

# Semileptonic $B \rightarrow D^{(*)} \ell \nu$ Decay Form Factors using the Oktay-Kronfeld Action

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We report recent progress in calculating semileptonic form factors for the  $\bar{B} \rightarrow D^* \ell \bar{\nu}$  and  $\bar{B} \rightarrow D \ell \bar{\nu}$  decays using the Oktay-Kronfeld (OK) action for bottom and charm quarks. We use the second order in heavy quark effective power counting  $\mathcal{O}(\lambda^2)$  improved currents in this work. The HISQ action is used for the light spectator quarks. We analyzed four 2 + 1 + 1-flavor MILC HISQ ensembles with  $a \approx 0.09$  fm, 0.12 fm and  $M_\pi \approx 220$  MeV, 310 MeV:  $a09m220$ ,  $a09m310$ ,  $a12m220$ ,  $a12m310$ . Preliminary results for  $B \rightarrow D^* \ell \nu$  decays form factor  $h_{A_1}(w)$  at zero recoil ( $w = 1$ ) are reported. Preliminary results for  $B \rightarrow D \ell \nu$  decays form factors  $h_\pm(w)$  over a kinematic range  $1 < w < 1.3$  are reported as well.

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## 1. Introduction

Semileptonic decays  $B \rightarrow D^{(*)} \ell \nu$  are interesting processes, because these are probes of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $|V_{cb}|$  involving heavy, the charm and bottom, quarks [1]. In the Standard Model (SM), the CKM matrix is unitary. Tests of the unitarity, e.g., global unitarity triangle analysis, using inputs from experiments and lattice calculations has become tight as the inputs are determined precisely. However, higher precision is required for a stringent test of SM in  $\varepsilon_K$  [2, 3, 4], which makes the importance of  $|V_{cb}|$  escalate even more. Currently,  $|V_{cb}|$  shows a  $3\sigma \sim 4\sigma$  difference between inclusive and exclusive determination using the CLN parameterization. Meanwhile the BGL parameterization gave the exclusive  $|V_{cb}|$  that is consistent with the inclusive determination, recent analysis with Belle untagged data for  $B \rightarrow D^* \ell \nu$  decays shifts the BGL result on top of the previous CLN result [2]. Ratios,  $R(D)$  and  $R(D^*)$ , of branching fractions of the semileptonic decays  $B \rightarrow D^{(*)} \tau \nu$  to  $B \rightarrow D^{(*)} \ell \nu$ , ( $\ell = e, \mu$ ) are also interesting in the test of the lepton flavor universality in the SM [1]. Thus, our focus is on a determination of form factors for  $B \rightarrow D^{(*)} \ell \nu$  decays in a sub-percent precision using an improved heavy quark discretization: the Oktay-Kronfeld (OK) action [5].

The OK action [5] further improves the Fermilab action [6] by including dimension 6 and 7 bilinear terms necessary for tree-level matching to QCD through order  $\mathcal{O}(\lambda^3)$  ( $\lambda \approx pa \approx \Lambda/(2m_Q)$ ) in heavy quark effective theory (HQET) power counting. The improvement coefficients are perturbatively calculated by matching on-shell amplitudes at tree-level between continuum QCD and lattice QCD. Heavy quark discretization error enters in  $\mathcal{O}(\Lambda/m_{b,c})^n$  so that the errors can be controlled with large lattice spacings  $am_0 > 1$ . In contrast, the Symanzik improved action has  $\mathcal{O}(am_0)^n$  discretization error, and thus,  $am_b \sim 1$  requires  $a \sim 0.045$  fm. At present, MILC HISQ ensemble is available down to  $a \sim 0.03$  fm [7].

Here, we present preliminary analysis on form factors  $h_{A_1}(w)$  at zero-recoil  $w = 1$  and  $h_{\pm}(w)$  over a kinematic range  $1 < w < 1.3$ . The differential decay rate for  $B \rightarrow D^{(*)} \ell \nu$  is parameterized by a conventional form factor  $\mathcal{F}(w)$  ( $\mathcal{G}(w)$ ):

$$\frac{d\Gamma}{dw}(B \rightarrow D^* \ell \nu) = \frac{G_F^2 M_{D^*}^3}{48\pi^3} (M_B - M_{D^*})^2 (w^2 - 1)^{1/2} \chi(w) |\eta_{EW}|^2 |V_{cb}|^2 |\mathcal{F}(w)|^2, \quad (1.1)$$

$$\frac{d\Gamma}{dw}(B \rightarrow D \ell \nu) = \frac{G_F^2 M_D^3}{48\pi^3} (M_B + M_D)^2 (w^2 - 1)^{3/2} |\eta_{EW}|^2 |V_{cb}|^2 |\mathcal{G}(w)|^2. \quad (1.2)$$

The  $\mathcal{F}(w)$  is factorized by the  $h_{A_1}(w)$  leaving the helicity amplitudes that are normalized at zero-recoil. We anticipate that the improved action and current could address the issue in the  $B \rightarrow D^* \ell \nu$  form factor parametrizations. The  $\mathcal{G}(w)$  is a linear combination of the two form factors  $h_{\pm}(w)$ .

The improvement by the OK action was explicitly demonstrated by calculating spectrum of heavy-light mesons and quarkonium [8]. The corrections in  $\alpha_s$  are partially taken into account by the tadpole improvement for both the action and the current [9]. Improved current operator  $J^{fg}$  for a flavor change  $f \rightarrow g$  can be written in terms of an improved (rotated) field  $\Psi = \mathcal{R}\psi$ :  $J^{fg} = \bar{\Psi}^g \Gamma \Psi^f$ . Tree-level matching up to  $\mathcal{O}(\lambda^3)$  is given in [9].

Three-point correlation functions  $C(t, \tau)$

$$C^{X(f) \rightarrow Y(g)}(t, \tau) = \langle \mathcal{O}_Y^\dagger(0) J^{fg}(t) \mathcal{O}_X(\tau) \rangle \quad (1.3)$$

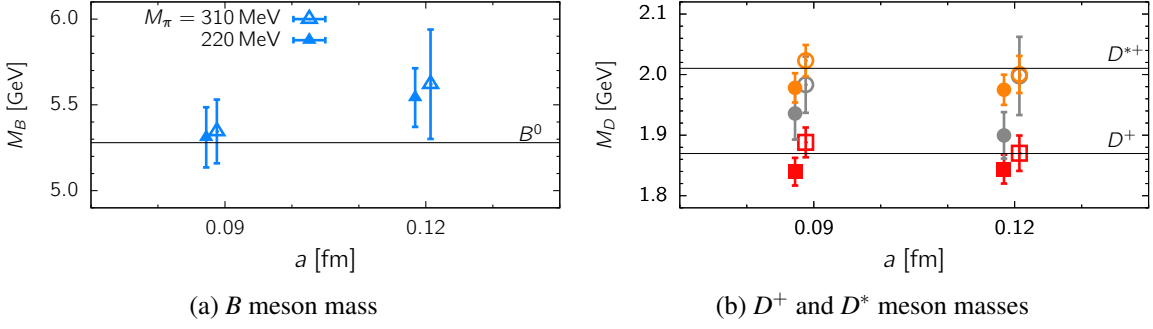


Figure 1: Masses of  $B^0$ ,  $D^*$  and  $D^+$ . The horizontal lines correspond to the physical masses from PDG. The orange (gray) circle represents the (alternative) kinetic meson mass  $M_2(D^*)$ . The filled (open) symbol corresponds to the  $M_\pi = 220$  MeV (310 MeV).

$$= \mathcal{A}_Y \mathcal{A}_X \frac{\langle Y | J^{fg} | X \rangle}{\sqrt{2M_Y} \sqrt{2M_X}} e^{-M_Y t} e^{-M_X(\tau-t)} + \dots \quad (1.4)$$

is calculated for multiple, four to six, source and sink separations  $\tau$ . Then, the correlators are analyzed with multistate fits to extract the matrix elements [10], which are decomposed into the form factors. See a companion proceeding [11] for the rest of details about tuned quark masses, smearing, the 2 + 1 + 1-flavor MILC HISQ ensembles used in this work, definitions of the OK action and the improved heavy quark field.

## 2. Meson Spectrum

Meson masses can be obtained from the kinetic mass  $M_2$ , which is extracted from fit to the dispersion relation

$$E = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{(\mathbf{p}^2)^2}{8M_4^3} - \frac{a^3 w_4}{6} \sum_i p_i^4 + \dots \quad (2.1)$$

In Fig. 1, the masses of  $B$ ,  $D^+$ ,  $D^*$  mesons from four ensembles are compared to the physical masses from PDG. Alternatively,  $M_2(D^*) = M_2(D) + M_1(D^*) - M_1(D)$  (the orange circles) gives another  $D^*$  mass, because the mass splitting between the rest masses is a physical quantity. This alternative kinetic mass of  $D^*$  is better consistent with the physical mass for all four ensembles. The rest mass is solely determined by the zero momentum meson correlator, and thus results in a smaller error than the  $M_2(D^*)$  determined from the fitting. Note that the vector meson correlator is noisier than the pseudoscalar meson correlator for a given momentum  $\mathbf{p}$ .

## 3. $B \rightarrow D^* \ell \nu$ form factor at zero recoil: $h_{A_1}(1)$

The form factor  $h_{A_1}(w=1)$  is obtained by taking the following double ratio:

$$\rho_{A_1}^2 \frac{\langle B | A^{bc} | D^* \rangle \langle D^* | A^{cb} | B \rangle}{\langle B | V^{bb} | B \rangle \langle D^* | V^{cc} | D^* \rangle} = |h_{A_1}(1)|^2, \quad \rho_{A_1}^2 = \frac{Z_A^{bc} Z_A^{cb}}{Z_V^{bb} Z_V^{cc}}. \quad (3.1)$$

In Fig. 2, the zero-recoil form factor  $h_{A_1}(1)$  is compared for different orders of current improvement. All of the results are obtained by assuming that the matching factor  $\rho_{A_1} = 1$  (*i.e.*  $\rho_{A_1}$

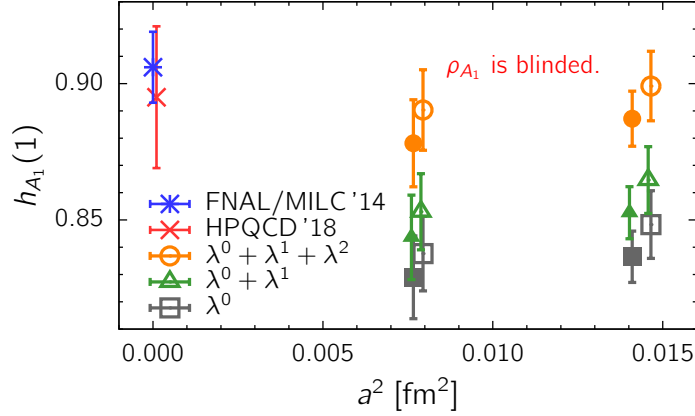


Figure 2:  $h_{A_1}(w)$  at zero recoil ( $w = 1$ ) from four ensembles. Current improvements up to order  $\mathcal{O}(\lambda^n)$ ,  $n = 0, 1, 2$ , are compared for each ensemble. The  $n = 0$  is with the unimproved current. The filled (open) symbol corresponds to the  $M_\pi = 220\text{MeV}$  ( $310\text{MeV}$ ).

is blinded). The size of the leading order  $\mathcal{O}(\lambda^1)$  correction to  $h_{A_1}(1)$  is small, as is expected from HQET. However, the second order correction of  $\mathcal{O}(\lambda^2)$  is larger than the leading order correction. This is unexpected from the HQET. Thus, the current improvement up to  $\mathcal{O}(\lambda^2)$  seems crucial, and a higher order improvement is interesting to check the convergence of HQET expansion. The third order  $\mathcal{O}(\lambda^3)$  improved current is being implemented.

Fig. 2 shows that the  $h_{A_1}(1)$  decreases by less than  $1\sigma$  as the pion mass changes from  $310\text{MeV}$  to  $220\text{MeV}$  and by about  $0.5\sigma$  as the lattice spacing changes from  $a = 0.12\text{ fm}$  to  $a = 0.09\text{ fm}$ . Since these differences are not significant from the standpoint of statistics, we need more data at various lattice spacings (*e.g.* superfine and ultrafine ensembles of the MILC HISQ ensembles) in order to address the issue properly. In addition, the change pattern with respect to the lattice spacing and pion mass are similar for all orders of the current improvement through  $\mathcal{O}(\lambda^n)$ , ( $n = 0, 1, 2$ ). Thus, it could be the light quark discretization that dominates the dependence on the lattice spacing and pion masses.

In Fig. 2, the blue cross symbol represents the result extrapolated to the physical limit from FNAL/MILC calculation [12] that was done by using the Fermilab action for charm and bottom valence quarks on  $2 + 1$ -flavor MILC asqtad staggered ensembles. In Fig. 2, the red cross symbol represents the extrapolated result from HPQCD [13]. The HPQCD calculation uses HISQ action for the light and charm valence quarks, and NRQCD action for bottom valence quark on  $2 + 1 + 1$ -flavor MILC HISQ ensembles.

The matching factor is  $\rho_{A_1} = 1 + \mathcal{O}(\alpha_s)$ . The correction term is not yet included, but is expected to be a subpercent effect because we anticipate a large cancellation among current renormalization factors between  $Z_A$ 's in the numerator and  $Z_V$ 's in the denominator in the double ratio.

#### 4. $B \rightarrow D \ell \nu$ Form Factors: $h_\pm(w)$

The hadronic matrix element of the  $\bar{B} \rightarrow D \ell \bar{\nu}$  decay amplitude can be expressed in terms of

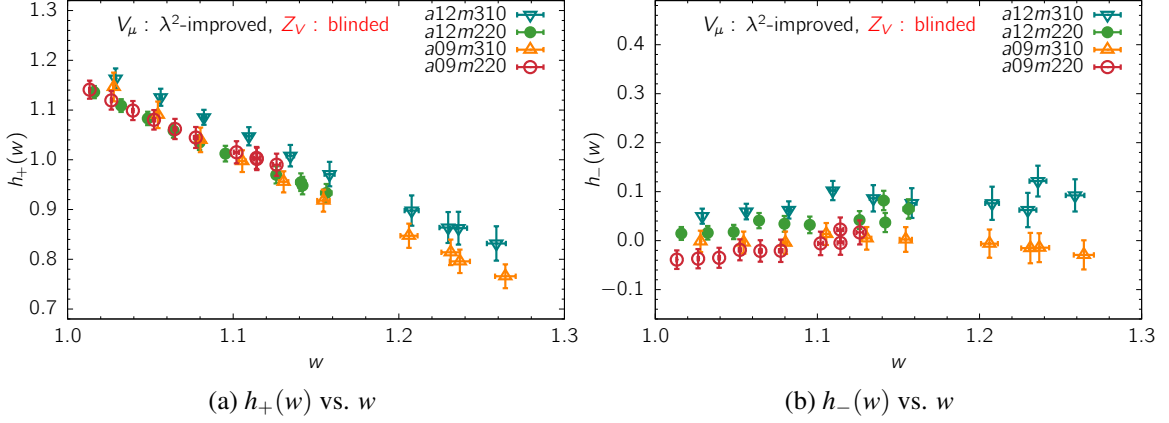


Figure 3:  $h_{\pm}(w)$  as a function of recoil parameter  $w$ . The current is improved up to the  $\lambda^2$  order.

semileptonic form factors  $h_{\pm}(w)$  as follows,

$$\frac{\langle D(M_D, \mathbf{p}') | V_{\mu} | B(M_B, \mathbf{0}) \rangle}{\sqrt{2M_D} \sqrt{2M_B}} = \frac{1}{2} \{ h_+(w)(v + v')_{\mu} + h_-(w)(v - v')_{\mu} \}, \quad (4.1)$$

where the four velocities  $v = p/M_B = (1, \mathbf{0})$ ,  $v' = p'/M_D = (E_D/M_D, \mathbf{p}'/M_D)$  and recoil parameter  $w = v \cdot v'$ .

In Fig. 3, we show form factors  $h_{\pm}(w)$  as a function of recoil parameter  $w$  for the two coarse ( $a12m310$  and  $a12m220$ ) and two fine ensembles ( $a09m310$  and  $a09m220$ ) of the MILC HISQ lattices. Here, we use the vector current improved up to the  $\lambda^2$  order. Tiny variation with respect to light quark mass and lattice spacing is observed for  $h_+$  except the  $a12m310$ . In contrast, the discretization effect is more visible for  $h_-$ .

In Fig. 4, we present form factors  $h_{\pm}(w)$  as a function of recoil parameter  $w$  for different orders of current improvement on  $a09m220$ . The first order correction ( $\mathcal{O}(\lambda^1)$  in HQET) to the unimproved ( $\mathcal{O}(\lambda^0)$ ) current is negligible for  $h_+$  and small for  $h_-$ . The second order correction of  $\mathcal{O}(\lambda^2)$  reduces  $h_+$  about 10% over the entire kinematic range. The shift in  $h_-$  is similar to that in  $h_+$ . The same pattern of current improvement is found for all four ensembles analyzed.

Non-perturbative calculation of renormalization factor  $Z_V$  for heavy quark current for  $b \rightarrow c$  transition is being developed. Tree-level renormalization  $Z_{V,cb}^{\text{tree}} = \exp[(m_{1b}a + m_{1c}a)/2]$  is applied in this work.

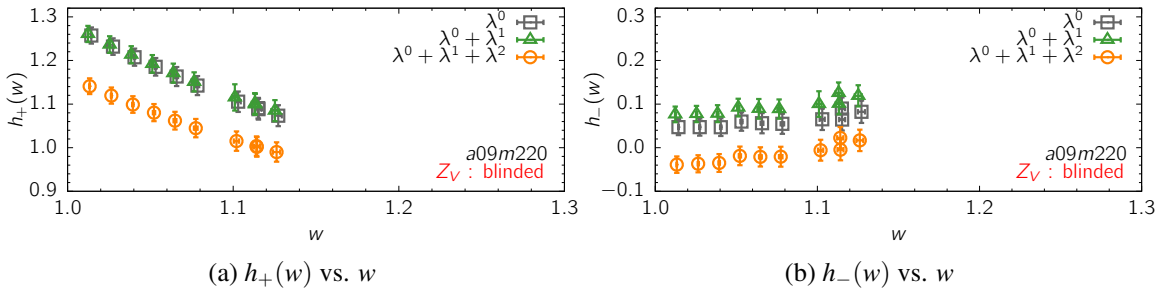


Figure 4: Form factors  $h_{\pm}(w)$  on  $a09m220$  as a function of recoil parameter  $w$ . Here, we show how  $h_{\pm}(w)$  changes as we improve the current up to the  $\lambda^n$  order for  $n = 0, 1, 2$ .

## 5. Summary & Outlook

It is crucial to improve the currents up to  $\mathcal{O}(\lambda^3)$ , the same level as the OK action in our work. We have implemented the  $\mathcal{O}(\lambda^2)$  improvement in this work and the  $\mathcal{O}(\lambda^3)$  current improvement is being implemented. We plan to calculate the matching factors  $\rho_{A_j}$  and  $Z_V$  in two independent ways: one is one-loop perturbation and the other is non-perturbative renormalization using the RI-MOM and RI-SMOM schemes.

We plan to analyze two more data sets measured,  $a12m130$  and  $a06m310$  [11], and to extend the measurement to include other physical pion mass and finer lattices. Statistics will be increased with the truncated solver method with bias correction. We plan to undertake the data analysis for the  $B \rightarrow D^* \ell \nu$  decays soon.

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