PROCEEDINGS OF SCIENCE

Muon Energy Reconstruction for Neutrino Detectors with Edepillim

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Large scale high energy neutrino detectors measure through going muons, which are reconstructed to infer the arrival direction of the originating neutrinos on the sky. In addition to this directional information, it is important to infer the energy of these muons, needed for discriminating astrophysical signals from atmospheric backgrounds in a diffuse analysis, or for increasing the ability to distinguish a point source of neutrinos from the backgrounds. Various methods exist to infer the muon energy, based on the fact that the light produced by each muon is on average proportional to its energy. Here, we describe the Edepillim algorithm, which interprets the entire pattern of stochastic muon energy losses along each track. The probability of each loss is used in a likelihood approach, taking account of the decreasing muon energy along the track, leading to a one-parameter fit for the initial muon energy. In this presentation, we will review the basic performance of this algorithm using idealised simulated muons, and discuss how the method might perform under real-world circumstances where the losses cannot be known precisely, but must be inferred by an unfolding of the observed light signals across a large array of detectors in a medium such as ice or water.

36th International Cosmic Ray Conference -ICRC2019-
July 24th - August 1st, 2019
Madison, WI, U.S.A.

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PoS(ICRC2019)909

1. Introduction

Neutrinos are an ideal tracer particle for high energy astrophysics as they are not deflected by magnetic fields. As such, they can be detected at Earth with their directional information intact. Experiments such as ANTARES and the IceCube Neutrino Observatory operate by observing Cherenkov radiation emitted from charged secondary interaction particles of neutrinos such as muons. With the increase in detection of astrophysical neutrinos by IceCube, the quality of muon energy reconstruction needs to be improved.

As a muon travels through a medium it experiences energy loss due to radiative processes: bremsstahlung, pair production and photonuclear reactions. These processes will cause energy loss of the muon in stochastic bursts thus the overall muon path can be represented by an energy loss pattern. The probability of these losses is dependent on the initial energy of the muon.

This work is based around the development of a new muon energy reconstruction method that, unlike previous methods, uses the whole energy loss pattern of the muon. This new method capitalises on the stochastic nature of muon interactions to produce an energy reconstruction that we will show in the idealised case has better energy resolution than any previous method (e.g. truncated energy (TE), [1]) when using the true energy loss pattern.

2. The Edepillim Method

The Edepillim method is a muon reconstruction that uses the probabilities of each observed muon energy loss within the energy loss pattern to achieve a high resolution energy reconstruction result [1].

A muon will undergo stochastic energy loss processes, which will result in an uneven pattern of energy loss along the muon track. The amplitude of each energy loss will be related to the muon's energy immediately prior to the loss. In general, high energy muons will have larger energy losses than those of low energy muons.

This idea can be expressed in terms of a probability likelihood function for a muon with an initial energy E_0 and an energy loss pattern $\vec{dE} = \{dE_0, dE_1, dE_2, ..., dE_N\}$, where dE_0 is the first energy loss, and dE_N is the Nth loss. The likelihood of the total observed loss pattern is based on successive probability distribution functions for each individual loss:

$$P(\vec{dE}; E_0) = P(dE_0; E_0)P(dE_1; E_1)...P(dE_N; E_N) \tag{2.1}$$

where there are N losses and each $P(dE_i; E_i)$ represents an individual probability of seeing an energy loss dE_i , over a segment length X, for a particular muon energy E_i.

The probability in Eq 2.1 can be simplified, as the muon energy after the first energy loss, E_1, is simply the difference between the initial muon energy, E_0, and the previous energy loss, dE_0. The progression of this along the muon track means the energies of the muon at any particular step can be expressed in terms of the initial muon energy and the sum of the previous energy losses,

[1] After our development of this method, we learned that Petrukhin and Kokoulin had proposed a similar idea in [2], and we thank them for several useful discussions.

$$E_N = E_0 - \sum_{i=0}^{N-1} dE_i \tag{2.2}$$

Because of this, successive muon energies can, for each probability, be put in terms of the original muon energy E_0. Thus, a probability likelihood function can be written as the following

$$P(\vec{dE}; E_0) = P(dE_0; E_0) \prod_{j=1}^{N} P(dE_j; E_0 - \sum_{i=0}^{j-1} dE_j) \tag{2.3}$$

A maximum likelihood approach using the log likelihood of Eq 2.3 can be used to compute the unknown variable; the muon's initial energy E_0.

$$\ln L = \sum_{i=0}^{N} \ln P(dE_i; E_i) \tag{2.4}$$

where dE_i is the energy loss in a bin i, E_i is the muon energy at the start of the bin i and N is the total number of energy loss segments along the path length.

In order to reconstruct an accurate muon energy, the Edepillim method requires an accurate Probability Distribution Function (PDF). The PDF needs to provide the probability of an energy loss occurring given a muon energy $P(dE;E)$, however in practice not every individual energy loss can be known, due to detector resolution. Instead, a total of all the energy losses would be known in a given segment length X, thus each PDF would need to be made for a required segment length $P_X(dE,E)$. To build the PDF, the simulation software PROPOSAL [3] is used to generate a muon at energy E and allow it to travel along a segment length X. Then the energy loss is taken as $dE = E_{initial} - E_{final}$, where $E_{initial}$ is the energy the muon was generated at and E_{final} is the generated muon's energy after a segment length X.

The PDF in Figure 1 displays features that may have effects on the performance of the reconstruction. The diagonal one-to-one line is the physical cut off for energy losses not to be greater than the initial energy of the muon. This is an important physical constraint to enforce in the reconstruction because if you have a muon with a given energy loss you must only consider muon energies greater than the given loss.

The PDF's vertical line represents the minimum value of energy loss for the given segment length X, and, as continuous losses are included, there is a minimum of approximately 200 MeV/m from ionisation. The use of continuous losses in the PDF impacts the implementation of Edepillim as all losses in the true or reconstructed energy loss pattern will need to have a continuous component to them. For each segment length of a muon that will be used a unique PDF needs to be produced.

3. Edepillim Energy Reconstruction Results in an Idealised Case

In order to test the Edepillim reconstruction method, a simple simulation of muons was produced using PROPOSAL [3] to propagate a muon of an initial energy over a path length of 1000 m with the energy losses reported for every segment length $X = 1$ m.

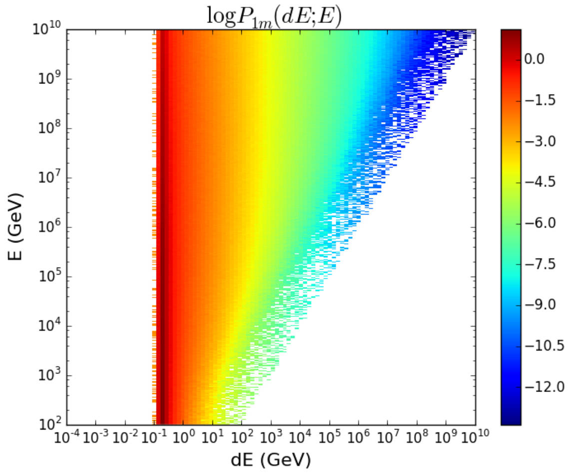


Figure 1: Probability distribution function built from simulated muon energy losses within a segment length $X = 1$ m, with each probability taking into account the bin width in dE, to convert the total counts per bin into a PDF, displayed in the log of probability. There is a diagonal line which represents the limit of the energy loss being less than the muon energy. The vertical line is the minimum probable energy loss $dE = 0.2$ GeV.

The energy loss patterns produced in this idealised simulation have a continuous loss at the minimum ionisation loss rate as well as larger stochastic losses that are produced due to the processes of Bremsstrahlung, pair production and photonuclear reactions. The energy loss patterns are consistent between events with the amplitude of the stochastic losses scaling with the muon's energy.

3.1 Muon Energy Loss Segment Length

In large scale neutrino detectors, accurate energy loss pattern reconstruction can be limited depending on the medium and the detector. In a detector such as IceCube the ice can cause scattering of the photons making the reconstruction of the location of the energy losses uncertain. For a detector such as ANTARES the water medium has a larger scattering length and thus the energy loss pattern can be estimated with more certainty in the segment length [4]. Because of this possible uncertainty in the location and size of reconstructed energy losses it is unlikely that the 1 m energy loss resolution as used in this simulation would be possible. To show the performance of Edepillim under different detector conditions the individual energy loss patterns of the simulated events were combined to make energy loss segment lengths of 10 m, 20 m, 50 m, 100 m, 200 m and 500 m. The purpose of larger segment lengths is to show how the performance of Edepillim changes as the expected resolution of the reconstructed energy loss pattern changes. The energy loss pattern's change with an $X = 100$ m segment length is shown in Figure 2 where the energy losses are more uniform with no clear distinction for stochastic energy losses.

The performance of the Edepillim energy reconstruction for segment length $X = 1$ m is shown in Figure 3 with the reconstruction distribution having a consistent peak that correlates to the true muon energy, E_{TRUE}, and the reconstructed muon energy, E_0. With the increase in segment length, the reconstruction distributions become wider but maintain a consistent peak. In Figure 3 when

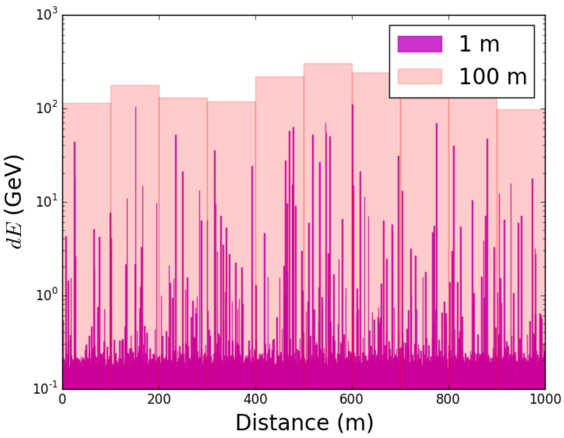
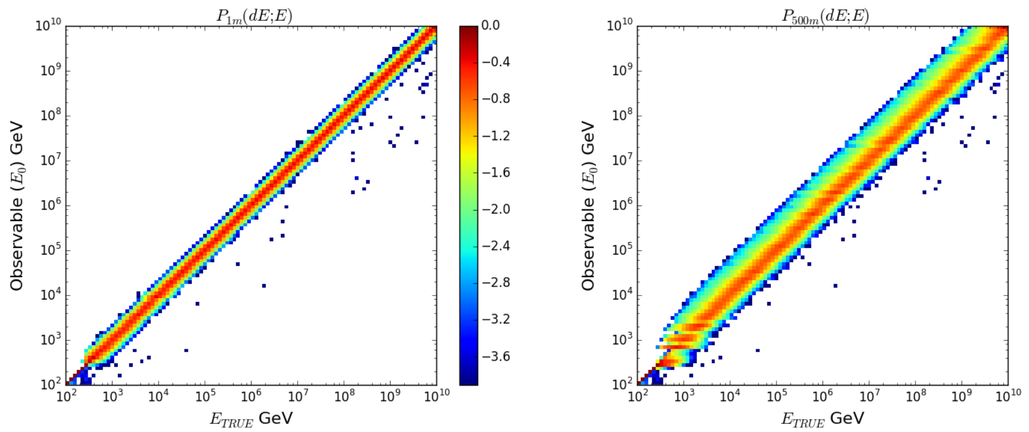


Figure 2: An example of an energy loss pattern for a muon event simulated at $E_{TRUE} = 10^4$ GeV with energy losses in a segment length $X = 1$ m combined into a segment length of $X = 100$ m. The combined segment length results in a more uniform energy loss pattern with difficulty distinguishing the stochastic energy losses.

using $X = 500$ m, which would result in only two losses, the reconstruction distribution has a clear smearing effect across true energies.



(a) $X = 1$ m

(b) $X = 500$ m

Figure 3: Reconstruction distributions of the observable muon reconstructed energy E_0 vs the true muon energy E_{TRUE} for muon pattern energy losses of segment length (a) $X = 1$ m (b) $X = 500$ m with a total path length $L = 1000$ m. Displayed in the logarithm of muon events, normalised per E_{TRUE}.

The reconstruction distribution can be interpreted as if there was a beam of muons at a particular true energy, and then the slice vertically would be the distribution of reconstructed energies that those muons could be reconstructed as. However, each reconstructed energy has a likelihood, taken as the horizontal slice of the plot, which gives the range of true energies that the given reconstructed energy could correspond to, i.e. the confidence interval of true energies given the observation. These confidence intervals need to be accounted for in the resolution by combining all the confidence intervals with a weight applied to each interval, each weight corresponding to the probability of getting that reconstructed energy given the true energy of the muon beam. Overall, this results in an average likelihood function for the target true energy. The one-sigma width is then taken as the resolution measure for that target true energy. Figure 4 shows the resolution obtained from the respective reconstructions, using the Average Likelihood Funcation (ALF). This technique was also used to present the resolution of the truncated mean energy loss rate in [5].

The Edepillim method has optimal resolution with energy losses binned in $X = 1$ m intervals. As the segment length increases the individual energy losses get grouped together and so the detail in the energy loss pattern is lost. This reduces the Edepillim reconstruction performance as there will be fewer probabilities in the likelihood with increasing segment length. In Figure 4 the larger binned energy loss lengths result in a poorer resolution. However, it is interesting to note that in the $X = 500$ m case in Figure 3 there are only two energy losses, and while this is not enough information for the Edepillim method to perform with high resolution, it is still a working reconstruction, despite only two input dE values.

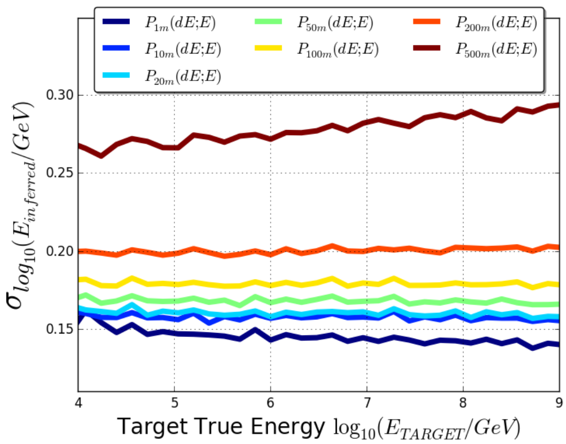


Figure 4: The Edepillim energy reconstruction ALF resolution measure for respective energy loss segment length over a range of target true muon energies. The reconstruction resolution increases as the energy loss segment length increases, indicating the best resolution is for energy loss patterns with the smallest segment length.

The results in Figure 4 show how the optimal resolution for energy reconstruction can be obtained. The level of resolution achieved in a realistic case with real reconstruction of the losses will depend on how accurately the energy loss pattern can be known. If we know the losses at a 1 m scale, then the energy reconstruction can be obtained with a resolution of $\sigma = 0.12$ in the logarithm of the target true energy.

3.2 Muon Energy Loss Pattern Path Length

In kilometre scale detectors such as IceCube, not all muon events will travel in a path that allows for a 1000 m energy loss pattern to be reconstructed. Through-going muons will more likely pass though at angles that do not pass through the centre of the detector and thus would have a path length of less than 1000 m. Also, in very rare cases muons may be produced inside the detector, giving a starting track.

To simulate the potential cases of shorter muon tracks, the muon energy loss patterns were limited to path lengths of 750 m, 500 m and 250 m. These path lengths do not mean that the muon has run out of energy, merely that the shorter length is all that is observed of the muon. The results for the case of $X = 1$ m segment length with muon track lengths less than 1000 m in Figure 5 displays the large loss in resolution power of the reconstruction with shortened track lengths.

These results show that the total muon path length has a large impact on the performance of the reconstruction. The effect is even greater than that shown by energy loss segment length, for as soon as only half the track is observed (path length 500 m for $P_{1m}(dE;E)$), the resolution becomes similar to using only two energy losses, $P_{500m}(dE;E)$ in Figure 4.

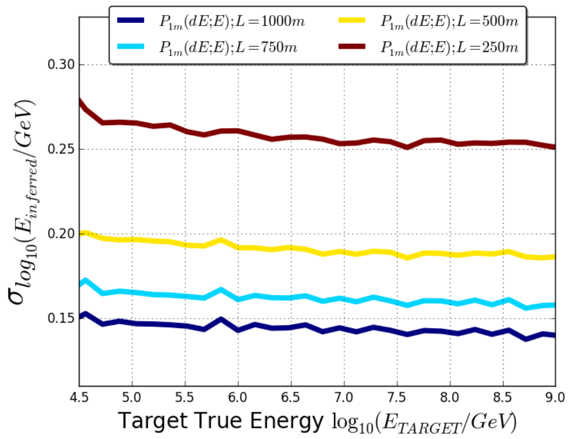


Figure 5: The Edepillim energy reconstruction ALF resolution distribution for segment length $X = 1$ m with varying muon path length L over a range of target true muon energies.

3.3 Comparison to Idealised Case of Truncated Mean Energy Loss Rate

The truncated energy method (TE) as implemented in IceCube analyses is the robust method described in [1]. For comparison, a simplified version of the TE method is tested against our Edepillim method. The TE method operates by removing a number of the largest losses, T, and then calculating the mean energy loss rate dE/dX using the remaining losses. The removal of the losses results in a more uniform energy loss pattern, with a reduction in the number of high energy stochastic losses. The resolution for idealised TE will not give the same resolution results as have been published in [1] as the IceCube method was optimised for detector conditions.

The results of the energy resolution can be compared for the two energy reconstruction methods, Edepillim and TE. The resolutions in Figure 6 are for the optimal segment length or truncated loss that was found for each method. For Edepillim the optimum is segment length $X = 1$ m, and TE removes the three highest losses $(T = 3)$.

In the idealised case, Edepillim has the best resolution of the two methods, however this resolution is limited by the level of bin size resolution in the energy loss pattern. If the losses were known to a smaller segment length than $X = 1$ m the reconstruction resolution would improve. While truncated energy with the three highest losses removed performs with a slightly larger resolution, it has been proven to be a robust and consistent method to perform within IceCube [1]. It is also important to note that the difference between Edepillim and TE is only $\sigma_{TE} - \sigma_{Edepillim} \sim 0.1$, which means all real-life effects of the detector response must be considered in order to decide on the best reconstruction in practice.

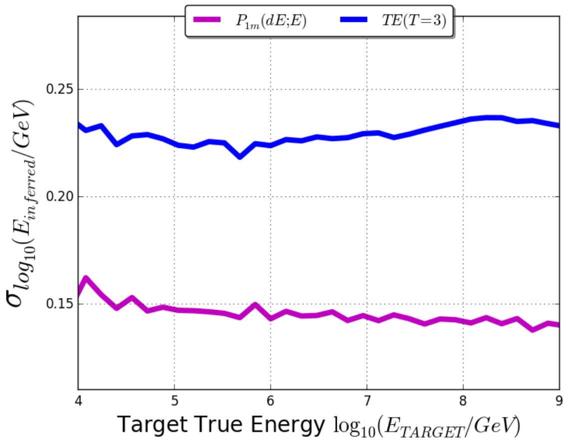


Figure 6: The ALF reconstruction resolution for comparison of the Edepillim method against the optimal TE method for muon energy pattern path length $L = 1000\,\mathrm{m}$.

3.4 Summary

The Edepillim method uses a maximum likelihood approach to reconstruct the muon energy at the beginning of an energy loss pattern. The method relies on accurate descriptions of the muon energy losses along the path length using probability distribution functions built from simulation to describe the energy losses given the muon energy. Using the energy loss pattern, Edepillim can perform a maximum likelihood fit over a range of test energies to find the reconstructed muon energy. This method produces a very good reconstruction resolution with tests having been made for various segment lengths showing that the smallest segment length will give the best result. Additionally, the total muon path length was tested showing that the longest path length gave the best result. The Edepillim reconstruction will continue to improve with more information about the true energy losses. In particular, that the more individual energy losses the better the reconstruction, with improvements seen for longer path lengths and smaller segment length rebinning.

References

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