Single-spin asymmetry $A_{UU}^{\sin \phi_{Sh}}$ of proton and $\Lambda$ production in SIDIS at subleading twist

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We investigate the single-spin asymmetry with a $\sin \phi_{Sh}$ modulation for the transversely polarized proton and $\Lambda$ production in semi-inclusive inelastic scattering process, where $\phi_{Sh}$ is the azimuthal angle of the transverse spin of the final hadron. Theoretically, the spin asymmetry can be interpreted by the convolution of the twist-3 transverse momentum dependent distributions and twist-2 fragmentation functions. In this work, three different origins in terms of the $hH_1$ term, the $f^\perp D_{1T}^\perp$ term and the $g^\perp G_1^T$ term are taken into account simultaneously for this asymmetry. We calculate the twist-3 quark transverse momentum dependent distributions $h$, $f^\perp$ and $g^\perp$ by using the quark spectator diquark model, and we investigate the role of the fragmentation functions $H_1$, $D_{1T}^\perp$ and $G_1^T$ in the $\sin \phi_{Sh}$ asymmetry as well. We also predict the numerical results of the asymmetries for the proton and the $\Lambda$ production at JLab with a 12 GeV beam and at COMPASS with a 160 GeV beam, separately. From the comparison of the different sources for the asymmetry, we find that, the distribution $h$ and the fragmentation function $H_1$ give the dominant contribution to the $\sin \phi_{Sh}$ asymmetry for proton production, while the distribution $f^\perp$ might be probed by the convolution with $D_{1T}^\perp$ in the $\Lambda$ production at JLab 12 GeV.

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1. Introduction

In this work, we extend the phenomenological study of the polarized hadron production in SIDIS at the twist-3 level within the transverse momentum dependent (TMD) framework. We focus on the single spin asymmetry (SSA) with a $\sin \phi_S$ modulation in the transversely polarized hadron production in SIDIS off an unpolarized nucleon: $\ell + N \rightarrow \ell' + h + X$, where $\phi_S$ is the azimuthal angle of the transverse spin $S_{hT}$ with respect to the lepton plane. Following Ref. [1,2], there are several contributions to this asymmetry from the convolutions of the twist-2 TMD fragmentation functions with twist-3 TMD distribution functions. We calculate the involved distributions and the fragmentation functions in a spectator diquark model [3,4]. Using the model results, we investigate the roles of the $hH_1$, $g^\perp G_{1T}$ and $f^\perp D_{1T}^\perp$ couplings in the SSA $A_{UU1T}^{\sin \phi_S}$ for the transversely polarized proton and the $\Lambda$ hyperon production in SIDIS at the JLab 12 GeV and at COMPASS.

2. Twist-3 TMD distribution functions in a spectator model

The three involved twist-3 TMD distributions in this work are the T-odd distribution $g^\perp$, the T-even distribution $f^\perp$ and the T-odd distribution $h$, respectively. We have presented the calculation on the T-odd distribution $g^\perp$ in a spectator model with both the scalar and the axial-vector diquark in Ref. [3,5]. Therefore, following the same method, we can calculate the other two twist-3 TMD distribution functions $h$ and $f^\perp$ from the quark-quark correlator $\Phi(x, p_T; S)$ via the traces

$$
\frac{1}{4} \text{Tr}(\Phi(x, p_T; S) + \Phi(x, -p_T; -S)) i\sigma^{\alpha\beta} \gamma_5] = -\frac{M}{p_T} e^{\alpha\beta} h, \quad (2.1)
$$

$$
\frac{1}{4} \text{Tr}(\Phi(x, p_T; S) + \Phi(x, -p_T; -S)) \gamma^\mu] = \frac{p_T^{\alpha}}{p_T} f^\perp. \quad (2.2)
$$

The corresponding expressions of $h$ and $f^\perp$ from the scalar component are

$$
h(x, p_T^2) = -\frac{N_2^2(1-x)^2}{16\pi^3 M} \frac{2mM(1-x)^2 - (1-x^2)M^2 + m_\perp^2}{(p_T^2 + L_{\perp}^2)^3}, \quad (2.3)
$$

$$
f^\perp(x, p_T^2) = -\frac{N_2^2(1-x)^2}{16\pi^3} \frac{2mM(1-x)^2 - (1-x^2)M^2 + m_\perp^2}{(p_T^2 + L_{\perp}^2)^3}, \quad (2.4)
$$

and the ones that from the axial-vector diquark component are

$$
h(x, p_T^2)_a = 0, \quad (2.5)
$$

$$
f^\perp(x, p_T^2)_a = \frac{N_2^2(1-x)}{16\pi^3} \frac{(xp_T^2 + 2mM(1-x)^2 + (x-1)m^2 + (x^2 - 2x^2 + 1)M^2 - m_\perp^2)}{L_{\perp}^2(p_T^2 + L_{\perp}^2)^3}. \quad (2.6)
$$

The detailed calculation of the distributions $h$ and $f^\perp$ has been performed in Ref. [7]. To construct the distributions for the $u$ and $d$ quarks, we adopt the following relation between the flavors and isospins of the diquark $f^u = c_u^a f^a + c_a^u f'^a$, $f^d = c_u^a f^a'$ [3]. $c_u$, $c_a$ and $c_a'$ are the free parameters of the model, $a$ and $a'$ denote the isoscalar and isovector states of the axial-diquark, respectively.

In Fig. [11] we show the dependence of the distributions $f^\perp$ and $h$ on the flavors, Bjorken $x$ and the active quark transverse momentum $p_T = 0.3$ GeV. As we can see, the distributions $f^\perp u$ and $f^\perp d$ are in similar sizes in the specified kinematic region ($x = 0.3$ or $p_T = 0.25$ GeV). While
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Figure 1: Model results for $xf^{1u}(x, p_T^2)$, $xh^{u}(x, p_T^2)$ (solid line) and $xf^{1d}(x, p_T^2)$, $xh^{d}(x, p_T^2)$ (dashed line).

for the T-odd distribution $h$, its size is smaller compared to those of the T-even distributions $f$. Particularly, we can verify that $h^u$ vanishes when one integrates out the transverse momentum $\int d^2 p^T h^u(x, p_T^2) = 0$, which is an expected result from the time-reversal invariance.

3. Twist-2 TMD fragmentation functions in spectator model

In this section, we will perform the model calculation on the relevant twist-2 TMD polarized fragmentation functions $H_1$, $G_{1T}$ and $D_1^{q/q}$. Since $D_1^{q}$ has been calculated for the $\Lambda$ hyperon by the same spectator diquark model in Ref. [8], we will focus on the proton fragmentation function $D_1^{p/q}$.

The calculation on the left two TMD fragmentation functions $H_1$ and $G_{1T}$ can be performed from the fragmentation correlation function $\Delta(z, k_T; S_{hT})$ by the traces

$$ S_T^{\mu} H_1(z, k_T^2) = \frac{1}{4} \text{Tr} [\Delta(z, k_T; S_{hT}) - \Delta(z, k_T; -S_{hT})] i \sigma^{\mu \nu} \gamma_{\nu}, \quad (3.1) $$

$$ S_{hT} G_{1L}(z, k_T^2) + \frac{k_T \cdot S_{hT}}{M_h} G_{1T}(z, k_T^2) = \frac{1}{4} \text{Tr} [\Delta(z, k_T; S_{hT}) - \Delta(z, k_T; -S_{hT})] \gamma^0 \gamma_5]. \quad (3.2) $$

The spin vector of the outgoing hadron $h_T$ has the form $S_h^\mu = S_{hL} (p_{h+} n^\mu - (p_{h-}) n^\mu) + S_{hT}^\mu$.

Assuming the SU(6) spin-flavor symmetry for the final state hadron [8, 13], the relations between quark flavors and diquark types for the proton and the $\Lambda$ hyperon can be established as

$$ D^{u \rightarrow p} = \frac{3}{2} D^{(s)}, \quad D^{d \rightarrow p} = D^{(s)}, \quad D^{s \rightarrow p} = 0 \quad (3.3) $$

$$ D^{u \rightarrow \Lambda} = D^{d \rightarrow \Lambda} = \frac{1}{4} D^{(s)} + \frac{3}{4} D^{(s)}, \quad D^{s \rightarrow \Lambda} = D^{(s)}. \quad (3.4) $$

Following the method and settings in Ref. [14], the modeling expression of the twist-2 TMD unpolarized fragmentation function $D_1$ can be obtained as

$$ D_1^{(s)}(z, k_T^2) = D_1^{(p)}(z, k_T^2) = \frac{g_D^s}{2(2\pi)^3} \frac{1}{z^2} \frac{(1-z)^2 k_T^2 + (M + zm)^2}{z^4 (k_T^2 + L_T^2)^2}, \quad (3.5) $$

which has already been calculated in Ref. [8]. According to Eq. (3.3) and Eq. (3.5), the unpolarized fragmentation function $D_1$ for proton satisfies the relation $D_1^{u \rightarrow p} = 2D_1^{d \rightarrow p}$, which is consistent with the HKNS parametrization of $D_1^p$ presented in Ref. [15].

Similarly, we can get the model results of the TMD fragmentation functions $H_1$ and $G_{1T}$ [8]

$$ H_1^p(z, k_T^2) = a_R \frac{g_D^p}{2(2\pi)^3} \frac{1}{z^2} \frac{(1-z)^2 [(zm + M_h)^2]}{z^4 (k_T^2 + L_T^2)^2}, \quad (3.6) $$

$$ G_{1T}^p(z, k_T^2) = a_R \frac{g_D^p}{2(2\pi)^3} \frac{1}{z^2} \frac{M_h (zm + M_h)(1-z)}{z^3 (k_T^2 + L_T^2)^2}, \quad (3.7) $$
with \( L_T^2 = \frac{1-\epsilon}{\epsilon} M_n^2 + m^2 + \frac{m_0^2 - m^2}{\epsilon} \). From Eqs. (4.3), (4.4) and (4.5), (4.7), one can find that for the fragmentation functions \( H_1 \) and \( G_{1T} \), both the \( u \) and \( d \) quark fragmenting to the \( \Lambda \) hyperon will vanish in our model, while the \( u \) and \( d \) quark fragmenting to the proton will be nonzero. That is, for the \( \Lambda \) production, we only need to consider the contribution from \( D_{1T}^\perp \).

In Fig. 2, we present our model results of the twist-2 TMD fragmentation functions \( H_1 \), \( G_{1T} \) and \( D_{1T}^\perp \) for the \( u \) and \( d \) quark fragmenting to the proton. As one can see, \( H_1 \) and \( G_{1T} \) decrease but \( D_{1T}^\perp \) increases with increasing \( z \); the magnitude of \( H_1 \) is larger than that of \( G_{1T} \), while \( D_{1T}^\perp \) is nonzero only in the large \( z \) region. From the view of the flavors, \( H_1^u \) and \( G_{1T}^u \) are positive in the model, while \( H_1^d \) and \( G_{1T}^d \) are negative. As for \( D_{1T}^\perp \), only \( D_{1T}^{d\perp} \) becomes sizable at large \( z \).

4. Prediction on the SSA of transversely polarized proton and \( \Lambda \) hyperon in SIDIS

The differential cross section of the process for the transverse polarized hadron in SIDIS off the unpolarized proton is generally expressed as [13]

\[
\frac{d\sigma}{dxdyd\phi d\psi dP_{h\perp}} = \frac{\alpha^2}{xyQ^2} \frac{\gamma^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2\epsilon} \right) \left( F_{UUU}^\perp + |S_{h\perp}| \sin \phi_{h\perp} \sqrt{2\epsilon(1+\epsilon)} F_{UUU}^{\sin \phi_{h\perp}} + \ldots \right).
\]

(4.1)

Here \( F_{UUU} \) and \( F_{UUU}^{\sin \phi_{h\perp}} \) are structure functions that contributes to the \( \sin \phi_{h\perp} \) azimuthal asymmetry, which \( P_{h\perp} \)-dependent expression can be defined as

\[
A_{UUU}^{\sin \phi_{h\perp}}(P_{h\perp}) = \frac{\int dx \int dy \int dz \frac{1}{2} \frac{\gamma^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2\epsilon} \right) \sqrt{2\epsilon(1+\epsilon)} F_{UUU}^{\sin \phi_{h\perp}}}{\int dx \int dy \int dz \frac{1}{2} \frac{\gamma^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2\epsilon} \right) F_{UUU}}.
\]

(4.2)

The \( x \)-dependent and the \( z \)-dependent asymmetries can be given in a similar way.

To perform the estimate, we adopt a more phenomenological approach by using the factorization at the tree-level [13] to give the expression for the structure function \( F_{UUU}^{\sin \phi_{h\perp}} \):

\[
F_{UUU}^{\sin \phi_{h\perp}} \approx \frac{2M}{Q} \mathcal{J} [-xhH_1 + \frac{k_T \cdot p_T}{2MM_h} (-x f_1 D_{1T}^\perp - xg_{1T} G_{1T})].
\]

(4.3)

where we introduce the convolution integral

\[
\mathcal{J}[\omega f D] = x \sum_q e_q^2 \int \frac{d^3 p_T}{p_T} \int d^2 k_T \delta^2(p_T - \frac{P_{h\perp}}{z} - k_T)w(p_T, k_T)f^q(x, p_T^2)D^q(z, k_T^2),
\]

(4.4)
and the Wandzur-Wilczek approximation \cite{35} to ignore the contribution from the twist-3 TMD fragmentation functions $\hat{D}_T, \hat{H}_T^\perp$ and $\hat{H}_T$ to $A_{UUT}^{\sin \phi_5}$.

Considering the following constraints on the transverse momenta of the initial quarks \cite{16}

\[
\begin{align*}
    p_T^2 &\leq (2-x)(1-x)Q^2, \quad \text{for } 0 < x < 1; \\
    p_T^2 &\leq \frac{x(1-x)}{(1-2x)^2}Q^2, \quad \text{for } x < 0.5;
\end{align*}
\]

and the kinematical cuts adopted at JLab 12 GeV

\[
0.1 < x < 0.6, \quad 0.3 < z < 0.7, \quad Q^2 > 1 \text{GeV}^2, \quad W^2 > 4 \text{GeV}^2, \quad p_{h\perp} > 0.05 \text{GeV},
\]

we estimate the SSA $A_{UUT}^{\sin \phi_5}$ of proton and $\Lambda$ hyperon production in SIDIS. The numerical results are shown in Fig. 3. For proton production, the magnitude of the asymmetry $A_{UUT}^{\sin \phi_5}$ is sizable and negative, and the size is around 4% at the kinematics of JLab. While for the $\Lambda$ hyperon production, the size is smaller than 1%, since it gets contribution only from the $f^+D_{\perp T}$ term in our model.

We also estimate this same asymmetry for proton and $\Lambda$ at COMPASS with the kinematics \cite{17}

\[
0.004 < x < 0.7, \quad 0.1 < y < 0.9, \quad z > 0.2, \quad Q^2 > 1 \text{GeV}^2, \quad p_{h\perp} > 0.1 \text{GeV}, \quad W > 5 \text{GeV}, \quad E_h > 1.5 \text{GeV},
\]

The responding asymmetries $A_{UUT}^{\sin \phi_5}$ for proton and $\Lambda$ hyperon are shown in Fig. 3. Interestingly, the size of the $hH_1$ contribution at COMPASS is clearly smaller than that of JLab. This is because the twist-3 effect is suppressed by a factor of $1/Q$, while $Q$ at COMPASS is larger than that at JLab.

5. conclusion

In this work, we predicted the contributions from three twist-3 terms to the SSA $A_{UUT}^{\sin \phi_5}$ in the transversely polarized proton as well as $\Lambda$ production in SIDIS at the kinematics of JLab 12...
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Figure 4: Predictions on the asymmetry $A_{UU}^{\sin \phi_{\Lambda}}$ for proton and $\Lambda$ in SIDIS at COMPASS.

GeV and COMPASS. The estimated contributions to the asymmetry for the proton is sizable and negative. Specifically, the magnitude is around 4% at JLab 12 GeV, but about 1% at COMPASS. The $\sin \phi_{\Lambda}$ asymmetry from the $f^{-1}D_{TT}^{\perp}$ term for the $\Lambda$ hyperon production is much smaller.

References