2019 Update on $\epsilon_K$ with lattice QCD inputs

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We present updated results for $\epsilon_K$ determined directly from the standard model (SM) with lattice QCD inputs such as $\hat{B}_K$, $|V_{cb}|$, $|V_{us}|$, $\xi_0$, $\xi_2$, $\xi_{LD}$, $f_K$, and $m_c$. We find that the standard model with exclusive $|V_{cb}|$ and other lattice QCD inputs describes only 65% of the experimental value of $|\epsilon_K|$ and does not explain its remaining 35%, which leads to a strong tension in $|\epsilon_K|$ at the $4.6\sigma \sim 4.2\sigma$ level between the SM theory and experiment. We also find that this tension disappears when we use the inclusive value of $|V_{cb}|$ obtained using the heavy quark expansion based on QCD sum rules.
1. Introduction

This paper is an update of our previous papers [1, 2, 3, 4, 5]. Here, we present recent progress in determination of $|\varepsilon_K|$ with updated inputs from lattice QCD.

2. Input parameters: $|V_{cb}|$

In Table 1 (a) and (d), we present updated results for exclusive $|V_{cb}|$ and inclusive $|V_{cb}|$ respectively. In Table 1 (a), we present results for exclusive $|V_{cb}|$ of BELLE [6] and BABAR [7]. They reported results obtained using both CLN and BGL methods, which turn out to be consistent with each other.

In Table 1 (c), we plot time evolution of the $|V_{cb}|$ results for the CLN analysis (blue line) as well as the BGL analysis (red line) for the $B \to D^* \ell \bar{\nu}$ decays. Here, the black cross symbols with label Gambino represent results from Refs. [8, 9, 10], respectively. The brown square symbol with label Grinstein represents results from Ref. [11]. The green circle symbol with label BELLE-17 represents results from Ref. [12]. The magenta circle symbols with label BELLE-18 represent results from Ref. [6]. The green triangle symbols with label BELLE-19 represent results from Ref. [13]. The orange diamond symbols with label BABAR represents results from Ref. [7]. In 2017 when Bigi, Gambino, and Schacht [8, 11] first raised a claim that there might be an inconsistency in exclusive $|V_{cb}|$ between the CLN and BGL analyses on the BELLE-2017 tagged data set of the $B \to D^* \ell \bar{\nu}$ decays, the BGL results seemed to be superficially consistent with those for inclusive $|V_{cb}|$. However, the 2019 analyses of both BELLE [6] (on the untagged data) and BABAR [7] show that the results of the BGL analysis might be consistent with those of the CLN analysis, which denies the previous claim of Refs. [8, 11]. The pink dashed line with label FLAG-19 represents preliminary results of BELLE-18 which the FLAG 2019 report took over to do their analysis on $|V_{cb}|$. Hence, if you are to use the $|V_{cb}|$ results of FLAG 2019, please do it with proper caution, since they might be out of date. The green dashed line with label SWME-19 represents results of BELLE-19 [6] which we use for the analysis on $\varepsilon_K$ in this paper.

In Table 1 (b), we show the plot made by HFLAV. Results on this plot are available on the web [14], but not published in any journal yet. Recently, there has been an interesting claim that the $|V_{cb}|$ puzzle might be resolved if we include $\mathcal{O}(1/m_c^2)$ corrections in the data analysis [15].

3. Input parameter $\xi_0$

The absorptive part of long distance effects on $\varepsilon_K$ is parametrized into $\xi_0$.

$$\xi_0 = \frac{\text{Im}A_0}{\text{Re}A_0}, \quad \xi_2 = \frac{\text{Im}A_2}{\text{Re}A_2}, \quad \text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \frac{\omega}{\sqrt{2}|\varepsilon_K|}(\xi_2 - \xi_0).$$

(3.1)

There are two independent methods to determine $\xi_0$ in lattice QCD: the indirect and direct methods. The indirect method is to determine $\xi_0$ using Eq. (3.1) with lattice QCD results for $\xi_2$ combined with experimental results for $\varepsilon'/\varepsilon$, $\varepsilon_K$, and $\omega$. The direct method is to determine $\xi_0$ directly using the lattice QCD results for $\text{Im}A_0$, combined with experimental results for $\text{Re}A_0$. In Table 1 (e), we summarize results for $\xi_0$ calculated by RBC-UKQCD. Here, we use the results of the indirect method for $\xi_0$ to evaluate $\varepsilon_K$, since its systematic and statistical errors are much smaller.
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\[ |V_{cb}| \times 10^3 \]

- \[ 39.13(59) \] HFLAV-17 [16]
- \[ 39.25(56) \] HFLAV-19 [14]
- \[ 38.3(3)(7)(6) \] BELLE-19 [6]
- \[ 38.40(84) \] BABAR-19 [7]
- \[ 38.36(90) \] BABAR-19 [7]

<table>
<thead>
<tr>
<th>Channel</th>
<th>Ref.</th>
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<tr>
<td>$\bar{B} \to D^* \ell \bar{\nu}$</td>
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<td>$\bar{B} \to D^* \ell \bar{\nu}$</td>
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<td>$\bar{B} \to D^* \ell \bar{\nu}$</td>
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<td>BGL</td>
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(a) Exclusive $|V_{cb}|$ in units of $10^{-3}$.

(b) $|V_{cb}|$ versus $|V_{ub}|$.

(c) CLN versus BGL.

(d) Inclusive $|V_{cb}|$ in units of $10^{-3}$.

(e) Results for $\xi_0$.

Table 1: Results for $|V_{cb}|$: (a) exclusive $|V_{cb}|$, (b) $|V_{cb}|$ versus $|V_{ub}|$, (c) Time evolution for exclusive $|V_{cb}|$ obtained using CLN and BGL, and (d) inclusive $|V_{cb}|$; (e) results for $\xi_0$.

4. Input parameters: Wolfenstein parameters, $\hat{B}_K$, $\xi_{LD}$, and others

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<th>UTfit</th>
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<td>0.22500(100)</td>
<td>0.2243(5)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.1577(96)</td>
<td>0.148(13)</td>
<td>0.146(22)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.3493(95)</td>
<td>0.348(10)</td>
<td>0.333(16)</td>
</tr>
</tbody>
</table>

(a) Wolfenstein parameters

Table 2: (a) Wolfenstein parameters and (b) QCD corrections: $\eta_{ij}$ with $i, j = c, t$.

In Table 2 (a), we present the Wolfenstein parameters on the market. As explained in Ref. [1, 5], we use the results of angle-only-fit (AOF) in Table 2 (a) in order to avoid unwanted correlation between $(\varepsilon_K, |V_{cb}|)$, and $(\rho, \eta)$. We determine $\lambda$ from $|V_{us}|$ which is obtained from the $K_{2\pi}$ and $K_{3\pi}$ decays using lattice QCD inputs for form factors and decay constants. We determine the $A$ parameter from $|V_{cb}|$.

In FLAG 2019 [25], they report lattice QCD results for $\hat{B}_K$ with $N_f = 2$, $N_f = 2 + 1$, and $N_f = 2 + 1 + 1$. Here, we use the results for $\hat{B}_K$ with $N_f = 2 + 1$, which is obtained by taking an average over the four data points from BMW 11, Laiho 11 RBC-UKQCD 14, and SWME 15 in Table 3 (a).
\[ \xi_{\text{LD}} = \frac{m_{t}^{4}}{\sqrt{2} \Delta M_{K}}, \quad m_{t}^{4} = -\text{Im} \left[ \mathcal{P} \sum_{C} \frac{\langle K^{0} | H_{u} | C \rangle \langle C | H_{u} | K^{0} \rangle}{m_{K}^{2} - E_{C}} \right] \]  

(4.1)

As explained in Refs. [1], there are two independent methods to estimate \( \xi_{\text{LD}} \): one is the BGI estimate [30], and the other is the RBC-UKQCD estimate [31, 32]. The BGI method is to estimate the size of \( \xi_{\text{LD}} \) using chiral perturbation theory as follows,

\[ \xi_{\text{LD}} = -0.4(3) \times \frac{\tilde{\xi}_{0}}{\sqrt{2}} \]  

(4.2)

The RBC-UKQCD method is to estimate the size of \( \xi_{\text{LD}} \) as follows,

\[ \xi_{\text{LD}} = (0 \pm 1.6)\%. \]  

(4.3)

Here, we use both methods to estimate the size of \( \xi_{\text{LD}} \).

In Table 2 (b), we present higher order QCD corrections: \( \eta_{ij} \) with \( i, j = t, c \). A new approach using \( u - t \) unitarity instead of \( c - t \) unitarity appeared in Ref. [33], which is supposed to have a better convergence with respect to the charm quark mass. Here, we have not incorporated this into our analysis yet, but will do it in near future.

In Table 3 (b), we present other input parameters needed to evaluate \( \varepsilon_{K} \). Here, the \( W \) boson mass \( M_{W} \) and the kaon decay constant \( F_{K} \) have been updated since Lattice 2018. In Table 4, we present the charm quark mass \( m_{c}(m_{c}) \) and top quark mass \( m_{t}(m_{t}) \). From FLAG 2019 [25], we take the results for \( m_{c}(m_{c}) \) with \( N_{f} = 2 + 1 \), since there is some discrepancy in those with \( N_{f} = 2 + 1 + 1 \). For the top quark mass, we use the PDG 2019 results to obtain \( m_{t}(m_{t}) \).

5. Results for \( \varepsilon_{K} \)

In Fig. 1, we show results for \( |\varepsilon_{K}| \) evaluated directly from the standard model (SM) with lattice QCD inputs given in the previous sections. In Fig. 1 (a), the blue curve represents the theoretical
evaluation of $|\epsilon_K|$ obtained using the FLAG-2019 results for $\hat{B}_K$, AOF for Wolfenstein parameters, the (BELLE-19, CLN) results for exclusive $|V_{cb}|$, and the RBC-UKQCD estimate for $\xi_{LD}$. The red curve in Fig. 1 represents the experimental results for $|\epsilon_K|$. In Fig. 1 (b), the blue curve represents the same as in Fig. 1 (a) except for using the 1S scheme results for the inclusive $|V_{cb}|$.

Our results for $|\epsilon_K|^{SM}$ are summarized in Table 5. Here, the superscript $SM$ represents the theoretical expectation value of $|\epsilon_K|$ obtained directly from the SM. The superscript $Exp$ represents the experimental value of $|\epsilon_K| = 2.228(11) \times 10^{-3}$. Results in Table 5 (a) are obtained using the RBC-UKQCD estimate for $\xi_{LD}$, and those in Table 5 (b) are obtained using the BGI estimate for $\xi_{LD}$. In Table 5 (a), we find that the theoretical expectation values of $|\epsilon_K|^{SM}$ with lattice QCD inputs (with exclusive $|V_{cb}|$) has $4.6 \sigma \sim 4.2 \sigma$ tension with the experimental value of $|\epsilon_K|^{Exp}$, while there is no tension with inclusive $|V_{cb}|$ (obtained using heavy quark expansion and QCD sum rules).

| $|V_{cb}|$ | method | reference | $|\epsilon_K|^{SM}$ | $\Delta \epsilon_K$ |
|---|---|---|---|---|
| exclusive | CLN | BELLE-19 | $1.456 \pm 0.172$ | $4.47 \sigma$ |
| exclusive | BGL | BELLE-19 | $1.443 \pm 0.181$ | $4.32 \sigma$ |
| exclusive | CLN | BABAR-19 | $1.456 \pm 0.169$ | $4.55 \sigma$ |
| exclusive | BGL | BABAR-19 | $1.451 \pm 0.175$ | $4.44 \sigma$ |
| exclusive | combined | HFLA V-19 | $1.576 \pm 0.154$ | $4.23 \sigma$ |
| inclusive | kinetic | HFLA V-19 | $2.060 \pm 0.212$ | $0.79 \sigma$ |
| inclusive | 1S | HFLA V-19 | $2.020 \pm 0.176$ | $1.18 \sigma$ |

(a) RBC-UKQCD estimate for $\xi_{LD}$

| $|V_{cb}|$ | method | reference | $|\epsilon_K|^{SM}$ | $\Delta \epsilon_K$ |
|---|---|---|---|---|
| exclusive | CLN | BELLE-19 | $1.501 \pm 0.174$ | $4.16 \sigma$ |
| exclusive | BGL | BELLE-19 | $1.488 \pm 0.183$ | $4.04 \sigma$ |

(b) BGI estimate for $\xi_{LD}$

Table 5: $|\epsilon_K|$ in units of $1.0 \times 10^{-3}$, and $\Delta \epsilon_K = |\epsilon_K|^{Exp} - |\epsilon_K|^{SM}$. 

Figure 1: $|\epsilon_K|$ with (a) exclusive $|V_{cb}|$ (left) and (b) inclusive $|V_{cb}|$ (right) in units of $1.0 \times 10^{-3}$. 

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In Fig. 2 (a), we show the time evolution of $\Delta \varepsilon_K$ starting from 2012 to 2019. In 2012, $\Delta \varepsilon_K$ was $2.5\sigma$, but now it is $4.5\sigma$ with exclusive $|V_{cb}|$ (BELLE-19, CLN). In Fig. 2 (b), we show the time evolution of the average $\Delta \varepsilon_K$ and the error $\sigma_{\Delta \varepsilon_K}$ during the period of 2012–2019.

Figure 2: Time history of (a) $\Delta \varepsilon_K/\sigma$, and (b) $\Delta \varepsilon_K$ and $\sigma_{\Delta \varepsilon_K}$.

At present, we find that the largest error ($\approx 50\%$) in $|\varepsilon_K|^{SM}$ comes from $|V_{cb}|$. Hence, it is of crucial importance to reduce the error in $|V_{cb}|$ significantly. To achieve this goal, there is an ongoing project to extract exclusive $|V_{cb}|$ using the Oktay-Kronfeld (OK) action for the heavy quarks to calculate the form factors for $\bar{B} \rightarrow D^{(*)}\ell\bar{\nu}$ decays [34, 35, 36, 37, 38].

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References

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