

Leptonic decays of $B_{(s)}$ and $D_{(s)}$ using the OK action

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We present recent progress in the lattice calculation of leptonic decay constants for $B_{(s)}$ and $D_{(s)}$ mesons using the Oktay-Kronfeld (OK) action for charm and bottom valence quarks, whose masses are tuned non-perturbatively. The calculations are done on 6 HISQ ensembles generated by the MILC collaboration with $N_f = 2 + 1 + 1$ flavors. We also use the HISQ action for the light spectator quarks. Results are presented for the ratios f_{B_s}/f_B and f_{D_s}/f_D , which reflect $SU(3)$ flavor symmetry breaking, and are independent of the renormalization constants of the axial currents.

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1. Introduction

The decay constant f_P of a pseudoscalar meson P is defined by

$$\langle 0|A^\mu|P\rangle = ip^\mu f_P, \quad (1.1)$$

where the external state $|P\rangle$ carries momentum p^μ , and the axial current $A^\mu = \bar{\psi}_h \gamma^\mu \gamma_5 \psi_l$. Here, the subscript h (l) represents heavy (light) flavors in the $B_{(s)}$ and $D_{(s)}$ states. We use the Oktay-Kronfeld (OK) action [1] for valence heavy quarks ψ_h with $h = b, c$, and the HISQ action [2] for valence light quarks χ that are recast into the naive quark field ψ_l . The calculations are done on MILC HISQ ensembles with $N_f = 2 + 1 + 1$ [3], whose parameters are summarized in Table 1.

The OK action S_{OK} improves on the Fermilab formulation of the Wilson clover action [4] by including $\mathcal{O}(\lambda^2)$ and $\mathcal{O}(\lambda^3)$ improvement terms in heavy quark effective theory (HQET) power counting.

$$S_{\text{OK}} = a^4 \sum_x \bar{\psi}(x) \left[\begin{aligned} & m_0 + \gamma_4 D_4 && \leftarrow \mathcal{O}(\lambda^0) \\ & -\frac{1}{2} a \Delta_4 + \zeta \boldsymbol{\gamma} \cdot \mathbf{D} - \frac{1}{2} r_s \zeta a \Delta^{(3)} - \frac{1}{2} c_B a \zeta i \boldsymbol{\Sigma} \cdot \mathbf{B} && \leftarrow \mathcal{O}(\lambda^1) \\ & -\frac{1}{2} c_E a \zeta \boldsymbol{\alpha} \cdot \mathbf{E} && \leftarrow \mathcal{O}(\lambda^2) \\ & + c_1 a^2 \sum_k \gamma_k D_k \Delta_k + c_2 a^2 \{ \boldsymbol{\gamma} \cdot \mathbf{D}, \Delta^{(3)} \} + c_3 a^2 \{ \boldsymbol{\gamma} \cdot \mathbf{D}, i \boldsymbol{\Sigma} \cdot \mathbf{B} \} \\ & + c_{EE} a^2 \{ \gamma_4 D_4, \boldsymbol{\alpha} \cdot \mathbf{E} \} + c_4 a^3 \sum_k \Delta_k^2 + c_5 a^3 \sum_k \sum_{j \neq k} \{ i \Sigma_k B_k, \Delta_j \} \end{aligned} \right] \psi(x) \left. \vphantom{S_{\text{OK}}} \right\} \leftarrow \mathcal{O}(\lambda^3) \quad (1.2)$$

The definition of the operators in Eq. (1.2) can be found in Ref. [1]. The bare quark mass m_0 is related to the hopping parameter κ as follows,

$$am_0 = \frac{1}{2} \left(\frac{1}{\kappa} - \frac{1}{\kappa_{\text{crit}}} \right). \quad (1.3)$$

ensemble ID	a (fm)	$N_s^3 \times N_t$	M_π (MeV)	am_l	am_s	am_c
a12m310	0.1207(11)	$24^3 \times 64$	305.3(4)	0.0102	0.0509	0.635
a12m220	0.1184(10)	$32^3 \times 64$	216.9(2)	0.00507	0.0507	0.628
a12m130	0.1191(7)	$48^3 \times 64$	131.7(1)	0.00184	0.0507	0.628
a09m310	0.0888(8)	$32^3 \times 96$	312.7(6)	0.0074	0.037	0.440
a09m220	0.0872(7)	$48^3 \times 96$	220.3(2)	0.00363	0.0363	0.430
a06m310	0.0871(6)	$48^3 \times 144$	319.3(5)	0.0048	0.024	0.286

Table 1: Parameters of the MILC HISQ ensembles with $N_f = 2 + 1 + 1$ [3] used in our calculations. The lattice spacing a is set by the Sommer scale r_1 and N_s (N_t) is the lattice size in the spacial (temporal) direction. M_π is the mass of Goldstone pions and am_l , am_s and am_c are sea quark masses for the light (up and down), strange and charm quarks in lattice units, respectively.

The non-perturbatively tuned hopping parameters for bottom and charm quarks, κ_b , κ_c , and the critical hopping parameter κ_{crit} [5] for each measurement are summarized in Table 2.

In order to achieve a better overlap with the wave functions of the $B_{(s)}$ and $D_{(s)}$ meson states, we apply the covariant Gaussian smearing (CGS), $\{1 + \sigma^2 \nabla^2 / (4N_{\text{GS}})\}^{N_{\text{GS}}}$ to the point source and sink as in Ref. [6]. The CGS parameters $\{\sigma, N_{\text{GS}}\}$ for each measurement are given in Table 2. Here, we apply the CGS only to the heavy quark fields of the pseudoscalar interpolating operators.

2. Correlator and current improvement

The meson-meson (MM) and meson-current (MC) 2-point correlators are defined as follows [7],

$$C_{\text{MM}}(t) = \sum_{\mathbf{x}} \langle \mathcal{O}_P^\dagger(t, \mathbf{x}) \mathcal{O}_P(0) \rangle = \sum_{\alpha=1}^4 \sum_{\mathbf{x}} \langle \mathcal{O}_\alpha^\dagger(t, \mathbf{x}) \mathcal{O}_\alpha(0) \rangle, \quad (2.1)$$

$$C_{\text{MC}}(t) = \sum_{\mathbf{x}} \langle A^{4\dagger}(t, \mathbf{x}) \mathcal{O}_P(0) \rangle = \sum_{\alpha=1}^4 \sum_{\mathbf{x}} \langle A_\alpha^{4\dagger}(t, \mathbf{x}) \mathcal{O}_\alpha(0) \rangle, \quad (2.2)$$

where the pseudoscalar heavy-light meson interpolating operator $\mathcal{O}_\alpha(t, \mathbf{x})$ and the axial current operator $A_\alpha^4(t, \mathbf{x})$ are

$$\mathcal{O}_\alpha(t, \mathbf{x}) = [\bar{\Psi}(t, \mathbf{x}) \gamma_5 \Omega(t, \mathbf{x})]_\alpha \chi(t, \mathbf{x}) \quad (2.3)$$

$$A_\alpha^4(t, \mathbf{x}) = [\bar{\Psi}(t, \mathbf{x}) \gamma^4 \gamma_5 \Omega(t, \mathbf{x})]_\alpha \chi(t, \mathbf{x}). \quad (2.4)$$

Here ψ is the OK heavy quark field, χ is the HISQ light quark field, and

$$\Omega(t, \mathbf{x}) \equiv \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3} \gamma_4^t, \quad (2.5)$$

and the subscript α represents the taste degree of the staggered light quarks. The rotated heavy quark field Ψ is introduced to improve the axial current A_α^4 up to $O(\lambda^3)$, the same level as the OK action.

$$\Psi(t, \mathbf{x}) = \left(\begin{array}{l} 1 \\ + d_1 a \boldsymbol{\gamma} \cdot \mathbf{D} \\ + d_2 a^2 \Delta^{(3)} + d_B a^2 i \boldsymbol{\Sigma} \cdot \mathbf{B} - d_E a^2 \boldsymbol{\alpha} \cdot \mathbf{E} \\ + d_{rE} a^3 \{ \boldsymbol{\gamma} \cdot \mathbf{D}, \boldsymbol{\alpha} \cdot \mathbf{E} \} - d_3 a^3 \sum_i \gamma_i D_i \Delta_i - d_4 a^3 \{ \boldsymbol{\gamma} \cdot \mathbf{D}, \Delta^{(3)} \} \\ - d_5 a^3 \{ \boldsymbol{\gamma} \cdot \mathbf{D}, i \boldsymbol{\Sigma} \cdot \mathbf{B} \} + d_{EE} a^3 \{ \gamma_4 D_4, \boldsymbol{\alpha} \cdot \mathbf{E} \} - d_6 a^3 [\gamma_4 D_4, \Delta^{(3)}] \\ - d_7 a^3 [\gamma_4 D_4, i \boldsymbol{\Sigma} \cdot \mathbf{B}] \end{array} \right) \psi(t, \mathbf{x}), \quad \left. \begin{array}{l} \leftarrow O(\lambda^0) \\ \leftarrow O(\lambda^1) \\ \leftarrow O(\lambda^2) \\ \leftarrow O(\lambda^3) \end{array} \right\} \quad (2.6)$$

where the improvement coefficients d_i are given in Ref. [8].

ensemble ID	m_x/m_s	κ_{crit}	κ_c	κ_b	$\{\sigma, N_{\text{GS}}\}$	$N_{\text{cfg}} \times N_{\text{src}}$
a12m310	1/5, 1	0.051211	0.048524	0.04102	{1.5, 5}	1053 × 3
a12m220	1/10, 1	0.051218	0.048613	0.04070	{1.5, 5}	1000 × 3
a12m130	1/27, 1	0.05119	0.048501	0.041343	{1.5, 5}	499 × 3
a09m310	1/5, 1	0.05075	0.04894	0.0429	{2.0, 10}	996 × 3
a09m220	1/10, 1	0.05077	0.04902	0.0431	{2.0, 10}	1001 × 3
a06m310	1/5, 1	0.050357	0.04924	0.0452	{3.0, 22}	1017 × 3

Table 2: The 2nd column gives the valence light quark masses m_x and the following columns are the hopping parameters, CGS parameters and the number of measurements. N_{cfg} represents the number of gauge configurations analyzed and N_{src} is the number of sources used for measurement on each gauge configuration.

3. Correlator fit

We fit the 2-point correlation functions $C_{\text{MM}}(t)$ and $C_{\text{MC}}(t)$ with three even time-parity and two odd time-parity states and label it the 3+2-state fit. The time parity is determined with respect to the shift operator in the Euclidean time direction. The fitting function is

$$\begin{aligned}
 C_Y(t) &= g_Y(t) \pm g_Y(T-t), \quad (+ \text{ for MM, } - \text{ for MC}) \\
 g_Y(t) &= A_0^Y e^{-M_0 t} \left[1 + R_2^Y e^{-\Delta M_2 t} + R_4^Y e^{-(\Delta M_2 + \Delta M_4)t} + \dots \right. \\
 &\quad \left. - (-1)^t R_1^Y e^{-\Delta M_1 t} - (-1)^t R_3^Y e^{-(\Delta M_1 + \Delta M_3)t} + \dots \right] \quad (3.1)
 \end{aligned}$$

where $Y=\text{MC}$ or MM , $\Delta M_i \equiv M_i - M_{i-2}$, $M_{-1} \equiv M_0$, and

$$A_i^{\text{MM}} \equiv \frac{1}{2M_i} \langle 0 | \mathcal{O}_P | P_i \rangle \langle P_i | \mathcal{O}_P | 0 \rangle, \quad A_i^{\text{MC}} \equiv \frac{1}{2M_i} \langle 0 | A^4 | P_i \rangle \langle P_i | \mathcal{O}_P | 0 \rangle, \quad R_i^Y \equiv \frac{A_i^Y}{A_0^Y} \quad (3.2)$$

Here, P_i represents the i -th excited meson state and P_0 the ground state. For a brevity, the subscript “0” for the ground state is dropped from now on. We take the following steps to analyze the 2-point correlation functions.

1. We fit the 2-point correlator, $C_{\text{MM}}(t)$, data using the 3+2-state fit given in Eq. (3.1) to extract the ground state pseudoscalar meson mass $M(\equiv M_0)$ and amplitude $A^{\text{MM}}(\equiv A_0^{\text{MM}})$ and control the excited states. We impose empirical Bayesian priors [9] on the excited state mass gaps ΔM_i and amplitude ratios R_i^{MM} to stabilize the fit. (See Fig. 1).
2. We feed the results for M_0 , and ΔM_i obtained in the previous step as inputs into the fit for $C_{\text{MC}}(t)$ to extract $A^{\text{MC}}(\equiv A_0^{\text{MC}})$ and the ratios R_i^{MC} . We use the same fit range and fit functional form as taken for $C_{\text{MM}}(t)$

An example of the effective mass plot with

$$m_{\text{eff}}^Y(t) \equiv \frac{1}{2} \log \left[\frac{C_Y(t)}{C_Y(t+2)} \right] \quad (3.3)$$

for the 2-point correlators $C_{\text{MM}}(t)$ and $C_{\text{MC}}(t)$ is shown in Fig. 2 along with the fits to them.

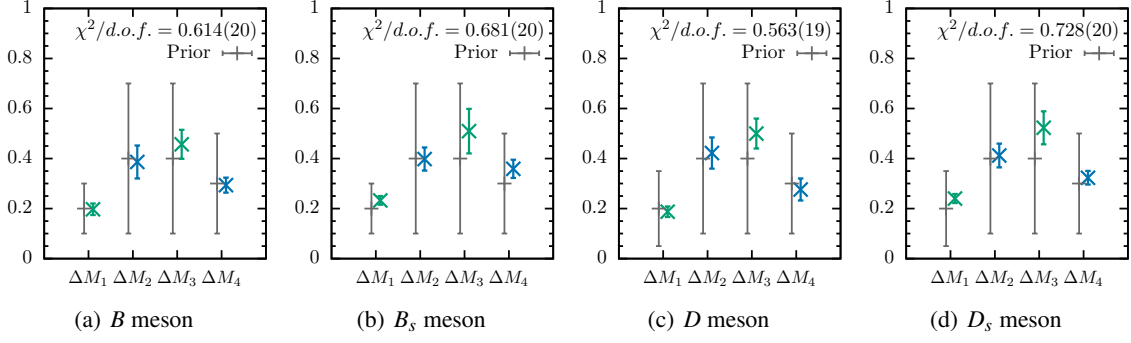


Figure 1: Fit results and Bayesian priors for ΔM_i from $a12m130$ ensembles.

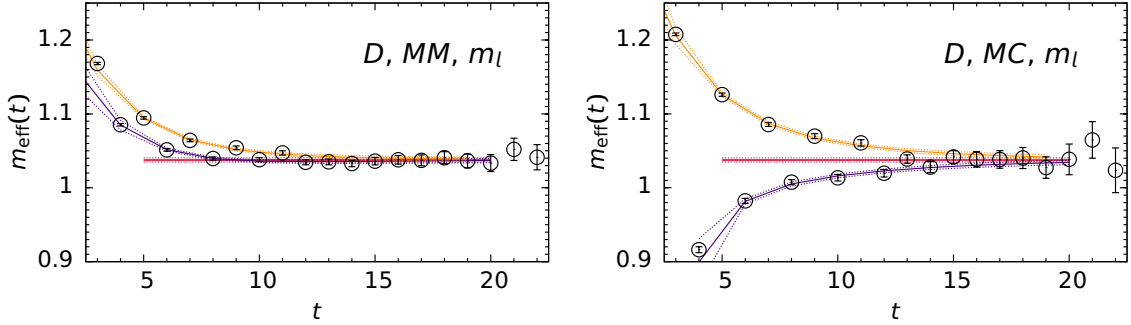


Figure 2: Effective mass plots for the $C_{MM}(t)$ and $C_{MC}(t)$ correlators of the D meson on the $a12m130$ ensemble with $m_l = m_s/27$. The orange (purple) curves connect 3+2-state fit results on the odd (even) time slices. The horizontal red line shows the ground state mass M_0 within the fit range $[t_{\min}, t_{\max}] = [5, 20]$. The axial current operators are improved up to the λ^3 order.

4. Results

The decay constant f_P defined in Eq. (1.1) can be expressed in terms of the ground state amplitudes A^{MM} and A^{MC} as follows,

$$f_P = Z_{A^4}^{hl} \sqrt{\frac{2}{M_P}} \frac{A^{\text{MC}}}{\sqrt{A^{\text{MM}}}}, \quad (4.1)$$

where we take the meson mass $M_P = M_0$ of the ground state obtained from the fits to C_{MM} . The tree-level renormalization factor is given as $Z_{A^4}^{hl, \text{tree}} = e^{m_1^h/2}$ where $m_1^h = \log(1 + m_0^h)$ is the rest mass and m_0^h is the bare mass for the heavy quark [4]. The perturbative and non-perturbative determination of $Z_{A^4}^{hl}$ is in progress. In this work, we present the flavor $SU(3)$ breaking ratio of decay constants:

$$f_{X_s}/f_X = \frac{Z_{A^4}^{hs}}{Z_{A^4}^{hl}} \sqrt{\frac{M_X}{M_{X_s}}} \sqrt{\frac{A^{\text{MM}}}{A_s^{\text{MM}}} \frac{A_s^{\text{MC}}}{A^{\text{MC}}}} \cong \sqrt{\frac{M_X}{M_{X_s}}} \sqrt{\frac{A^{\text{MM}}}{A_s^{\text{MM}}} \frac{A_s^{\text{MC}}}{A^{\text{MC}}}}, \quad (4.2)$$

for $X_{(s)} = B_{(s)}$ and $D_{(s)}$ mesons. Here, A_s^{MM} and A_s^{MC} are the ground state amplitudes for the heavy-strange mesons. In this ratio, we assume that the light quark mass m_l dependence of $Z_{A^4}^{hl}$ is negligible [7], so the ratio $Z_{A^4}^{hs}/Z_{A^4}^{hl} \cong 1$.

Fig. 3 shows that in the ratios f_{B_s}/f_B and f_{D_s}/f_D , the effect of the current improvement applied to the heavy quark field (as given in Eq. (2.6)) cancels up to $O(\lambda^3)$.

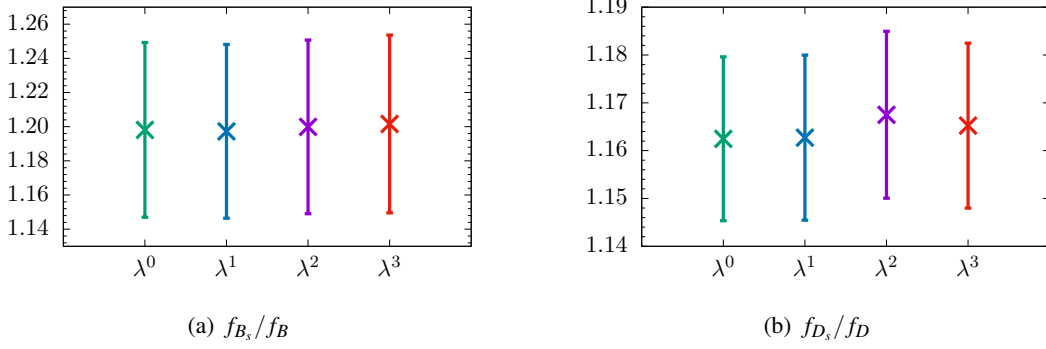


Figure 3: The ratios of decay constants, f_{B_s}/f_B and f_{D_s}/f_D , on the a12m130 ensemble as a function of current improvement order in HQET power counting.

In Fig. 4, we present preliminary results on f_{B_s}/f_B and f_{D_s}/f_D calculated on 6 different HISQ ensembles, and compare them with the continuum limit value given in the FLAG 2019 review [10]. The statistical errors in f_{B_s}/f_B (Fig. 4 (a)) are much larger than in f_{D_s}/f_D (Fig. 4 (b)). As a result, discerning a chiral or discretization effect in f_{B_s}/f_B is not clear, other than to note that the result from the physical ensemble a12m130 is consistent with the FLAG 2019 value. The results for f_{D_s}/f_D show no significant discretization effect on the three lattices with $M_\pi \approx 310$ MeV and the two with $M_\pi \approx 220$ MeV. On the other hand, there is a shift upwards towards the FLAG result as M_π is lowered towards the physical value. Presuming that the OK action has significantly eliminated the heavy quark discretization error even on the coarsest lattice spacing $a \approx 0.12$ fm [11], the leading effect to quantify is the pion mass dependence. For the $a \approx 0.12$ fm data, the current trend is anchored by the physical ensemble with a value close to the FLAG 2019 result. In near future, we plan to add measurements on more ensembles to check for discretization effects and on more physical pion mass ensembles to improve the chiral-continuum extrapolation.

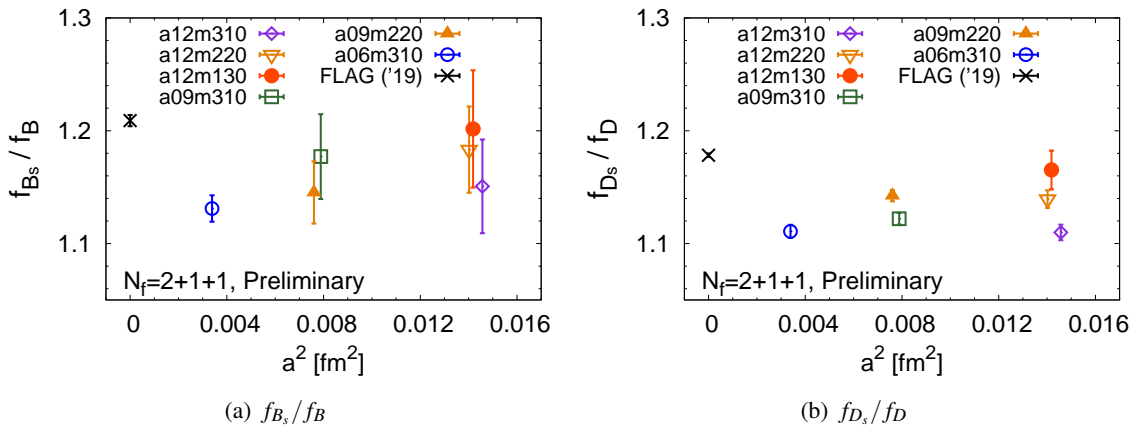


Figure 4: The ratios of decay constants, f_{B_s}/f_B and f_{D_s}/f_D on six ensembles. The errors are purely statistical. The FLAG 2019 [10] results are given for the physical value.

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