag{2.2}$$

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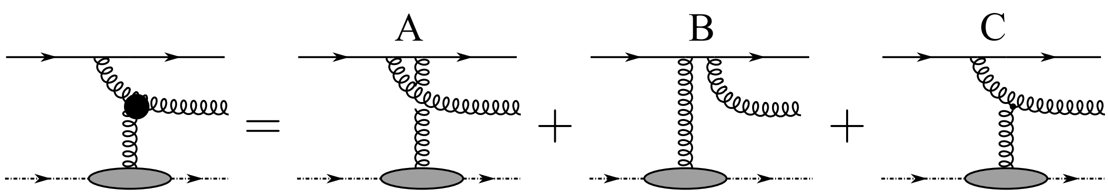

Figure 1: Diagrams that contribute to the computation of the Lipatov vertex. The black dot represents the Lipatov vertex which is the sum of all real diagrams for gluon production shown on the right hand side of the equation.

We would like to mention that this result is not new by itself and similar calculations and expressions can be found in the literature [5, 6]. But the identification of this building block for its use to include non-eikonal corrections in CGC calculations is done in [4] for the first time.

3. Non-eikonal single and double inclusive gluon production

By using the expression for the non-eikonal Lipatov vertex Eq. (2.2), one can easily calculate the non-eikonal single and double inclusive gluon production cross-section in pp collisions with the following recipe. The single and double inclusive gluon production in pA collisions in the eikonal limit has been calculated in [7]. For the single inclusive production the cross section reads

$$\frac{d\sigma}{d^2k d\eta} = 4\pi\alpha_s \int_{z\bar{z}} e^{ik(z-\bar{z})} \int_{xy} A^i(x-z)A^i(\bar{z}-y)\Big\langle \rho^a(x)\rho^b(y)\Big\rangle_P \Big\langle \big[U_z - U_x\big]^{ac}\big[U^\dagger_{\bar{z}} - U^\dagger_y\big]^{cb}\Big\rangle_T, \quad (3.1)$$

where k is the transverse momenta and η is the rapidity of the produced gluon, U_z and $\rho(z)$ are the eikonal Wilson line and the color charge density of the projectile at transverse position z, respectively, and $\langle\cdots\rangle_{P,T}$ denote the averages on projectile and target configurations. By using the MV model for the two projectile color charge correlator ($\langle\rho^a(x)\rho^b(y)\rangle_P$), expanding the Wilson lines in powers of the target background field to first order in the amplitude and in the complex conjugate amplitude and Fourier transforming the expression to the momentum space, one gets the following expression for the single inclusive production in the dilute limit:

$$\begin{aligned}\frac{d\sigma}{d^2k d\eta}\bigg|_{\text{dilute}} &= 4\pi\alpha_s C_A g^2 \int dx_1^+ dx_2^+ \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\delta^{c\bar{c}}\Big\langle A^-_c(x_1^+,q_1)A^-_{\bar{c}}(x_2^+,q_2)\Big\rangle_T \\ &\times\ \mu^2[k-q_1,q_2-k]L^i(k,q_1)L^i(k,q_2). \end{aligned} \quad (3.2)$$

In order to go from the eikonal to the non-eikonal case, one replaces the eikonal Lipatov vertex by the non-eikonal one,

$$L^i(k,q) \to L^i_{\text{NonEik}}(k^+,k,q;x^+) \quad (3.3)$$

and employs

$$\Big\langle A^-_c(x_1^+,q_1)A^-_{\bar{c}}(x_2^+,q_2)\Big\rangle_T = \delta^{c\bar{c}}\frac{n(x_1^+)}{2\lambda^+}\theta\Big(\lambda^+ - |x_1^+ - x_2^+|\Big)(2\pi)^2\delta^{(2)}(q_1-q_2)|a(q_1)|^2 \quad (3.4)$$

for the two field correlator. Here, λ^+ is the color correlation length in the target that we assume to be much smaller than the total longitudinal width of the target L^+. Moreover, function $n(x^+)$ defines

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the one dimensional target density along the longitudinal axis. For simplicity of the calculation, we take this function constant with a finite support, $n(x^+) = n_0$ for $0 \le x^+ \le L^+$ and 0 elsewhere. Finally, function $a(q)$ that appears in the definition of the two field correlator is the functional form of the potential in momentum space which is usually taken to be a Yukawa type potential in jet quenching calculations ($|a(q)|^2 = m^2/(q^2+m^2)^2$). All said and done, one gets the non-eikonal single inclusive gluon production cross section in pp collisions:

$$\frac{d\sigma}{d^2k d\eta}\bigg|^{\rm NE}_{\rm dilute} = 4\pi\alpha_s C_A (N_c^2-1) g^2 \mathscr{G}_1^{\rm NE}(k^-;\lambda^+) \int \frac{d^2q}{(2\pi)^2} \mu^2[k-q,q-k] L^i(k,q) L^i(k,q) |a(q)|^2, \tag{3.5}$$

where $k^- = k^2/2k^+$. Here, all the non-eikonal effects are encoded in the function $\mathscr{G}_1^{\rm NE}(k^-;\lambda^+)$ whose explicit expression reads

$$\mathscr{G}_1^{\rm NE}(k^-;\lambda^+) = \frac{1}{k^-\lambda^+} \sin(k^-\lambda^+), \tag{3.6}$$

which goes to unity in the eikonal limit ($\lambda^+ \to 0$). One can adopt the same procedure to calculate the non-eikonal double inclusive gluon production cross section in pp collisions in the CGC framework. The result can be written in the following compact way (see [4] for details of the calculation, and also for the triple inclusive gluon cross section):

$$\begin{aligned} \frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2}\bigg|^{\rm NE}_{\rm dilute} &= \alpha_s^2 (4\pi)^2 g^4 C_A^2 (N_c^2-1) \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_1}{(2\pi)^2} |a(q_1)|^2 |a(q_2)|^2 \\ &\times \mathscr{G}_1^{\rm NE}(k_1^-;\lambda^+) \mathscr{G}_1^{\rm NE}(k_2^-;\lambda^+) \left\{ I_{\rm 2tr}^{(0)} + \frac{1}{N_c^2-1}\left[I_{\rm 2tr}^{(1)} + I_{\rm 1tr}^{(1)} \right] \right\}, \end{aligned} \tag{3.7}$$

where the explicit expressions for the terms $I_{\rm 2tr}^{(0)}, I_{\rm 2tr}^{(1)}$ and $I_{\rm 1tr}^{(1)}$ can be found in [4]. It is important to note that some of the terms that appear in the double inclusive gluon production cross section are accompanied by a new function $\mathscr{G}_2^{\rm NE}(k_1^-,k_2^-;L^+)$ which is defined as

$$\mathscr{G}_2^{\rm NE}(k_1^-,k_2^-;L^+) = \left\{ \frac{2}{(k_1^- - k_2^-)L^+} \sin\left[\frac{(k_1^- - k_2^-)}{2} L^+ \right] \right\}. \tag{3.8}$$

Before we conclude this section, we would like to comment on the nature of the non-eikonal double inclusive gluon production cross section. In the eikonal limit, double inclusive gluon production cross section calculated in the CGC framework is known to obey the so-called "accidental symmetry". Effectively, this corresponds to the fact that the double inclusive gluon production cross section is symmetric under $k_2 \to -k_2$. However, in the non-eikonal case, the cross section can be written by using $(k_2^+, k_2) \to (-k_2^+, -k_2)$. It is obvious from the definition of function $\mathscr{G}_2^{\rm NE}(k_1^-,k_2^-;L^+)$ given in Eq. (3.8) that this function is not symmetric under $(k_2^+, k_2) \to (-k_2^+, -k_2)$ which leads to the same lack of symmetry in the non-eikonal double inclusive production cross section.

4. Discussion

Let us now discuss the implications of the non-eikonal corrections on single and double inclusive gluon production cross sections.

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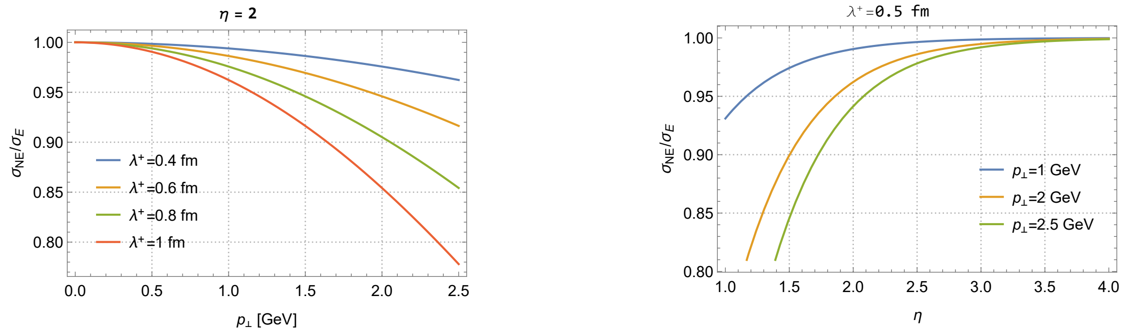

Figure 2: Left: ratio of non-eikonal to eikonal single inclusive gluon production cross sections as a function of the transverse momenta of the produced gluon for different values of the correlation length λ^+, at fixed pseudorapidity $\eta = 2$. Right: the same ratio plotted as a function of the pseudorapidity of the produced gluon for different values of its transverse momenta at a fixed correlation length $\lambda^+ = 0.5$ fm.

In Fig. (2) on the left, we have plotted the ratio of the non-eikonal to eikonal single inclusive gluon production cross sections as a function of the transverse momenta of the produced gluon. The ratio shows up to 20% relative weight of the non-eikonal corrections for $\lambda^+ = 1$ fm, while for smaller values of λ^+ the results show a suppression from a few to up to 10%. On the right, we have plotted the ratio of the non-eikonal to eikonal single inclusive gluon production cross sections as a function of pseudorapidity for different values of the transverse momenta of the produced gluon at a fixed correlation length $\lambda^+ = 0.5$ fm. The results show that up to pseudorapidity $\eta = 2.5$, depending on the value of the transverse momenta of the produced gluon, the relative weight of the non-eikonal corrections can vary roughly between 15% and 2%. These results confirm our analytical predictions for the importance of the non-eikonal corrections in certain kinematical regions.

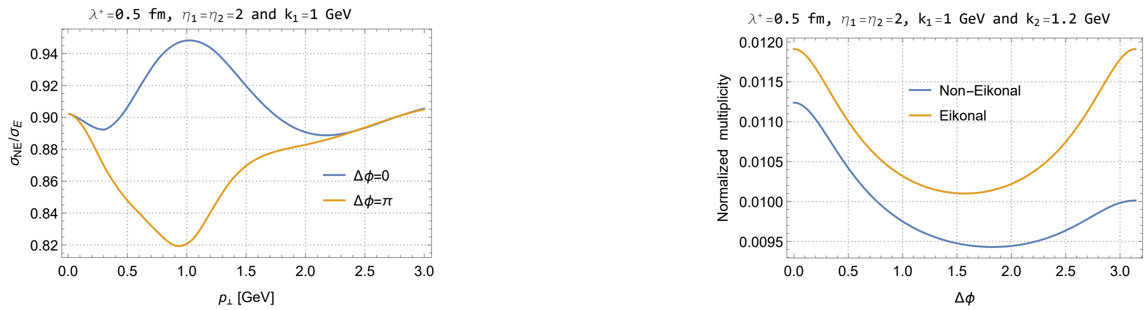

Figure 3: Left: behavior of the ratio of non-eikonal to eikonal cross sections at $\Delta\phi = 0$ and $\Delta\phi = \pi$ as a function of the transverse momenta of the second gluon for a correlation length $\lambda^+ = 0.5$ fm, $L^+ = 6$ fm, rapidities of the produced gluons $\eta_1 = \eta_2 = 2$ and transverse momenta of the first gluon $k_1 = 1$ GeV. Right: non-eikonal and eikonal normalized double inclusive gluon production cross sections as a function of azimuthal angle between the two produced gluons $\Delta\phi$ for $\lambda^+ = 0.5$ fm, $L^+ = 6$ fm, and rapidities $\eta_1 = \eta_2 = 2$ and transverse momenta $k_1 = 1$ GeV and $k_2 = 1.2$ GeV of the two produced gluons.

In Fig. (3) on the left, we show the ratio of the non-eikonal to eikonal double inclusive gluon

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production cross sections as a function of the transverse momenta of the second produced gluon while keeping the transverse momenta of the first gluon fixed $k_1 = 1$ GeV, for $\Delta\phi = 0$ and $\Delta\phi = \pi$ with $\Delta\phi$ the azimuthal angle between the two produced gluons. The result shows that the ratio is enhanced for $\Delta\phi = 0$ and suppressed for $\Delta\phi = \pi$. The relative modification is peaked when the transverse momenta of the second gluon is the same as the transverse momenta of the first gluon and it varies roughly between 4% and 10% for values of the transverse momenta of the second gluon 0.5 GeV $< k_2 <$ 1.5 GeV. On the right, we plot the normalized non-eikonal and eikonal double inclusive gluon production cross sections as a function of the azimuthal angle between the two produced gluons $\Delta\phi$. The results are completely symmetric with respect to $\Delta\phi = \pi/2$ in the eikonal case, while an asymmetric behavior is seen for the non-eikonal case.

To sum up, the results indicate that including the non-eikonal corrections in the double inclusive gluon production cross section has a direct consequence. In the double inclusive production, changing the azimuthal angle from $\Delta\phi = 0$ to $\Delta\phi = \pi$ modifies the magnitude of the non-eikonal effects which causes the breaking of the accidental symmetry of the CGC, which may lead to non-zero odd azimuthal harmonics [8].

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