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Towards transverse-momentum-dependent splitting functions in k_T-factorization

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We present a calculation of transverse momentum dependent real emission contributions to splitting functions within k_T-factorization [1–3], giving special attention to the gluon-to-gluon splitting. The calculation is performed in a formalism that generalizes the framework of [4, 5]. The advantage of the presented approach is that the obtain splitting functions fulfill appropriate limits. In particular, the gluon-to-gluon splitting function reduces to the leading order BFKL kernel in the low z limit, to the DGLAP gluon-to-gluon splitting function in the collinear limit as well as to the CCFM kernel in the angular ordered region. In the last part of the contribution we comment also on the calculation of virtual corrections to the obtain splitting functions.

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Parton distributions functions (PDFs) are crucial elements of collider phenomenology. In presence of a hard scale M with $M \gg \Lambda_{\rm QCD}$, factorization theorems allow to express cross-sections as convolutions of parton densities (PDFs) and hard matrix elements, where the latter are calculated within perturbative QCD. This was first achieved within the framework of collinear factorization, where the incoming partons are taken to be collinear with the respective mother hadron. Calculating hard matrix elements to higher orders in the strong coupling constant, one can systematically improve the precision of the theoretical prediction, by incorporating more loops and more emissions of real partons. These extra emissions allow to improve the kinematic approximation inherent to the leading order (LO) description. As an alternative to improving the kinematic description through the calculation of higher order corrections, one may attempt to account for the bulk of kinematic effects already at leading order. An important example of such kinematic effects is the transverse momentum k_T of the initial state partons. Schemes which provide an improved kinematic description already at the leading order involve in general Transverse-Momentum-Dependent (TMD) or 'unintegrated' PDFs [6]. TMD PDFs arise naturally in regions of phase space characterized by a hierarchy of scales. A particularly interesting example is provided by the so called low x region, where x is the ratio of the hard scale M^2 of the process and the center-of-mass energy squared s. The low x region corresponds therefore to the hierarchy $s \gg M^2 \gg \Lambda_{\rm QCD}^2$. In such a kinematical setup, large logarithms $\ln 1/x$ can compensate for the smallness of the perturbative strong coupling α_s and it is necessary to resum terms $(\alpha_s \ln 1/x)^n$ to all orders to maintain the predictive power of the perturbative expansion. Such a resummation is achieved by the BFKL [7–9] evolution equation. Its formulation is based on the so called k_T (or high-energy) factorization [10] which is strictly speaking valid in the high energy limit, $s \gg M^2$. In this approach one obtains QCD cross-sections as convolutions of unintegrated gluon density and k_T-dependent perturbative coefficients.

While high energy factorization provides a well defined calculational framework the applicability of the results is naturally limited to the low x limit. If the ensuing formalism is naively extrapolated to intermediate or large x, the framework is naturally confronted with a series of short-comings, e.g. contributions of quarks to the evolution arise as a pure next-to-leading order (NLO) effect and elementary vertices violate energy conservation. One can account for such effects by including a resummation of terms which restore subleading, but numerically relevant, pieces of the DGLAP splitting functions [11–14]. These resummations allow to stablize the low x evolution in the region of intermediate $x \sim 10^{-2}$, nevertheless, extrapolations to larger x values are still prohibited. Moreover, by merely resumming and calculating higher order corrections within the BFKL formalism, one essentially repeats the program initially outlined for collinear factorization: higher order corrections are calculated to account for kinematic effects which are beyond the regarding factorization scheme. To arrive at a framework which avoids the need to account for kinematic effects through the calculation of higher order corrections, it is therefore necessary to devise a scheme which accounts for both DGLAP (conservation of longitudinal momentum) and BFKL (conservation of transverse momentum) kinematics. Note that the mere definition of such a scheme is difficult: neither the hard scale of the process (as in DGLAP evolution) nor x (BFKL evolution) provides at first a suitable expansion parameter, if one desires to keep exact kinematics in both variables. To overcome these difficulties, we follow here a proposal initially outlined in [5]. There, the low x resummed DGLAP splitting functions have been constructed following the definition of DGLAP splittings by Curci-Furmanski-Petronzio (CFP) [4]. The authors of [5] were able to define a TMD gluon-to-quark splitting function $\tilde{P}_{qg}$, both exact in transverse momentum and longitudinal momentum fraction.[1] Following observation of [15] two of us generalized this scheme to calculate the remaining splittings which involve quarks, $\tilde{P}_{gq}$, $\tilde{P}_{qg}$ and $\tilde{P}_{qq}$ [1]. The computation of the gluon-to-gluon splitting $\tilde{P}_{gg}$ required a further modification of the formalism used in [1,5] which we did recently in [3]. In this contribution we summarize the most relevant results obtained in this work and we comment on calculation of virtual corrections.

The calculation of the $\tilde{P}_{gg}$ splitting function follows the prescription used in our papers [1,3]. We are working in the high energy kinematics where the momentum of the incoming parton is off-shell (see Fig. 1) and given by: $k^\mu = yp^\mu + k_\perp^\mu$, where p is a light-like momentum defining the direction of a hadron beam and n defines the axial gauge with $n^2 = p^2 = 0$ (that is necessary when using CFP inspired formalism). Additionally, the outgoing momenta is parametrized as: $q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2+\mathbf{q}^2}{2xp\cdot n} n^\mu$ and we also use: $\tilde{\mathbf{q}} = \mathbf{q} - z\mathbf{k}$ with $z = x/y$.

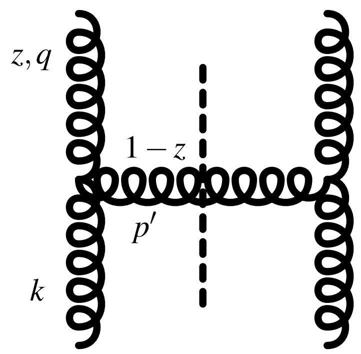

Figure 1: Diagram contributing to the real $\tilde{P}_{gg}$ splitting function at leading order.

The TMD splitting function, $\tilde{P}_{gg}$, is defined as

$$\hat{K}_{gg}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \varepsilon\right) = z \int \frac{d^{2+2\varepsilon}\mathbf{q}}{2(2\pi)^{4+2\varepsilon}} \underbrace{\int dq^2 \, \mathbb{P}_{g,\mathrm{in}} \otimes \hat{K}_{gg}^{(0)}(q,k) \otimes \mathbb{P}_{g,\mathrm{out}}}_{\tilde{P}_{gg}^{(0)}(z,\mathbf{k},\tilde{\mathbf{q}},\varepsilon)} \Theta(\mu_F^2 + q^2), \tag{1}$$

with $\mathbb{P}_g$ being appropriate projection operators and $\hat{K}_{gg}$ the matrix element contributing to the kernel, which at LO is given by the diagram of Fig. 1.

In order to calculate $\tilde{P}_{gg}$ splitting we needed to extend formalism of [1,5] to the gluon case. This was achieved in [3] by generalizing definition of projector operators and defining appropriate generalized 3-gluon vertex that is gauge invariant in the presence of the off-shell momentum k. Definition of the generalized vertex follows from application of the spinor helicity methods to the high-energy factorization [16–21] and can be obtained by summing the diagrams of Fig. 2, giving:

$$\Gamma^{\mu_1\mu_2\mu_3}_{g^*g^*g}(q,k,p') = \mathscr{V}^{\lambda\kappa\mu_3}(-q,k,-p')\, d^{\mu_1}{}_{\lambda}(q)\, d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'}, \tag{2}$$

with $\mathscr{V}^{\lambda\kappa\mu_3}(-q,k,-p')$ being the ordinary 3-gluon QCD vertex, and $d^{\mu_1\mu_2}(q) = -g^{\mu_1\mu_2} + \frac{q^{\mu_1}n^{\mu_2}+q^{\mu_2}n^{\mu_1}}{q\cdot n}$ is the numerator of the gluon propagator in the light-cone gauge. More details on the exact procedure can be found in [3].

The definition of the projectors is more involved. We need to ensure that in the collinear limit the new projectors reduce to the ones introduced by the CFP [4] and that the appropriate high-energy limit for the gluon splitting is also obtained. A natural approach is to modify only the incoming projector $\mathbb{P}_{g,\mathrm{in}}$, as the kinematics of the incoming momentum is more general, and keep the collinear outgoing projector $\mathbb{P}_{g,\mathrm{out}}$ unchanged (as the kinematics of the outgoing momentum is

[1]Hereafter, we will use the symbol $\tilde{P}$ to indicate a transverse momentum dependent splitting function.

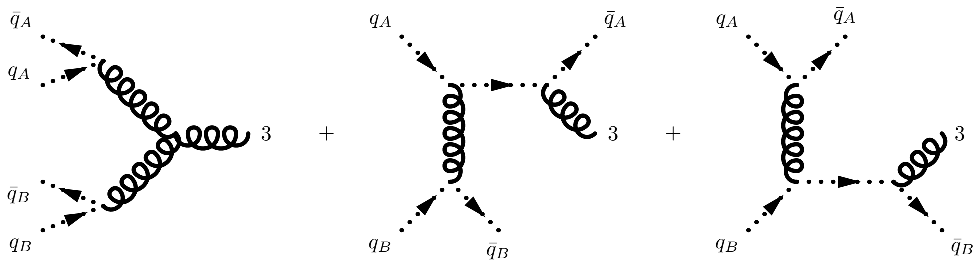

Figure 2: Feynman diagrams contributing to the amplitude used to define the generalized 3-gluon vertex.

the same). This is what was done by Catani and Hautmann [5,10] and later by us [1] when defining the projector for the calculation of the quark splitting functions. In that case we used a transverse projector: $k_\perp^\mu k_\perp^\nu/k_\perp^2$ which, however, due to the more complicated structure of the 3-gluon vertex $\Gamma^{\mu_1\mu_2\mu_3}_{g^*g^*g}$ can not be used any more. Instead we use, a more natural (for high-energy factorization), longitudinal projector given by: $\mathbb{P}^{\mu\nu}_{g,\mathrm{in}} = y^2 p^\mu p^\nu/k_\perp^2$. However, in order to satisfy the requirements of $\mathbb{P}^s_g = \mathbb{P}_{g,\mathrm{in}}\mathbb{P}_{g,\mathrm{out}}$ being a projector operator ($\mathbb{P}^s_g \otimes \mathbb{P}^s_g = \mathbb{P}^s_g$) we also need to modify the outgoing projector. In the end we end up with the following gluon projectors:

$$\mathbb{P}^{\mu\nu}_{g,\mathrm{in}} = -y^2\,\frac{p^\mu p^\nu}{k_\perp^2}\,, \qquad \mathbb{P}^{\mu\nu}_{g,\mathrm{out}} = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n} - k^2\,\frac{n_\mu n_\nu}{(k\cdot n)^2}\,. \tag{3}$$

It is easy to check that now $\mathbb{P}^s_g \otimes \mathbb{P}^s_g = \mathbb{P}^s_g$ holds. Also the new outgoing projector is consistent with the collinear case and one can show that in the collinear limit the difference between $y^2 p^\mu p^\nu/k_\perp^2$ and $k_\perp^\mu k_\perp^\nu/k_\perp^2$ vanishes when contracted into the relevant vertices. More details are provided in [3].

Using the elements introduced in the previous section we can compute the transverse momentum dependent gluon-to-gluon splitting function. We present here only the final result:

$$\begin{aligned}\tilde{P}^{(0)}_{gg}(z,\tilde{\mathbf{q}},\mathbf{k}) = 2C_A \Bigg\{ & \frac{\tilde{\mathbf{q}}^4}{(\tilde{\mathbf{q}}-(1-z)\mathbf{k})^2\,[\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2]}\left[\frac{z}{1-z}+\frac{1-z}{z}+\right. \\ & \left. +(3-4z)\frac{\tilde{\mathbf{q}}\cdot\mathbf{k}}{\tilde{\mathbf{q}}^2}+z(3-2z)\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right] + \frac{(1+\varepsilon)\tilde{\mathbf{q}}^2 z(1-z)[2\tilde{\mathbf{q}}\cdot\mathbf{k}+(2z-1)\mathbf{k}^2]^2}{2\mathbf{k}^2[\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2]^2}\Bigg\}\,,\end{aligned} \tag{4}$$

or after angular averaging (and setting $\varepsilon = 0$):

$$\begin{aligned}\bar{P}^{(0)}_{gg}\left(z,\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right) = {} & C_A\,\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2}\left[\frac{(2-z)\tilde{\mathbf{q}}^2+(z^3-4z^2+3z)\mathbf{k}^2}{z(1-z)\,|\tilde{\mathbf{q}}^2-(1-z)^2\mathbf{k}^2|}\right. \\ & \left. + \frac{(2z^3-4z^2+6z-3)\tilde{\mathbf{q}}^2+z(4z^4-12z^3+9z^2+z-2)\mathbf{k}^2}{(1-z)(\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2)}\right].\end{aligned} \tag{5}$$

Now we can explicitly check the corresponding kinematic limits. In the collinear case this is straightforward, since the transverse integral in Eq. (1) is specially adapted for this limit. In particular, one easily obtains the real part of the DGLAP gluon-to-gluon splitting function:

$$\lim_{\mathbf{k}^2\to 0}\bar{P}^{(0)}_{ij}\left(z,\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right) = 2C_A\left[\frac{z}{1-z}+\frac{1-z}{z}+z\,(1-z)\right]. \tag{6}$$

In order to study the high-energy and soft limit it is convenient to change to the following variables: $\tilde{\mathbf{p}} = \frac{\mathbf{k}-\mathbf{q}}{1-z} = \mathbf{k} - \frac{\tilde{\mathbf{q}}}{1-z}$, then in the high-energy limit ($z \to 0$) we obtain:

$$\lim_{z\to 0} \hat{K}_{gg}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \varepsilon, \alpha_s\right) = \frac{\alpha_s C_A}{\pi(e^{\gamma_E}\mu^2)^\varepsilon} \int \frac{d^{2+2\varepsilon}\tilde{\mathbf{p}}}{\pi^{1+\varepsilon}} \Theta\left(\mu_F^2 - (\mathbf{k}-\tilde{\mathbf{p}})^2\right) \frac{1}{\tilde{\mathbf{p}}^2}$$

where the term under the integral is easily identified as the real part of the LO BFKL kernel. Additionally, one can check that in the angular ordered region of phase space, where $\tilde{\mathbf{p}}^2 \to 0$, we reproduce the real/unresummed part of the CCFM kernel [22–24]: $\frac{1}{z} + \frac{1}{1-z} + \mathscr{O}\left(\frac{\tilde{\mathbf{p}}^2}{\mathbf{k}^2}\right)$.

The main result of this paper is the calculation of a transverse momentum dependent gluon-to-gluon splitting function. The splitting function reduces both to the conventional gluon-to-gluon DGLAP splitting in the collinear limit as well as to the LO BFKL kernel in the low *x*/high energy limit; moreover the CCFM gluon-to-gluon splitting function is re-obtained in the limit where the transverse momentum of the emitted gluon vanishes, *i.e.* if the emitted gluon is soft. The derivation of this result is based on the Curci-Furmanski-Petronzio formalism for the calculation of DGLAP splitting functions in axial gauges. To address gauge invariance in the presence of off-shell partons, high energy factorization adapted for axial gauges has been used to derive an effective production vertex which then could be shown to satisfy current conservation. The next step in completing the calculation of TMD splitting functions is the determination of the still missing virtual corrections. For this purpose we will use the same approach as for real emissions. In order to construct gauge invariant amplitudes in the presence of off-shell external particles we embed them in a more general process featuring only on-shell external legs [16–21]. In case of the P_{qq} splitting function the non-vanishing contributions are given by the diagrams depicted in Fig. 3 Once these contributions are

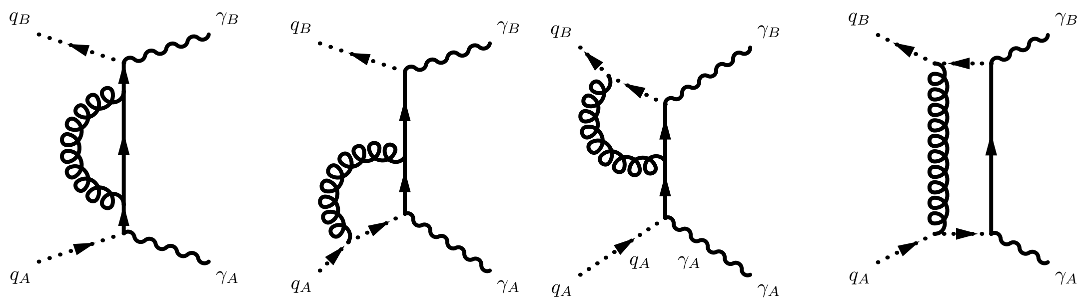

Figure 3: Feynman diagrams contributing to the virtual P_{qq} splitting function.

computed (including renormalization of ultra-violet singularities following a standard prescription of the $\overline{\text{MS}}$-scheme), the projectors defined in [3] can be applied to obtain the virtual correction. This work is ongoing but more cross-checks is needed before presenting the results.

With the complete set of splitting functions at hand, it will be finally possible to formulate an evolution equation for the unintegrated (TMD) parton distribution functions including both gluons and quarks.

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