

Disentangling excited states

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In lattice gauge theory, understanding contributions from excited states is imperative for achieving high precision calculations. A variety of methods are available to extract excited states, such as fitting to multiple exponentials, Prony's method, and Matrix Prony, correlator matrices, and generalized eigenvalue problems. A generic problem faced by all these methods is that the resulting states tend to have overlapping error ellipses (e.g. jackknife, bootstrap, cross-validation, etc.) making identification of states ambiguous. The problem may be alleviated somewhat by expert guidance in operator selection to minimize overlap for a few low-lying states, but this defeats the overall design goal of an automated black-box method. Instead, we face the overlapping states labeling problem directly. For example, using the bootstrapping method, resolving excited state energies and their error bars requires finding the most probable set of state labels for each bootstrap sample. We investigate several variants of expectation maximization clustering in attempt to find an efficient algorithm for bootstrap labeling and therefore state identification.

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In this work, we investigated the labeling problem which occurs when using an algebraic method of extracting excited states from lattice correlation functions. The algebraic method discussed here is Prony's method, derived from the original work [1, 2] for use in lattice correlation functions [3–7]. In our presentation, we mainly address the problem using clustering algorithms, following the work in [8], but we find this approach to be insufficient. Recently, we have found a suitable solution to the labeling problem which we call *automated label flows*, based on an idea originating in [9]. The shortcomings of our original approach and a discussion of the automated label flows solution and how it overcomes the labeling problem is presented in [10], which can be found at <https://arxiv.org/abs/1912.08205>.

References

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