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QED with heavy ions: on the way from strong to supercritical fields

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The current status of tests of quantum electrodynamics with heavy ions is reviewed. The theoretical predictions for the Lamb shift and the hyperfine splitting in heavy ions are compared with available experimental data. Recent achievements and future prospects in studies of the g factor with highly charged ions are also reported. These studies can provide precise determination of the fundamental constants and tests of QED within and beyond the Furry picture at the strong-coupling regime. Theoretical calculations of the electron-positron pair creation probabilities in low-energy heavy-ion collisions are also considered. Special attention is paid to tests of QED in supercritical-field regime, which can be accessed in slow collisions of two bare nuclei with the total charge number larger than the critical value, $Z_{\rm crit} \approx 173$. In the supercritical field, the initially neutral vacuum can spontaneously decay into the charged vacuum and two positrons. It is demonstrated that this fundamental phenomenon can be observed via impact-sensitive measurements of the pair-production probabilities.

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1. Introduction

Bound-state quantum electrodynamics (QED) has been tested for many decades with light atomic systems, such as hydrogen, helium, positronium, and muonium. As the result of these tests, nowadays nobody doubts the basic principles of QED, including the quantization of the interacting electron-positron and electromagnetic fields as well as the renormalization procedure. However, this does not mean that any further QED tests are not needed. First, the studies of light atomic systems restrict tests of the QED methods to few low orders in the parameters α and αZ, where α is the fine structure constant and Z is the nuclear charge number. Even if we believe that these methods should also work to higher orders, this can not be fully guaranteed unless it is proven by comparison of experiment and theory. This means that any further advance in precision of both theory and experiment is of great importance. Moreover, high-precision measurements with heavy few-electron ions, which became possible in the last few decades, provide a unique opportunity to significantly expand the region of QED tests. Namely, the study of these systems allows us to test bound-state QED in the nonperturbative in αZ regime (in other words, to all orders in αZ). Again, although it is expected that the standard QED formalism can be naturally adapted for calculations of these systems, the corresponding tests in the nonperturbative regime must be performed before these studies can be used for various practical applications. As the main reference points for the QED tests in the nonperturbative in αZ regime, one may consider the Lamb shift, the hyperfine splitting, and the g factor of heavy few-electron ions. The current status of these tests is briefly reviewed in the first part of the present paper.

The methods developed for precise QED calculations of heavy few-electron ions can also serve as an important intermediate step towards tests of QED in supercritical Coulomb field. Such field can be created in low-energy collisions of two heavy ions with the total nuclear charge number exceeding the critical value, $Z_1 + Z_2 > Z_{\rm crit} \approx 173$. In the supercritical field the lowest one-electron quasimolecular level enters into the negative-energy continuum and becomes a resonance. If this level is empty, its “diving” into the negative-energy continuum should result in the decay of the originally neutral vacuum into the charged vacuum and two positrons. In the second part of the paper, we demonstrate that this fundamental phenomenon can be observed in studying the impact-sensitive pair-production probabilities.

2. Strong field QED with heavy ions

2.1 Lamb shift

The Lamb shift in H-like ions is mainly determined by the QED contributions evaluated within the Furry picture. In this picture, the nucleus is considered as a source of the external Coulomb field and the standard QED formalism in the presence of static classical field can be used. The main QED contributions to the Lamb shift are defined by one-loop self-energy and vacuum-polarization diagrams which are presented in Fig. 1. In contrast to light atoms, where the parameter αZ is small, in highly charged ions these diagrams must be calculated without any expansion in αZ. Nowadays these calculations cause no problem. Much more challenging is the calculation of two-loop QED corrections which are defined by the diagrams depicted in Fig. 2. To date, most of these diagrams have been calculated and the total uncertainty of the two-loop contribution is determined by the

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last two diagrams in Fig. 2, which have been evaluated in the free-electron-loop approximation only. In addition to the Furry-picture QED contributions, one has to take into account the nuclear recoil, nuclear size, and nuclear polarization effects. The evaluation of the nuclear recoil effect requires using the QED formalism in the nonperturbative in αZ regime beyond the Furry picture. On the one hand, this provides us a rather serious conceptual problem, but, on the other hand, it gives a unique opportunity to test QED in a new region: strong-coupling regime beyond the Furry picture. The nuclear size effect can be easily evaluated by solving numerically the Dirac equation with an extended nucleus. Finally, one should take into account the nuclear polarization effect which can be described by two-photon-exchange electron-nucleus interaction diagrams, in which intermediate nuclear states are excited. The references to the calculations of all aforementioned contributions can be found, e.g., in Ref. [1]. In Table 1, we present the individual contributions to the ground-state Lamb shift in H-like uranium, which should be considered as the main reference point for the QED tests at strong fields. Here the Lamb shift is defined as the difference between the total binding energy and the Dirac binding energy calculated for the pure Coulomb field of the infinite mass nucleus, $V_{\rm C} = -\alpha Z/r$. It should be noted that in Ref. [1] the leading recoil correction was not included in the Lamb shift to make the definition suitable for both light and heavy ions. The nuclear charge radius was taken from Refs. [2, 3]. The finite nuclear size contribution was evaluated taking into account the nuclear deformation effect [2]. It can be seen that the present status of theory and experiment on the Lamb shift in H-like uranium provides a test of QED in the nonperturbative regime at the 2% level.

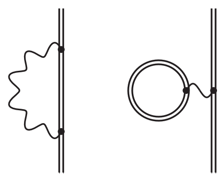

Figure 1: One-loop QED diagrams.

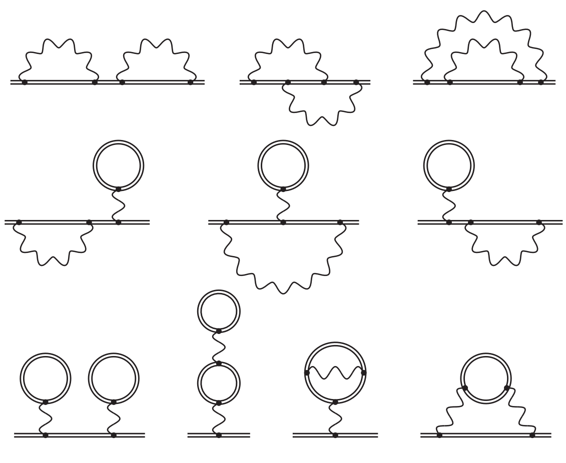

Figure 2: Two-loop QED diagrams.

Table 1: Theoretical contributions to the $1s$ Lamb shift in H-like ^{238}U, in eV.

Contribution	Value
Furry picture QED	265.07(48)
Finite nuclear size	198.51(19)
Nuclear recoil	0.46(1)
Nuclear polarization	−0.20(10)
Total theory	463.84(53)
Experiment [4]	460.2(4.6)

A higher experimental accuracy was achieved for heavy lithiumlike ions [5, 6, 7]. The comparison of theory and experiment on the $2p_{1/2} - 2s$ transition energy in Li-like uranium provides a test of QED at the 0.2% level (see Ref. [2] and references therein).

2.2 Hyperfine splitting

The hyperfine splitting (HFS) of atomic levels is caused by the interaction of the atomic electrons with the magnetic field induced by a nonzero nuclear magnetic moment. For heavy few-electron ions this field becomes extremely strong. For instance, in the case of H-like ^{209}Bi, the electron experiences on average the magnetic field of about 30 000 T. This value is three orders of magnitude greater than that induced by the strongest superconducting magnets. It means that the study of the HFS with heavy few-electron ions could provide tests of QED in a unique combination of the strongest electric and magnetic fields. This has triggered a great interest to measurements of the HFS in H-like ions [8, 9, 10, 11, 12]. However, the calculations of the HFS in H-like ions [13] revealed that the uncertainty due to the nuclear magnetization distribution correction (so-called Bohr-Weisskopf effect) is generally of the same order of magnitude as the QED correction and, therefore, tests of QED effects on the HFS by the direct comparison of theory and experiment for H-like ions are not possible. To circumvent this problem, in Ref. [14] it was proposed to study a specific difference of the HFS values in H- and Li-like ions of the same heavy isotope,

$$\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)}, \tag{2.1}$$

where $\Delta E^{(1s)}$ is the HFS in the H-like ion, $\Delta E^{(2s)}$ is the HFS in the corresponding Li-like ion, and ξ is chosen to cancel the Bohr-Weisskopf effect. In Ref. [14] it was shown that both the parameter ξ and the specific difference $\Delta' E$ are very stable with respect to possible variations of the nuclear models and nuclear parameters. This means that both ξ and $\Delta' E$ can be evaluated to a high accuracy. For instance, in the case of ^{209}Bi the precise calculation leads to ξ = 0.16886. This method has a potential to test QED on the level of a few percent, provided the HFS is measured to accuracy $\sim 10^{-6}$. This proposal has initiated precise experiments on the HFS in Li-like bismuth. As a result, in Ref. [15] the experimental value for the specific difference in ^{209}Bi was reported, but the result was 7σ off from the latest theoretical prediction [16]. This discrepancy established a "hyperfine puzzle" [17], which was resolved in Ref. [18]. New calculations of the magnetic shielding factor in ^{209}Bi(NO$_3$)$_3$ and in ^{209}BiF$_6^-$ performed in Ref. [18] clearly showed that the nuclear magnetic

Table 2: Theoretical contributions to $\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)}$ in ^{209}Bi (in meV) for $\mu/\mu_N = 4.092(2)$ [18]. All theoretical values are obtained by scaling the related contributions from Ref. [16].

Contribution	Value
Dirac value	−31.665
Interelectronic interaction, $\sim 1/Z$	−29.859
Interelectronic interaction, $\sim 1/Z^{2+}$	0.254(3)
One-electron QED	0.036
Screened QED	0.192(2)
Total theory	−61.043(5)(30)
Experiment [15]	−61.012(5)(21)

moment of ^{209}Bi widely used in literature is incorrect. The new calculations and measurements for $^{209}\mathrm{BiF}_6^-$ [18] lead to a new value of the nuclear magnetic moment: $\mu/\mu_N = 4.092(2)$. The individual contributions to the specific HFS difference, obtained with the new magnetic moment, are presented in Table 2. It can be seen that the theoretical result agrees well with the experiment. However, the present status of the accuracy of the nuclear magnetic moment and the HFS experiment limits the QED tests to the 15% level. More precise measurements are needed to provide stringent QED tests.

2.3 Bound-electron *g* factor

First high-precision measurements of the *g* factor of highly charged ions were performed for H-like carbon two decades ago [19]. This experiment has triggered a great interest to precise QED calculations of the bound-electron *g* factor in highly charged ions (see, e.g., Refs. [20, 21] for reviews). The measurements of the bound-electron *g* factors in low- and middle-*Z* H-like ions and comparison with the corresponding theory lead to the most precise determination of the electron mass [22, 23, 24]. Recent measurements for two isotopes of Li-like calcium [25] allowed the first test of the relativistic theory of the nuclear recoil effect with highly charged ions in presence of magnetic field [26]. High-precision measurements of the g factors of heavy few-electron ions are anticipated in the near future at the Max-Planck Institut fuer Kernphysik (MPIK) in Heidelberg and at the HITRAP/FAIR facilities in Darmstadt. In addition to the strong-field QED tests within the Furry picture, these measurements, combined with the corresponding calculations, can provide a test of the QED theory of the nuclear recoil effect on a few-percent level [27]. This would mean the first test of bound-state QED at the strong-coupling regime beyond the Furry picture. Measurements of the g factor of ions with nonzero nuclear spin, which are planned at the same facilities, will give an access to the nuclear *g* factors. Precise values of the nuclear magnetic moments, which can be determined from these experiments by means of the related theory [20, 28], would be of great importance for the HFS study discussed above. Finally, one should mention a possibility for an independent determination of the fine structure constant from the g-factor experiments with highly charged ions, provided the corresponding theoretical calculations are performed to the required accuracy [29, 30].

3. QED in supercritical fields

In accordance with the presently accepted QED formalism, strong static electric field can create electron-positron pairs, provided its strength exceeds a critical value. This effect was predicted many decades ago but has never been observed experimentally. Starting with Refs. [31, 32, 33] predicted spontaneous creation of electron-positron pairs by a strong uniform time-independent electric field (so-called Schwinger mechanism) and Refs. [34, 35] predicted a similar effect in a supercritical Coulomb field, great efforts were undertaken to find feasible scenarios for experimental observation of this fundamental phenomenon. In particular, it was expected that the desired field strength could be achieved with new laser technologies (see, e.g., Ref. [36] and references therein). However, even in the most optimistic scenarios for the developments of these technologies, the field strength which can be achieved in the not too distant future is by two orders of magnitude smaller than the value needed to observe the Schwinger effect. In what follows, we consider the pair creation by supercritical Coulomb field.

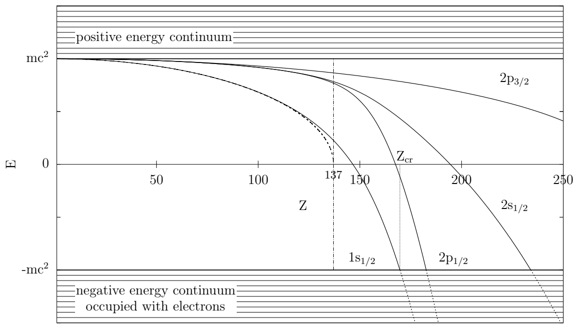

Figure 3: The low-lying energy levels of a H-like ion as functions of the nuclear charge number Z.

In Fig. 3 we display the low-lying energy levels of a H-like ion as functions of the nuclear charge number Z. In the case of the point-charge nucleus, the $1s$ level exists up to $Z \approx 137$ and then disappears. For an extended nucleus, it goes continuously down and reaches the negative-energy continuum at $Z \approx 173$. If this level is empty, its "diving" into the negative-energy continuum should lead to the decay of the originally neutral vacuum into the charged vacuum and two positrons. Since there are no nuclei with so high Z, the only way to access the supercritical regime is to study low-energy collisions of two ions with the total nuclear charge larger than the critical value, $Z_1 + Z_2 > Z_{\rm crit} \approx 173$. The corresponding experiments were first performed about three decades ago at GSI (Darmstadt). However, these experiments could not prove the spontaneous pair creation. One of the reasons preventing the study of the effect of interest consists in the fact that the experiments were performed with many-electron systems. The investigations could be much more successful if they were performed for collisions of bare nuclei. New studies of low-energy heavy-ion collisions at the supercritical regime are anticipated at the upcoming accelerator facilities in Germany, Russia, and China [37, 38, 39]. These facilities will allow to perform experiments on low-energy collisions of heavy bare nuclei. However, there is another problem that poses a serious obstacle to observing the vacuum decay. This is a too short period of time of the supercritical regime. For instance, in U-U collisions at the energy near the Coulomb barrier the supercritical regime emerges for about 10^{-21}s only. This is by two orders of magnitude smaller than the time required for the spontaneous pair creation. The world-leading Frankfurt's group, which worked on this topic for more than 20 years, finally concluded that the spontaneous pair creation could only be observed in collisions with nuclear sticking, in which the nuclei are bound to each other for some period of time by nuclear forces [40, 41]. Since to date there is no evidence of existence of the nuclear sticking in collisions of interest, this scenario does not seem promising. One may expect, however, that the detailed study of quantum dynamics of the electron-positron field in low-energy heavy-ion collisions can result in finding some signatures for the supercritical-field regime. To perform these studies, first of all new methods, which would allow the calculations beyond the approximations employed by the Frankfurt's group, were needed. To this end, new efforts were started a decade ago by the St. Petersburg's group with the developments of new methods for the relativistic calculations of the charge-transfer, electron excitation and ionization processes [42, 43, 44, 45, 46]. These methods can be easily adapted to calculations of the pair-creation probabilities within the monopole approximation, which was mainly used by the Frankfurt's group [47]. In this approximation, the two-center nuclear potential is expanded in spherical harmonics around the center of mass and then only the zero-order spherical harmonic term is used in the calculations (see, e.g., Ref. [48] and references therein). However, till recently it was not completely clear if the monopole approximation is good enough to be used for studying in detail the pair-creation processes in subcritical and supercritical regimes. A significant progress in this direction was made recently in Refs. [49, 50, 51], where the effects beyond the monopole approximation were evaluated by different methods. These calculations revealed that, for the nuclear charge numbers and the energies of interest, this approximation works very well at small impact parameters. In particular, the calculations at the energy near the Coulomb barrier showed that the difference between the exact and the monopole-approximation results varies from about 6% at the zero impact parameter, $b = 0$, to about 10% at $b = 10$ fm. Thus, this approximation should work well for studying the energy and impact-parameter dependences of the pair-creation probabilities for the near-head-on collisions.

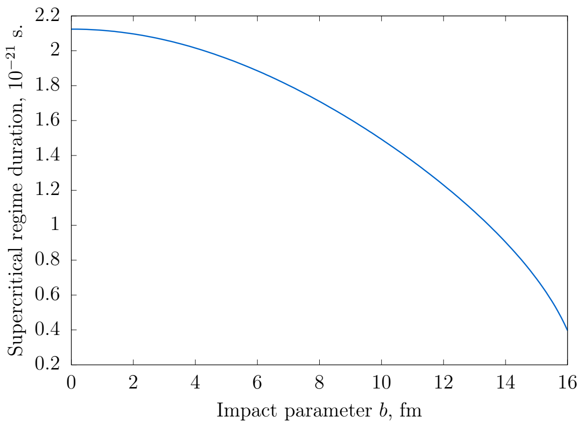

Figure 4: The time period of the supercritical regime in collisions of bare uranium nuclei as a function of the impact parameter b for a given minimal distance of the nuclear approach for all b, $R_{\rm min} = 16.5$ fm.

Let us now discuss how the impact-sensitive measurements of the pair-production probabilities can provide observation of the qualitative difference between the subcritical and supercritical regimes [52]. We consider a low-energy collision of two heavy nuclei with nuclear charge numbers Z_1 and Z_2 and the total charge number larger than the critical one, $Z_1 + Z_2 > 173$. To separate the spontaneous pair creation from the dynamical (induced) one, let us consider the trajectories which correspond to the same minimal distance between the nuclei ($R_{\rm min}$) but different impact parameters (b). According to the Rutherford kinematics, these parameters are related by

$$b^2 = R_{\rm min}^2 - R_{\rm min}\frac{Z_1 Z_2}{E}\,, \tag{3.1}$$

where E is the collision energy. At given value of $R_{\rm min}$, the minimal energy corresponds to the head-on collision and reads as

$$E_0 = \frac{Z_1 Z_2}{R_{\rm min}}\,. \tag{3.2}$$

Let us consider the U-U collision, for which the supercritical regime emerges when the nuclei approach each other at the distance closer than 32.6 fm. With $R_{\rm min} = 16.5$ fm, which corresponds to the distance at which the nuclei are in 1-2 fm away from touching each other, the time period of the supercritical regime as a function of the impact parameter b is presented in Fig. 4. We note that, according to Eq. (3.1), for a given value of $R_{\rm min}$ the impact parameter b and the energy E are related to each other. As it can be seen from Fig. 4, the more is b, the less is the supercritical-regime time period. This corresponds to increasing the energies and, therefore, the relative velocity of the colliding nuclei with increasing b. It is evident that the dynamical pair creation must monotonically decrease with decreasing the velocity (and, therefore, b), provided $R_{\rm min}$ is fixed. At the same time, the spontaneous pair creation must monotonically increase with decreasing b, since the supercritical-regime time period increases. This means that any increase of the pair-creation probability as a function of $\eta = E/E_0$ at $\eta \to 1$ would reveal the effect of the spontaneous pair creation, which is possible only in the supercritical regime. Moreover, even a qualitative change in the behaviour of the derivative of the pair-production probability $P(\eta)$ with respect to η when going from the subcritical to supercritical regime would demonstrate the effect of the spontaneous pair creation. In Fig. 5 we present the behaviour of $dP(\eta)/d\eta|_{\eta=1}$ as a function of the nuclear charge number Z for symmetric collisions ($Z = Z_1 = Z_2$). It can be seen that even in the case of the U-U collision the effect of the decrease of the derivative compared to the subcritical case ($Z < 88$) is very pronounced. We note also that an estimate of the bound quasimolecular level occupation probability for the collision of a bare uranium nucleus with a neutral uranium atom, based on the methods developed in Refs. [42, 43, 53], shows that the filled K shell of the atom can only lead to a few times decrease of the pair-production probability, leaving the main conclusions qualitatively unchanged. This makes feasible the experimental study of the pronounced decrease of $dP(\eta)/d\eta|_{\eta=1}$ as a function of Z with the near future facilities [54, 55]. The observation of this decrease must be considered as a clear proof of the fundamental phenomenon: the vacuum decay in supercritical Coulomb field.

4. Conclusion

We have reviewed the present status of tests of QED with heavy ions. The high-precision measurements of the Lamb shifts in heavy H- and Li-like ions serve as the main reference points

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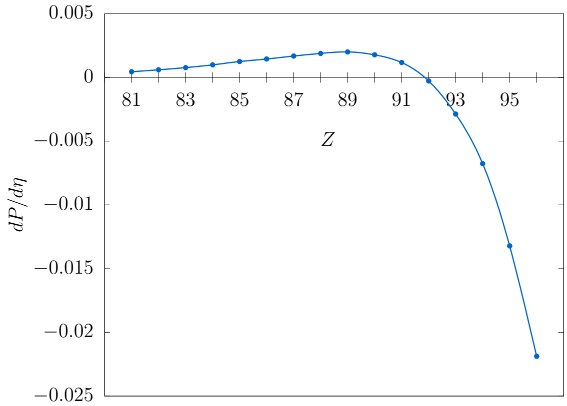

Figure 5: The derivative of the pair-production probability $dP(\eta)/d\eta|_{\eta=1}$ as a function of the nuclear charge number Z for symmetric collisions, $Z = Z_1 = Z_2$. Here $\eta = E/E_0$, E is the collision energy, and E_0 is the head-on collision energy. All the values correspond to a given minimal distance of the nuclear approach, $R_{\rm min} = 16.5$ fm (see the text).

of the QED tests at strong Coulomb field. The high-precision HFS and g-factor experiments with heavy few-electron ions, which are anticipated in the near future, should provide the QED tests in a unique combination of the strongest electric and magnetic fields as well as tests of QED at the strong-coupling regime beyond the Furry picture. Given these tests are performed, the experiment and theory with heavy ions can be employed for precise determinations of various nuclear parameters and fundamental constants. The study of the pair-creation probabilities in low-energy heavy-ion collisions near the Coulomb barrier energy should provide a unique access to QED in supercritical field. The precise measurements of the pair-creation probabilities for different impact parameters but with a given minimal distance of approach of the colliding nuclei and comparison with the theory will result either in discovery of the vacuum decay in supercritical Coulomb field or in finding new physics, which is beyond the standard QED formalism.

Acknowledgements

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