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Z_b tetraquark channel and $B\bar{B}^*$ interaction

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Two tetraquark candidates $Z_b(10610)$ and $Z_b(10650)$ with flavor structure $\bar{b}b\bar{d}u$ were discovered by Belle experiment in 2011. We present a preliminary $N_f = 2$ lattice study of the $\bar{b}b\bar{d}u$ system in the approximation of static b quarks, where the total spin of heavy quarks is fixed to one. The ground and the excited eigen-energies are determined as a function of separation r between b and $\bar{b}$. The lower eigenstates are related to a bottomonium and a pion. One of the higher eigenstates is dominated by $B\bar{B}^*$: its energy is significantly below $m_B + m_{B*}$ for r=[0.1,0.4] fm, which suggests sizable attraction. The attractive potential $V(r)$ between B and $\bar{B}^*$ is extracted assuming that this eigenstate is related exclusively to $B\bar{B}^*$. Assuming a certain form of the potential at small $r < 0.1$ fm and solving non-relativistic Schrodinger equation, we find a virtual bound state pole 32^{-29}_{+5} MeV below $B\bar{B}^*$ threshold. This pole leads to a narrow peak in the cross-section just above threshold that could be perhaps related to experimental Z_b resonances. Given all these approximations, we surprisingly find also a deep bound state 403 ± 70 MeV below $B\bar{B}^*$ threshold. If such a Z_b state exists, it could be experimentally searched in the accurate dependence of rates on $\Upsilon(1S)\pi^+$ invariant mass.

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1. Introduction

The Belle experiment observed two Z_b^+ states with exotic quark content $\bar{b}b\bar{d}u$ and $J^P = 1^+$ in 2011 [1, 2]. The lighter state $Z_b(10610)$ lies slightly above $B\bar{B}^*$ threshold and the heavier $Z_b(10650)$ just above $B^*\bar{B}^*$ threshold. The experimentally discovered decay modes are [1, 2, 3]

$$Z_b^+ \to B\bar{B}^*,\ B^*\bar{B}^*, \Upsilon(1S)\pi^+,\ \Upsilon(2S)\pi^+,\ \Upsilon(3S)\pi^+,\ h_b(1S)\pi^+,\ h_b(2S)\pi^+ \tag{1.1}$$

where the $B\bar{B}^*$ and $B^*\bar{B}^*$ largely dominate $Z_b(10610)$ and $Z_b(10650)$ decays, respectively.

The only preliminary lattice QCD study of these states has been reported in [4, 5] and will be briefly reviewed below. No other lattice results are available since this channel present a severe challenge. The proper method would require determination of scattering matrices for at least 7 coupled channels (1.1) using the rigorous Lüscher's method. Poles of such scattering matrix would render information on possible Z_b states. Following this path seems too challenging for the moment.

In the present study we consider simplified Born-Oppenheimer approximation, inspired by the study of this system in [4, 5]. This approach is based on distinction between heavy and light degrees of freedom and finds enormous application in molecular physics. It is valuable also for hadron systems with heavy quarks (see for example [6, 7]) and represents a good approximation for Z_b system since m_b is large. In the first step, b and $\bar{b}$ are treated as static and fixed at distance r (Figure 1a). We determine the eigen-energies $E_n(r)$ for the light degrees of freedom (light u/d quarks and gluons) in presence of these two static sources as function of r by lattice QCD. The low-lying eigenstates (relevant for quantum numbers discussed in Section 2) are related to two-hadron states illustrated in Figs. 1 (b-d)

$$B(0)\bar{B}^*(r),\ \ \Upsilon(r)\ \pi(\vec{p}=0),\ \ \Upsilon(r)\ \pi(\vec{p}\neq 0),\ \ \Upsilon(r)\ b_1(\vec{0})\ , \qquad \vec{p}=\vec{n}\tfrac{2\pi}{L}\ . \tag{1.2}$$

The energy of $B\bar{B}^*$ in Fig. 1b is of most interest since Z_b resonances lie near $B\bar{B}^*$ threshold. This eigen-energy $E(r)$ provides the potential $V(r) = E(r) - m_B - m_{B*}$ between B and $\bar{B}^*$ within certain simplifying assumptions mentioned below. The ground state of this system is not represented by $B\bar{B}^*$ ([1]), but by the bottomonium-pion states $\Upsilon(r)\ \pi(\vec{p})$. The $\Upsilon(r)$ denotes spin-one bottomonium where $\bar{b}$ and b are separated by r and its energy is given by the well-known static potential $V_{\bar{b}b}(r)$. Pion can have zero or non-zero momentum $\vec{p} = \vec{n}\frac{2\pi}{L}$ since total momenta of light degrees of freedom is not conserved in presence of static quarks, i.e. the momentum of light meson is not conserved when it scatters on infinitely heavy bottomonium. Our task is to extract energies of all these eigenstates and extract the potential $V(r)$ between B and $\bar{B}^*$.

In the second step of the Born-Oppenheimer approach, the heavy degrees of freedom are relaxed from their static positions. The B and $\bar{B}^*$ mesons with finite masse $m_{B^{(*)}}^{exp}$ evolve in the extracted potential $V(r)$. The non-relativistic Schrodinger equation is solved to look for possible Z_b bound states, virtual bound states or resonances in this system.

The only previous lattice study of this system [4] presents preliminary results based on Fock components $B\bar{B}^*$ and $\Upsilon\pi(0)$. The presence of states $\Upsilon\pi(\vec{p}\neq 0)$ was mentioned in [5], but not included in the simulation.

[1] In contrast to open beauty channel $bb\bar{u}\bar{d}$ where BB^* is the lowest two-hadron channel.

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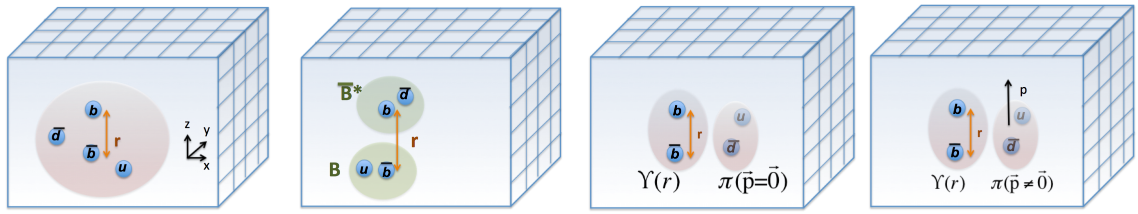

(a) (b) (c) (d)

Figure 1: (a) System considered. (b-d) Two-hadron Fock components relevant in the system with quantum numbers (2.1).

2. Quantum numbers and operators

The quantum numbers of neutral Z_b^0 in experiment are $I=1$, $I_3=0$ and $J^{PC}=1^{+-}$. Certain quantum numbers are somewhat different in the systems with two static particles and are reminiscent of diatomic molecule. We study the system with quantum numbers

$$I=1,\ I_3=0,\quad S^{heavy}=1,\ S_z^{heavy}=0,\quad J_z^{light}=\lambda=0,\quad \varepsilon=-1,\quad C\cdot P=-1\ , \tag{2.1}$$

where the neutral system is considered so that C-conjugation can be applied (Fig. 1 shows the charged partner). Only the z-component of angular momenta for light degrees of freedom (J_z^{light}) is conserved. The ε is a quantum number related to the reflection over the yz plane. P refers to inversion with respect to mid-point between b and $\bar{b}$ and C is charge conjugation, where only their product is conserved.

The spin of infinitely heavy quark can not flip via interaction with gluons, so S^{heavy} of $\bar{b}b$ is conserved. We choose to study system with $S^{heavy}=1$, $S_z^{heavy}=0$, where decays to Υ are allowed, while decays to η_b and h_b are forbidden. Note that physical system Z_b and $B\bar{B}^*$ with finite m_b can be a linear combination of $S^{heavy}=1$ as well as $S^{heavy}=0$ and we study only $S^{heavy}=1$ component. We have in mind this component including $B\bar{B}^*$, $\bar{B}B^*$, $\bar{B}^*B^*$ (O_1 in Eq. 2.2) when we refer to "$B\bar{B}^*$".

We employ 6 operators with quantum numbers (2.1) which resemble Fock components (1.2) in Figs. 1 (b-d)

$$\begin{aligned}
O_1 = O^{B\bar{B}^*} &\propto \sum_{a,b}\sum_{A,B,C,D} \Gamma_{BA}\tilde{\Gamma}_{CD}\ \bar{b}_C^a(0)q_A^a(0)\ \bar{q}_B^b(r)b_D^b(r)\ , \quad \Gamma=P_-\gamma_5\ \tilde{\Gamma}=\gamma_z P_+, \qquad (2.2)\\
&\propto [\bar{b}(0)P_-\gamma_5 q(0)]\ [\bar{q}(r)\gamma_z P_+ b(r)] + [\bar{b}(0)P_-\gamma_z q(0)]\ [\bar{q}(r)\gamma_5 P_+ b(r)]\\
&\quad - [\bar{b}(0)P_-\gamma_x q(0)]\ [\bar{q}(r)\gamma_y P_+ b(r)] + [\bar{b}(0)P_-\gamma_y q(0)]\ [\bar{q}(r)\gamma_x P_+ b(r)]\\
O_2 = O^{\Upsilon\pi(0)} &\propto \Upsilon_z\ \pi_{p=000} = [\bar{b}(0)\gamma_z P_+ b(r)]\ [\bar{q}\gamma_5 q]_{p=000}\\
O_3 = O^{\Upsilon\pi(1)} &\propto \Upsilon_z\ (\pi_{p=001}+\pi_{p=00-1}) = [\bar{b}(0)\gamma_z P_+ b(r)]\ \big([\bar{q}\gamma_5 q]_{p=001}+[\bar{q}\gamma_5 q]_{p=00-1}\big)\\
O_4 = O^{\Upsilon\pi(2)} &\propto \Upsilon_z\ (\pi_{p=002}+\pi_{p=00-2}) = [\bar{b}(0)\gamma_z P_+ b(r)]\ \big([\bar{q}\gamma_5 q]_{p=002}+[\bar{q}\gamma_5 q]_{p=00-2}\big)\\
O_5 = O^{B\bar{B}^*} &\propto \sum_{a,b}\sum_{A,B,C,D} \Gamma_{BA}\tilde{\Gamma}_{CD}\ \bar{b}_C^a(0)\nabla^2 q_A^a(0)\ \bar{q}_B^b(r)\nabla^2 b_D^b(r)\ , \quad \Gamma=P_-\gamma_5\ \tilde{\Gamma}=\gamma_z P_+,\\
O_6 = O^{\Upsilon\, b1(0)} &\propto \Upsilon_z\ (b1_z)_{p=000} = [\bar{b}(0)\gamma_z P_+ b(r)]\ [\bar{q}\gamma_x\gamma_y q]_{p=000}
\end{aligned}$$

and color singlets are denoted by [..]. First line in $O_{1,5}$ decouples spin indices of light and heavy quarks, so that transformation under J_z^{light} is more transparent [4] , while the second line in O_1 is obtained via Fierz transformation. $O_{3,4}$ have pion momenta in z direction due to J_z^{light} and have two terms due to $C\cdot P$. The $\Upsilon\, b_1$ is not a decay mode for finite m_b where C and P are separately conserved, but it is has quantum numbers (2.1) for $m_b\to\infty$. All light quarks $q(x)$ are smeared around the central position x using full distillation [8] with radius $\simeq 0.3$ fm, while heavy quarks are point-like.

We verified that there are no other two-hadron Fock components in addition to (1.2) with quantum numbers (2.1) and with non-interacting energies below $m_B+m_{B^*}+0.2$ GeV.

3. Lattice details

Lattice simulation with $N_f=2$, $m_\pi\simeq 266$ MeV and $a\simeq 0.124$ fm is performed. We take an ensemble with small $N_L=16$ and $L\simeq 2$ fm so that only $\Upsilon\pi(p_z)$ with $p_z\le 2\frac{2\pi}{L}$ appear in the energy region below $m_B+m_{B*}+0.2$ GeV; larger volumes would require additional operators like $O_{3,4}$ for higher pion momenta.

Correlation matrices $\langle O_i(t)O_j^\dagger(0)\rangle$ for operators $O_{i,j=1,..,6}$ (2.2) are evaluated using full distillation method [8] and $\bar{b}b$ annihilation is omitted. The eigen-energies of the system are extracted from the correlation matrices using the well-known GEVP variational approach.

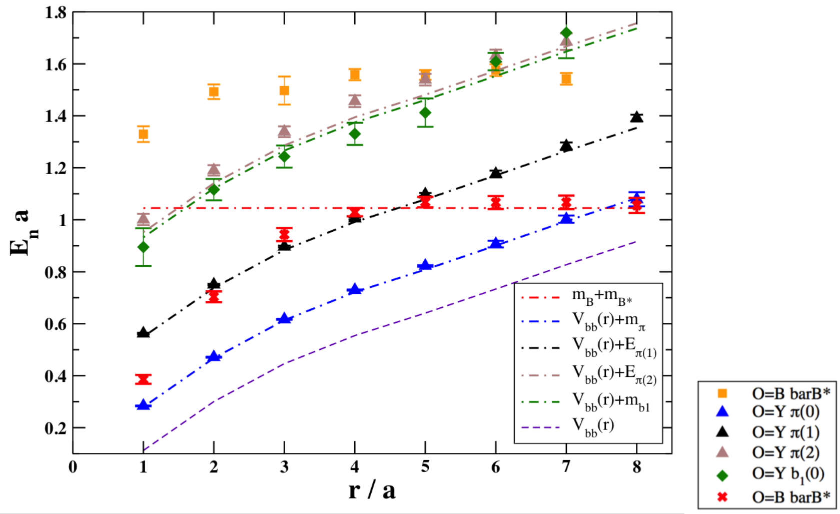

Figure 2: Eigen-energies of $\bar{b}b\bar{d}u$ system (Fig. 1a) for various separations r between static quarks b and $\bar{b}$ are shown by points. The label on the right indicates which two-hadron Fock component dominates each eigenstate. The dot-dashed lines indicate related two-hadron energies $E^{n.i.}$ (4.1) in the limit when two hadrons (1.2) do not interact. The most important conclusion based on this spectra is that $B\bar{B}^*$ eigenstate (red crosses) has energy significantly below m_B+m_{B*}, therefore shows sizable strong attraction for $r=[0.1,0.4]$ fm. Lattice spacing is $a\simeq 0.124$ fm.

4. Eigen-energies of $\bar{b}b\bar{d}u$ system as a function of r

The main result of our study are the eigen-energies of the $\bar{b}b\bar{d}u$ system (Fig. 1a) with static b and $\bar{b}$ separated by r, that are shown by points in Figure 2. The colors of points indicate which

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Fock-components among (1.2) dominates an eigenstate, as determined from overlaps $\langle O_i|n\rangle$ of an eigenstate $|n\rangle$ to operator O_i.

The dashed lines provide related non-interacting (n.i.) energies E_n of two-hadron states (1.2)

$$E^{n.i.}_{B\bar{B}^*} = m_B + m_{B*}, \quad E^{n.i.}_{\Upsilon\pi(\vec{p})} = V_{\bar{b}b}(r) + E_{\pi(\vec{p})} = V_{\bar{b}b}(r) + \sqrt{m_\pi^2 + \vec{p}^2}, \quad E^{n.i.}_{\Upsilon b_1(0)} = V_{\bar{b}b}(r) + m_{b_1} \quad (4.1)$$

where $\bar{b}b$ static potential $V_{\bar{b}b}(r)$, $m_B = m_{B^*}$ (for $m_b \to \infty$), m_π and m_{b_1} are determined on the same ensemble.

The eigenstate dominated by $B\bar{B}^*$ has energy close to $m_B + m_{B^*}$ for $r > 0.5$ fm, but has it has significantly lower energy for $r \simeq [0.1, 0.4]$ fm (red crosses in Fig. 2). This indicates sizable strong attraction between B and $\bar{B}^*$ in this system - something that might be related to the existence of Z_b tetraquarks. This is the most important and robust result of our lattice simulation.

Other eigenstates are dominated by $\Upsilon\pi(\vec{p})$ and Υb_1 states. Their energies E lie close to non-interacting energies $E^{n.i.}$ (4.1) given by dot-dashed lines, so $E \simeq E^{n.i.}$. We point out that our simulation is not accurate enough to claim nonzero energy shifts $E - E^{n.i.}$ for $\Upsilon\pi$ and Υb_1 states, although Fig. 2 shows small deviation from zero in some cases.

5. Towards masses of Z_b states within several simplifying approximations

The total energy of the $\bar{b}b\bar{d}u$ system is composed of the energy $E_n(r)$ for static b and $\bar{b}$, determined in the previous section, plus the kinetic energy of heavy quarks, which presents a small perturbation within Born Oppenheimer approximation. The heavy quarks are now relaxed from their static positions and evolve in the potentials determined from $E_n(r)$.

We apply two serious simplifying approximations in order to shed some light on the possible existence of Z_b based on energies in Figure 2. The first assumption is that the eigenstate indicated by red crosses in Fig. 2 is related exclusively to $B\bar{B}^*$ Fock component and does not contain other Fock components in (1.2). This is supported by our lattice results to a good approximation, since this eigenstate couples almost exclusively to $O^{B\bar{B}^*}$ and has much smaller coupling to $O^{\Upsilon\pi}$ and $O^{\Upsilon b_1}$ ([2]). In this case, the energy $E(r)$ of this eigenstate provides potential $V(r) = E(r) - m_B - m_{B*}$ between B and $\bar{B}^*$, given in Fig. 3. The potential shows sizable attraction for small distances and is compatible with zero for $r \geq 0.6$ fm within sizable statistical errors of our result. Lattice study that would probe whether one-pion exchange dominates at large r would therefore need much higher accuracy.

The form of the potential $V(r)$ at $r < a \simeq 0.124$ fm is not known and it might be affected by discretization effects at $r \simeq a$. This brings us to the second simplifying approximation, where we assume a certain form to fit our potential in Fig. 3

$$V(r) = -A\, e^{-(r/d)^p}, \quad p = 3/2,\ A = 0.99(5),\ d = 1.84(10) \quad (5.1)$$

where A and d follow from the fit for $r/a = [1, 4]$. We note that more physically motivated forms of the potentials at small r will be considered in the forthcoming publication.

The possible (virtual) bound states or resonances of the $B\bar{B}^*$ system with determined potential (5.1) are obtained by solving the non-relativistic 3D Schrodinger equation $[-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} +$

[2] The overlap $\tilde{Z}^{B\bar{B}^*}_{2,3,4,6}/\tilde{Z}^{B\bar{B}^*}_1 < 0.02$ where $\tilde{Z}^n_i = Z^n_i/max_m(Z^m_i)$ is normalized overlap $Z^n_i = \langle O_i|n\rangle$.

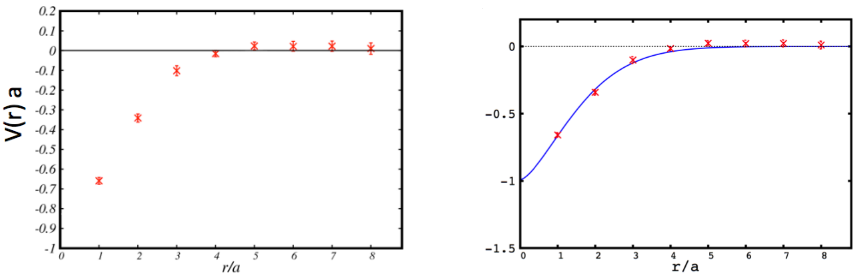


Figure 3: Left: The potential $V(r)$ between B and $\bar{B}^*$ as function of separation r extracted from our lattice simulation. It is based on a simplifying approximation discussed in Section 5 and can be prone to lattice discretization errors for $r/a \simeq 1$. Right: Fit assuming the form (5.1) for central values of parameters. Lattice spacing is $a \simeq 0.124$ fm.

$V(r)]u(r) = Wu(r)$ for finite (measured) $B^{(*)}$ meson masses and $1/\mu = 1/m_B^{exp} + 1/m_{B^*}^{exp}$. Scattering matrix $S_l(W) = e^{2i\delta_l(W)}$ for partial wave l is obtained from the phase shifts $\delta_l(W)$.

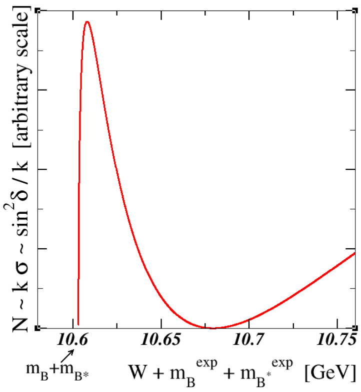
The potential (5.1) leads to s-wave virtual bound state $W_B = -32^{+29}_{-5}$ MeV below $B\bar{B}^*$ threshold in s-wave. Virtual bound state pole in the scattering matrix occurs below threshold for imaginary cmf momenta $k = -i|k|$ of $B^{(*)}$. This pole leads to a narrow peak in the cross section above threshold shown in Fig. 4a. The peak resembles the observed $Z_b \to B\bar{B}^*$ peak by Belle in Fig. 4b [3]. The virtual bound state found by our lattice simulation might therefore be related to Z_b from experiment. Z_b was found as a virtual bound state pole in $B\bar{B}^*$ few MeV below threshold also by the analysis of the experimental data [9] when the coupling to bottomonium light-meson channels was turned off; the position of the pole if only slightly shifted when this small coupling is taken into account [9].

Surprisingly, the strongly attractive potential $V(r)$ (5.1) leads also to a deep bound state $W_B = -403 \pm 70$ MeV below $B\bar{B}^*$ threshold in s-wave (pole for $k = +i|k|$). Such a state was never reported by experiments. If it exists, it could be searched for in $Z_b \to \Upsilon(1S)\pi^+$ decays. The invariant mass distribution observed by Belle in Fig. 4 [2] is indeed not flat, so it would be valuable to experimentally explore if some structure becomes significant at better statistics.

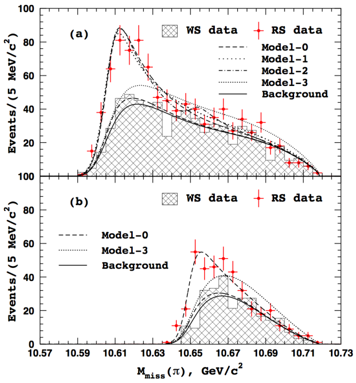
6. Conclusions and outlook

We find a sizable attractive interaction between B and $\bar{B}^*$ in the Z_b^+ channel for separations $r = [0.1, 0.4]$ fm and this is the most robust conclusion of our lattice QCD simulation. Using further severe simplifying assumptions, we find a virtual bound state $W_B = -32^{+29}_{-5}$ MeV below $B\bar{B}^*$ threshold, which could be related to Z_b resonances by Belle. We surprisingly find also a deep bound state $W_B = -403 \pm 70$ MeV below threshold, which could be experimentally searched the $Z_b \to \Upsilon(1S)\pi^+$ invariant-mass distribution.

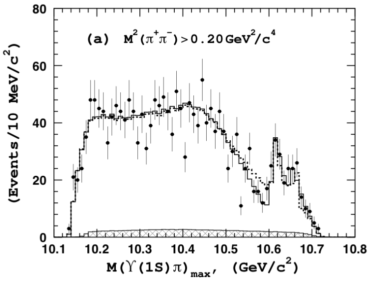
The more physically motivated form of the potential between $B\bar{B}^*$ at small r will be considered in the forthcoming publication. Another assumption of this work is that a certain eigenstate is exclusively related to $B\bar{B}^*$ channel. The more challenging problem of extracting a matrix of potentials for coupled $B\bar{B}^*$ and $\Upsilon\pi$ channels is left for the future.



(a) $N_{B\bar{B}^*}$ from lattice.



(b) $N_{B\bar{B}^*}$ from Belle [3].



(c) $N_{\Upsilon(1S)\pi^+}$ from Belle [2].

Figure 4: (a) The expected rate $N_{B\bar{B}^*} \propto k\sigma_{B\bar{B}^*} \propto \sin^2\delta/k$ based on our lattice results for the potential (5.1) with $A = 1.02,\ d = 1.9$ that are within the uncertainty range. (b) Rate related to $Z_b \to B\bar{B}^*$ and $N_{B\bar{B}^*}$ by Belle [3] (c) Rate related to $Z_b \to \Upsilon(1s)\pi$ by Belle (Fig. 4 of [2]).

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