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Chiral Condensate and Susceptibility of SU(2) $n_f = 8$ Naive Staggered System

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The SU(2) gauge theory with 8 fundamental fermions is studied using unimproved staggered regularization. A phase transition or a crossover at strong coupling, which can be a bulk transition. By using chiral random matrix model we analyze the chiral condensate of this system. We also report the chiral susceptibility and its volume dependence near the transition point.

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1. Introduction

Identifying gauge theories in the conformal window has been attracting a lot of attention, in the context of composite Higgs model as well as purely theoretical interests. One of the simplest candidates is SU(2) gauge theory with fundamental fermions. According to the Helsinki group with their Wilson fermion simulations, the system is in the conformal window with $n_f = 6$ and $n_f = 8$ flavors [1, 2]. We also studied the $n_f = 8$ system with naive staggered fermion and surveyed the bulk phase structure [3, 4]. In [4], we used chiral symplectic Random Matrix Theory (RMT with the Dyson index $\beta_D = 4$) to analyze the chiral condensate of the system. In order to obtain the smallest eigenvalue distribution of the RMT, we used a finite matrix rank $N = 400$ and used Hybrid Monte Carlo method. We observed that the chiral symmetry is broken in the strong coupling side, while it is restored in the weak coupling side. The Polyakov loop analysis suggested that the transition is weakly first order [3], but from the chiral condensate, the order was unclear.

Recently, one of the authors (I.K.) together with H. Fuji and S. M. Nishigaki has provided a new numerical tool to estimate individual eigenvalue distribution for $\beta_D = 4$ RMT [5]. In this talk, we apply this new formula to SU(2) $n_f = 8$ staggered system and analyze chiral symmetry breaking pattern. Combining HMC estimation of the all the eigenvalue distribution of RMT with larger N and the revised estimation of the chiral condensate, we further estimate the chiral susceptibility, as from its volume dependence we can argue the nature of the bulk phase transition.

In the next section, we briefly review the relation of eigenvalue spectra between RMT and QCD(-like) theory. Then we present the revised result of the chiral condensate. The RMT analysis of the chiral susceptibility is given in Sec. 3. Section. 4 is the conclusion.

2. Chiral Condensate

We fit the value of chiral condensate with RMT by using the following relation:

$$\rho_{\mathrm{QCD}}(m;\lambda_i) = \rho_{\mathrm{RMT}}(\mu = V\Sigma_{\mathrm{param}} m, \zeta_i = V\Sigma_{\mathrm{param}}\lambda_i). \tag{2.1}$$

In the simulation for the QCD-like gauge theory, we input the fermion mass, m, and the four-volume and extract the $i-$th smallest eigenvalue of the Dirac operator. The distributions of these small eigenvalues can also be determined. From the RMT, we have a mass parameter μ, i-th smallest eigenvalue ζ_i and its distribution ρ_{RMT}. Equation (2.1) tells us that the distributions of the eigenvalues are identical with a rescaling by $V\Sigma_{\mathrm{param}}$ of the eigenvalues and the mass parameters. Here, Σ_{param} is the chiral condensate of the QCD side. Note that the above relation holds in the broken phase of chiral symmetry and for λ_i smaller than (the correspondence of) the Thouless energy. In such a situation, we can fit the value of Σ_{param}. If the fit does not work to obtain Σ_{param}, it implies the QCD side is in the symmetric phase. Note that we can safely assume that the smallest λ_i is smaller than the Thouless energy even the system is not in the ε-regime.

Our lattice setting is the following. The action is plaquette gauge action with unimproved staggered fermions. The gauge group is SU(2) and the number of fermions is $n_f = 8$ in the fundamental representation, for which no rooting trick is needed. That is, we use two staggered flavors. Our fermion mass analyzed here are $am = 0.003, 0.005, 0.010$, and the lattice volume is in $L^3 \times T = 6^3 \times 6 - 16^3 \times 16$. We use the periodic boundary condition in all directions for both gauge field and fermions. The distribution of the eigenvalues depends on the topological charge ν so that we choose to work with $\nu = 0$ for the RMT, and use gauge configurations in the same topological sector in the fitting. The number of fermions for RMT, that is the degeneracy of the eigenvalues, is rather non-trivial. As pointed out in [6], it is $2n_f$ for n_f flavor system due to the double-fold degeneracy coming form the pseudo reality of SU(2) gauge group. In addition, we have observed no eigenvalue degeneracy for the staggered taste for our parameters with naive staggered fermion. As a result, we compare our QCD-like simulation with RMT for the number of flavors $n_f^{\mathrm{RMT}} = 2n_f/4 = 4$.

We fit the smallest eigenvalue from the lattice simulation by using the RMT eigenvalue distribution obtained in [5][1]. We plot our fitted result of the chiral condensate in Fig. 1. In the plot, light colored symbols are for fit with large chi squared per degrees of freedom (χ^2/d.o.m > 1.5). The errors are obtained by jackknife analysis. We observe finite chiral condensate at strong coupling, i.e., small $\beta = 4/g^2$. It disappears at around $\beta = 1.4$–1.5. As reported in [3], there is a bulk transition at the same β value, where no four-volume dependence of the transition point appears in the plaquette variables. We associate the transition between broken and symmetric phase of the chiral symmetry to the same bulk transition. In order to access to the conformal window, we must perform the simulation in the symmetric phase, i.e., in the weak coupling side. Although our previous result in [4] is based on a subset of data used in this analysis, it is essentially the same as the revised result.

3. Chiral Susceptibility

By noting that $Z_{\mathrm{QCD}}(m;\lambda_i) = Z_{\mathrm{RMT}}(\mu = V\Sigma_{\mathrm{param}} m; \zeta_i = V\Sigma_{\mathrm{param}}\lambda_i)$ in the ε-regime, we have an expression of chiral susceptibility χ:

$$\chi = -\frac{1}{n_f}\frac{1}{V}\frac{\partial^2}{\partial^2 m}\ln Z_{\mathrm{QCD}} = -\frac{1}{n_f}V\Sigma_{\mathrm{param}}\frac{\partial^2}{\partial\mu^2}\ln Z_{\mathrm{RMT}} \tag{3.1}$$

$$= V\Sigma_{\mathrm{param}}^2\left\{\langle A(\mu)\rangle_{\mathrm{RMT}} + n_f\left(\langle B(\mu)B(\mu)\rangle_{\mathrm{RMT}} - (\langle B(\mu)\rangle_{\mathrm{RMT}})^2\right)\right\} \tag{3.2}$$

$$= \frac{\Sigma_{\mathrm{param}}}{m}\mu\left\{\langle A(\mu)\rangle_{\mathrm{RMT}} + n_f\left(\langle B(\mu)B(\mu)\rangle_{\mathrm{RMT}} - (\langle B(\mu)\rangle_{\mathrm{RMT}})^2\right)\right\}, \tag{3.3}$$

where $A(\mu) = \sum_i\left[\frac{2}{\zeta_i^2+\mu^2} - \frac{4\mu^2}{(\zeta_i^2+\mu^2)^2}\right]$ and $B(\mu) = \sum_i \frac{2\mu}{\zeta_i^2+\mu^2}$. The expectation value $\langle\bullet\rangle_{\mathrm{RMT}}$ is that in the RMT. The same formula was used for the Dyson index $\beta_{\mathrm{D}} = 2$ system [7]. Since we have the fitted result of the chiral condensate Σ_{param} and thus the value of μ, we can calculate the chiral susceptibility χ. Although the parameter Σ_{param} gives the chiral condensate at the infinite volume, the partition function Z_{QCD} is the one at finite volume V. That is, the susceptibility χ obtained by eq. (3.3) has a volume dependence.

It is important to note that this formula is based on the equivalence of the whole partition function. That is, we must use the lattice data in the ε-regime so that the value of μ must be

[1] In [5], it is shown that our previous estimate with $N = 400$ [4] has a sizable finite N effects to the estimation of the smallest eigenvalue distribution. Compared with the error coming from lattice data in [4], however, it is the same order or smaller so that the systematic error coming from finite N was under control.

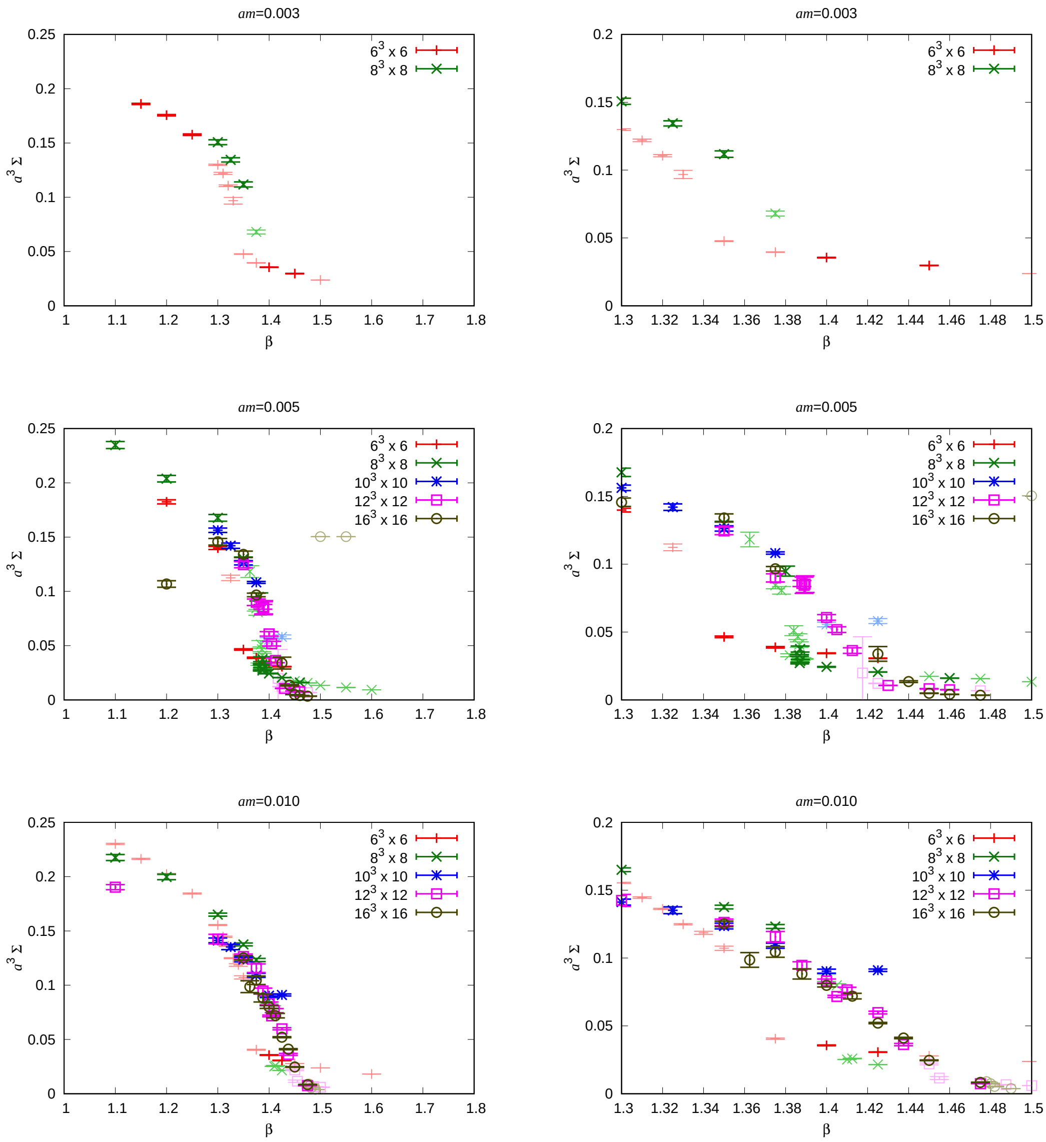

Figure 1: Chiral condensates $\Sigma = \Sigma_{\rm param}$ in the lattice unit obtained by using RMT (left column) and the zoom-ups (right column). The fermion mass is $am = 0.003$, 0.005 and 0.010, respectively, from the top panels to the bottom.

order 1 or smaller. This is different from fitting the chiral condensate, for which we need only the information of the smallest eigenvalue, and gives a restriction on using eq. (3.3). For the $\beta_{\rm D} = 2$ quenched system, it was shown in [7] that the formula leads to a good agreement with the result obtained by lattice simulation at μ is $O(1)$ or smaller. Since we need all the eigenvalues of the RMT, and it is not practically feasible to employ the new method in [5] to achieve this, we resort to a numerical approach by formulating the problem as a one-dimensional field theory and simulating the theory using the Hybrid Monte Carlo (HMC) algorithm with the matrix rank $N = 2000$. The details of the HMC on this system is found in [5].

Figure 2 shows the rescaled chiral susceptibility $m\chi/\Sigma_{\rm param}$ against $\mu = m\Sigma_{\rm param}V$. For a fixed value of fermion mass m and the chiral condensate Σ_{param}, which are equivalent to a fixed value of β (see Fig. 1), μ is proportional to the volume. Therefor the linear raising of $m\chi/\Sigma_{\text{param}}$ near the origin in Fig. 2 implies a linear growth of the susceptibility in volume. In this small μ region, however, no peak structure appears and thus it is not clear whether this linear behavior is also true for the peak of the susceptibility or not.

In Fig. 3, we plot the susceptibility obtained by using eq. (3.3). Although we only have limited data points at $\mu < 2$, it is clear that there is no peak which grows as the volume becomes larger. That is, there is no indication of the first order transition.

We need some care to interpret this result. As the volume becomes larger with fixed fermion mass, the system moves away from the ε-regime so that we need to use small enough fermion mass to stay in the ε-regime. From the smallest mass plot ($am = 0.003$, top panel in Fig. 3), the larger ($8^3 \times 8$) volume gives the larger susceptibility. This volume dependence might imply the first order transition, but since no peak structure is observed, our data cannot be used to draw this conclusion. This is different from SU(3) case, where first order bulk phase transitions related to the S^4 symmetry for staggered fermion exist [8].

If the transition is not first order, it can be either a crossover or a second order phase transition. The latter case implies possible existence of a non-trivial continuum limit in the ε-regime. Since we can discuss only the broken side of the phase transition with our analysis, independent estimations of the chiral susceptibility with different method such as direct lattice calculation are required to give a conclusive result.

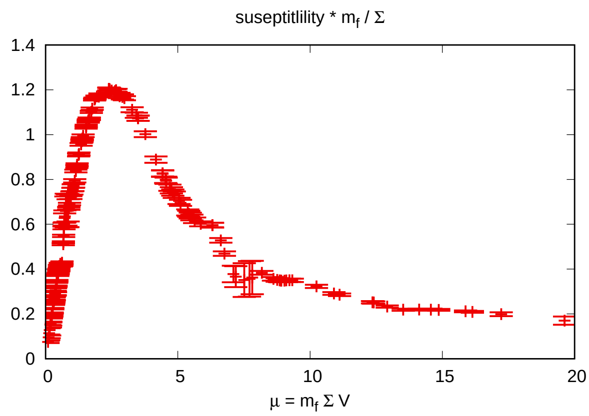

Figure 2: The rescaled chiral susceptibility $m\chi/\Sigma = m\chi/\Sigma_{\text{param}}$ obtained by using eq. (3.3).

4. Conclusions

We revised the chiral condensate of $n_f = 8$ naive staggered system in SU(2) fundamental representation by fitting the smallest Dirac eigenvalue with symplectic chiral RMT. We used a new numerical estimation of the distribution of individual eigenvalues of the RMT. As previously reported, at strong coupling, we observe a bulk phase in which chiral symmetry is broken. The weak coupling side is the symmetric phase so that it is consistent with the scenario that the theory is in the conformal window. By further using RMT, we also have estimated chiral susceptibility.

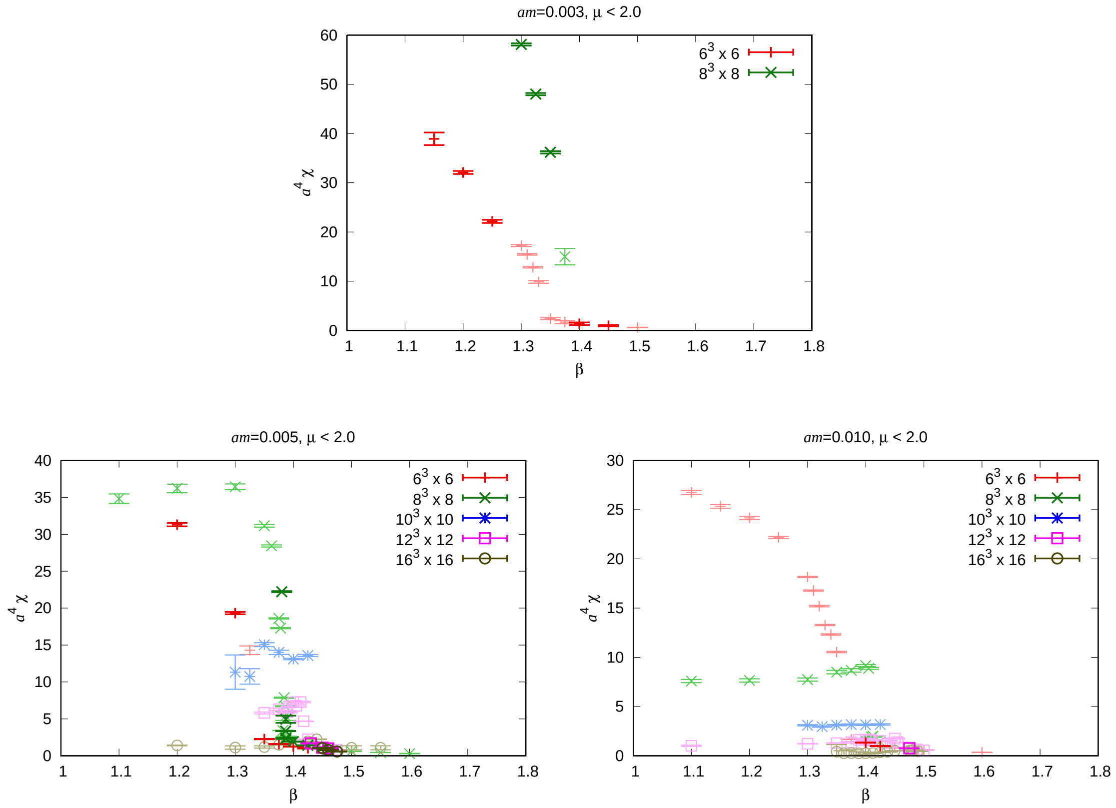

Figure 3: Chiral susceptibility. Light color symbols correspond to $\mu \geq 2$ data, for which eq. (3.3) may not apply.

Although data in the ε-regime, in which our estimation is justified, is limited, we have not observed any peak which grows with the volume in the susceptibility. This implies that the bulk transition is not of first order.

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