Inclusive production of two rapidity-separated heavy quarks as a probe of BFKL dynamics

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The inclusive photoproduction of two heavy quarks, separated by a large rapidity interval, is proposed as a new channel for the manifestation of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) dynamics. The extension to the hadroproduction case is also discussed.
1. Introduction

Semihard processes \( s \gg Q^2 \gg \Lambda_{\text{QCD}}^2 \), with \( s \) the squared center-of-mass energy, \( Q \) the process hard scale and \( \Lambda_{\text{QCD}} \) the QCD mass scale) represent a challenge for high-energy QCD. Fixed-order perturbative calculations miss the effect of large energy logarithms, which must be resummed to all orders. The theoretical tool for this resummation is the BFKL approach [1], valid both in the leading logarithmic approximation (LLA) (all terms \( \mathcal{O}(\alpha(s) \ln(s)^n) \)) and in the next-to-LLA (NLA) (all terms \( \mathcal{O}(\alpha(s) \ln(s)^n) \)). In this approach, the (possibly differential) cross section factorizes into two process-dependent impact factors and a process-independent Green’s function. Only a few impact factors have been calculated with next-to-leading order accuracy: parton to parton [2], parton to hadron [4], \( \gamma^* \) to light vector meson [5] and \( \gamma^* \) to \( \gamma^* \) [6]. They were used to build predictions for a few exclusive processes: \( \gamma^* \gamma^* \) to two light vector mesons [7] and the \( \gamma^* \gamma^* \) to all [8], which can be studied in future high-energy linear colliders. They enter, however, a lot of inclusive processes, accessible at LHC: Mueller-Navelet jet production [9], three and four jets, separated in rapidity [10], two identified rapidity-separated hadrons [11], forward identified light hadron and backward jet [12], forward \( J/\Psi \)-meson and backward jet [13], forward Drell-Yan pair and backward jet [14]. Here we present another possible BFKL probe: the inclusive production of two heavy quarks, separated in rapidity, in \( \gamma\gamma \) collisions (photoproduction),

\[
\gamma(p_1) + \gamma(p_2) \rightarrow Q(q_1) + X + Q(q_2),
\]

where \( Q \) here stands for a \( c \)- or \( b \)-quark (see Fig. 1(left)). This process can be studied either at \( e^+e^- \) or in nucleus-nucleus collisions via the interaction of two quasi-real photons. Here we focus on \( e^+e^- \) collisions, but we briefly discuss also the case of production in proton-proton collisions (hadroproduction), via a gluon-initiated subprocess.

2. Theoretical setup: photoproduction case

The impact factor relevant for the process given in (1.1) reads [15], at leading order \(^1\)

\[
d\Phi = \frac{\alpha_0 e_Q^2}{\pi} \sqrt{N_c^2 - 1} \left[ m^2 R^2 + \vec{\mathbf{p}}^2 \left( z^2 + \bar{z}^2 \right) \right] d^2 q \, dz,
\]

\[
R = \frac{1}{m^2 + \vec{q}^2} - \frac{1}{m^2 + (\vec{q} - \vec{k})^2}, \quad \vec{\mathbf{p}} = \frac{\vec{q}}{m^2 + \vec{q}^2} + \frac{\vec{k} - \vec{q}}{m^2 + (\vec{q} - \vec{k})^2}.
\]

Here \( \alpha \) and \( \alpha_s \) denote the QED and QCD couplings, \( N_c \) the number of colors, \( e_Q \) the electric charge of the heavy quark, \( m \) its mass, \( z \) and \( \bar{z} \equiv 1 - z \) the longitudinal fractions of the quark and antiquark produced in the same vertex and \( \vec{k}, \vec{q} - \vec{q} \) the transverse momenta with respect to the photons collision axis of Reggeized gluon, produced quark and antiquark, respectively.

Similarly to the dihadron production processes (see [11]), we have

\[
\frac{d\sigma_{\gamma\gamma}}{dy_1 dy_2 d|\vec{q}_1| d|\vec{q}_2| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2} \left[ \mathcal{C}_0 + 2 \sum_{n=1}^{\infty} \cos(n\phi) \mathcal{C}_n \right],
\]

\(^1\)In Ref. [16] the factor \( \sqrt{N_c^2 - 1} \) was forgotten; plots and tables in the present paper take it properly into account and, thus, overwrite the corresponding ones in Ref. [16].
where $\Phi = \phi_1 - \phi_2 - \pi$, with $\phi_{1,2}$ and $y_{1,2}$, respectively, the azimuthal angles, the transverse momenta and the rapidities of the produced quarks. The $C_n$ coefficients encode the two leading-order impact factors and the NLA BFKL Green’s function (see Fig. 1(right)). For brevity, the expression for $C_n$ is not presented here; it can be found in Ref. [16], to which we refer for all details concerning the present paper.

For the process initiated by $e^+e^-$ collisions, we must take into account the flux of quasi-real photons $dn/dx$ emitted by each of the two colliding particles. The cross section, differential in the rapidity gap $\Delta Y$ between the two tagged heavy quarks, reads then

$$
\frac{d\sigma_{e^+e^-}}{d(\Delta Y)} = \int_{q_{\min}}^{q_{\max}} dq_1 \int_{q_{\min}}^{q_{\max}} dq_2 \int_{y_{1\max}}^{y_{1\min}} dy_1 \int_{y_{2\max}}^{y_{2\min}} dy_2 \delta (y_1 - y_2 - \Delta Y)
\times \int_{e}^{1} \left(\frac{1}{y_{1\min} - y_1}\right) \frac{dn_1}{dx_1} \int_{e}^{1} \left(\frac{1}{y_{2\min} - y_2}\right) \frac{dn_2}{dx_2} \frac{d\sigma_{\gamma\gamma}}{d(\Delta Y)},
$$

with $y_{1\max} = \ln \sqrt{\frac{m_1^2 + q_1^2}{s}}$ and $y_{2\max} = \ln \sqrt{\frac{s}{m_2^2 + q_2^2}}$, where $s$ is the squared center-of-mass energy of the colliding $e^+e^-$ pair. In the following we will present results for the integrated azimuthal coefficients $C_n$, defined through

$$
\frac{d\sigma_{e^+e^-}}{d(\Delta Y) d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2} \left[ C_0 + 2 \sum_{n=1}^{\infty} \cos (n\varphi) C_n \right].
$$

Table 1: $C_0$ [pb] $vs. \Delta Y$ for $q_{\min} = 0$ GeV and $\sqrt{s} = 200$ GeV; $C$ stands for $\mu_R^2/(s_1s_2)$, with $s_{1,2} = m_{1,2}^2 + q_{1,2}^2$.

<table>
<thead>
<tr>
<th>$\Delta Y$</th>
<th>Box $q\bar{q}$</th>
<th>LLA $C = 1/2$</th>
<th>LLA $C = 1$</th>
<th>LLA $C = 2$</th>
<th>NLA $C = 1/2$</th>
<th>NLA $C = 1$</th>
<th>NLA $C = 2$</th>
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<tbody>
<tr>
<td>1.5</td>
<td>98.26</td>
<td>415.0(1.3)</td>
<td>65.24(31)</td>
<td>28.94(14)</td>
<td>16.96(10)</td>
<td>11.237(73)</td>
<td>10.289(74)</td>
</tr>
<tr>
<td>2.5</td>
<td>42.73</td>
<td>723.7(2.1)</td>
<td>88.64(36)</td>
<td>34.58(17)</td>
<td>17.580(91)</td>
<td>9.581(57)</td>
<td>8.504(56)</td>
</tr>
<tr>
<td>3.5</td>
<td>14.077</td>
<td>1203.4(3.4)</td>
<td>113.33(43)</td>
<td>39.01(16)</td>
<td>18.522(92)</td>
<td>7.989(43)</td>
<td>6.637(36)</td>
</tr>
<tr>
<td>4.5</td>
<td>3.9497</td>
<td>1851.6(5.0)</td>
<td>133.64(52)</td>
<td>40.42(19)</td>
<td>18.412(90)</td>
<td>6.210(31)</td>
<td>4.893(25)</td>
</tr>
<tr>
<td>5.5</td>
<td>0.9862</td>
<td>2559.4(7.1)</td>
<td>140.23(55)</td>
<td>37.18(17)</td>
<td>16.971(83)</td>
<td>4.329(21)</td>
<td>3.138(15)</td>
</tr>
</tbody>
</table>

Figure 1: (Left) Heavy-quark pair photoproduction. (Right) BFKL factorization: crosses denote the tagged quarks, whose momenta are not integrated over for getting the cross section.
we show pure LLA and NLA BFKL predictions for PoS(DIS2019)067 shown in Fig. 3. Numerical analysis

We consider only the case of c-quark with mass \( m = 1.2 \) GeV/c\(^2\) and fix \( q_{\text{max}} = 10 \) GeV and \( q_{\text{min}} = 0, 1, 3 \) GeV. We take \( \sqrt{s} = 200 \) GeV, as in LEP2, with \( 1 < \Delta Y < 6 \), and \( \sqrt{s} = 3 \) TeV, as in the future e\(^+\)e\(^-\) CLIC linear accelerator, with \( 1 < \Delta Y < 11 \).

In Tables 1-2 we show pure LLA and NLA BFKL predictions for \( C_0 \) with \( q_{\text{min}} = 0 \) GeV and \( \sqrt{s} = 200 \) GeV and \( 3 \) TeV, respectively, and compare them with the exclusive photoproduction of a \( c\bar{c} \) pair, given by two “box” diagrams. We see that at LEP2 energies the “box” cross section dominates, but at CLIC energies BFKL takes over.

Results for \( C_0, R_{10}, \) and \( R_{20} \equiv C_2/C_0 \) with \( q_{\text{min}} = 1, 3 \) GeV and \( \sqrt{s} = 200 \) GeV and \( 3 \) TeV are shown in Fig. 2. We see that the cross section increases from LEP2 to CLIC energies and decreases from LLA to NLA. Azimuthal correlations are in all cases much smaller than one and decrease when \( \Delta Y \) increases, as it must be due to the larger emission of undetected partons. Moreover, the inclusion of NLA effects increases the correlations, which can only be explained with the larger suppression of \( C_0 \) with respect to \( C_{1,2} \) when these effects are included.

4. Theoretical setup: hadroproduction case

The hadroproduction case can be studied in a similar fashion as the photoproduction one, with the role of photons played by gluons and the photon flux replaced by the gluon parton distribution function in the proton. The differential impact factor in this case takes the form

\[
\frac{d\Phi}{d^2q\,dz} = \frac{\alpha_s^2 N_c^2}{2\pi N_c} \left[ \left( m^2 (R + R)^2 + (\bar{P} + \bar{P})^2 (z^2 + \bar{z}^2) \right) - 2 \frac{N_c^2}{N_c^2 - 1} \left( m^2 R + \bar{P} \bar{P} (z^2 + \bar{z}^2) \right) \right],
\]

\[
R = \frac{1}{m^2 + \bar{q}^2} - \frac{1}{m^2 + (\bar{q} - \bar{k}z)^2}, \quad \bar{P} = \frac{\bar{q}}{m^2 + \bar{q}^2} + \frac{\bar{k}z - \bar{q}}{m^2 + (\bar{q} - \bar{k}z)^2},
\]

\[
\bar{R} = \frac{-1}{m^2 + (\bar{q} - \bar{k})^2} + \frac{1}{m^2 + (\bar{q} - \bar{k}z)^2}, \quad \bar{P} = \frac{\bar{k} - \bar{q}}{m^2 + (\bar{q} - \bar{k})^2} - \frac{\bar{k}z - \bar{q}}{m^2 + (\bar{q} - \bar{k}z)^2}.
\]

Color and coupling prefactors enhance hadroproduction cross section by some \( 10^3 \) with respect to photoproduction, but photon flux \( dn/dx \) dominates over \( g(x) \) for \( x \to 0 \) and \( x \to 1 \) so that it is not easy to estimate the size of the cross section without a detailed numerical analysis.
Figure 2: $\Delta Y$-dependence of $C_{0}, R_{10}$, and $R_{20}$ for $q_{\text{min}} = 1, 3$ GeV, $\sqrt{s} = 200$ GeV and 3 TeV, and for different values of $C = \mu_{R}^{2}/\sqrt{s_{1}s_{2}}$, with $s_{1,2} = m_{T,2}^{2} + q_{T,2}^{2}$.

References


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