PROCEEDINGS OF SCIENCE

The meson spectrum of large N gauge theories

Margarita García Pérez
Instituto de Física Teórica UAM/CSIC, Nicolás Cabrera 13-15,
E-28049 Universidad Autónomade Madrid, Madrid, Spain
E-mail: margarita.garcia@uam.es

Antonio González-Arroyo*
Departamento de Física Teórica, Módulo 15, Cantoblanco,
E-28049 Universidad Autónoma deMadrid, Madrid, Spain
and
Instituto de Física Teórica UAM-CSIC, Nicolás Cabrera 13-15,
E-28049 Universidad Autónomade Madrid, Madrid, Spain
E-mail: antonio.gonzalez-arroyo@uam.es

Masanori Okawa
Graduate School of Science, Hiroshima University,
Higashi-Hiroshima 739-8526, Japan
E-mail: okawa@hiroshima-u.ac.jp

We present our preliminary results on the determination of the low lying meson spectrum for pure gauge theory in the large N limit. Some results are also shown for the theory with two flavours of quarks in the adjoint representation.

37th International Symposium on Lattice Field Theory - Lattice2019
16-22 June 2019
Wuhan, China

*Speaker.

PoS(LATTICE2019)113

1. Introduction

The study of gauge theories in the large N limit is of fundamental importance for the understanding of the dynamics of these theories, which are at the core of the Standard Model and many of its extensions. The advantage of the large N limit rests in its simplicity, which comes at no price regarding the richness of phenomena encompassed by these theories. The simplification appears already in perturbation theory, since only planar diagrams survive the limit. Large N gauge theories are also sitting at the crux of several new methodologies emerging from string theory, such as holography. Some results obtained by these new methods refer to non-perturbative quantities as the meson spectrum and the string tension. This provides a challenge to lattice gauge theories (LGT), which is the most solid first principles approach to the study of the non-perturbative behaviour of gauge theories. Hence, we believe that, besides the obvious interest of computing non-perturbative observables of the standard model with direct phenomenological impact, a thorough study of large N gauge theories should be addressed. This should be done within the standards of present day LGT, where all errors, statistical and systematic, are controllable and estimated. This is the program that we have set ourselves to accomplish. We have already obtained some results in this respect [1] and the present talk reports on the present status of the calculation of the lowest lying meson spectrum. A complete set of results on masses and decay constants will appear soon [2].

2. Methodology

The most important tool used in obtaining non-perturbative results on the lattice is the Monte Carlo method. As such, it demands dealing with a large but finite number of degrees of freedom. This implies a finite lattice volume, which indirectly also provides a minimal value of the lattice spacing at which the results are not seriously affected by the finite physical volume. Fortunately, these questions are part of the daily procedures of the LGT community and we know how to test and estimate the errors involved, by performing measurements at various values of the lattice volume and the lattice spacing. For the case of large N gauge theories the finite volume does not suffice and one needs to restrict to finite values of N and extrapolate the results. This is the standard methodological procedure that many authors have used to obtain non-perturbative values in the large N limit. From that perspective large N does not provide a simplification of lattice gauge theories.

Fortunately, there is indeed a simplification that takes place in the large N limit within the lattice approach. This follows from the observation of Eguchi and Kawai [3] who argued that finite volume corrections go to zero in the large N limit. Indeed, in its more standard version, the proposal does not hold. There are several alternatives proposed over the years to transform this idea into a reality (We refer the reader to our previous and forthcoming publications for a full list of references). The method that we are using is based on a fairly slight modification of the original proposal of Eguchi and Kawai, which was introduced by two of the present authors [4, 5, 6]. The idea is to use ‘t Hooft twisted boundary conditions (TBC) instead of purely periodic ones. TBC can be easily incorporated to the lattice regularization [7]. Furthermore, as observed in [4], Eguchi-Kawai arguments are valid for TBC as well, but the main assumptions involved in the vanishing of finite volume corrections should work better for well-chosen twist fluxes.

PoS(LATTICE2019)113

The basis of our methodology is the statement that physical observables in the large N limit, on the lattice and in the continuum, take the same value irrespective of the lattice volume, provided the limit is taken with appropiately chosen twisted boundary conditions. Although, not at all compulsory, we will take the extreme case of reducing the volume to a 1-point lattice, defining a matrix model known as the Twisted Eguchi-Kawai model (TEK) [5]. The partition function of this model, which is used to generate gauge field configurations, is given by:

$$Z_{\rm TEK} = \prod_{\mu} \left(\int dV_\mu \right) \exp\{bN \sum_{\mu \neq \nu} z_{\mu\nu} \mathrm{Tr}(V_\mu V_\nu V_\mu^\dagger V_\nu^\dagger)\} \tag{2.1}$$

The V_μ are 4 SU(N) matrices integrated with the Haar measure. The constant b is the lattice version of the inverse 't Hooft coupling, that is kept fixed when taking the large N limit. The constants $z_{\mu\nu} \in$ Z(N) are precisely the fluxes that appear in the definition of the twisted boundary conditions ($z_{\mu\nu} = z^*_{\nu\mu}$). To ensure the validity of the reduction mechanism one should make an appropriate choice. The original Eguchi-Kawai model was based in taking $z_{\mu\nu} = 1$, which has problems at weak coupling where the continuum limit is taken [8]. There are many possible choices but our prefered choice is the one that treats all lattice directions in the most symmetric way. For that purpose one should take N to be the square of an integer $N = \hat{L}^2$ and $z_{\mu\nu} = \exp\{2\pi i k_N \varepsilon_{\mu\nu}/\hat{L}\}$, where $\varepsilon_{\mu\nu} = 1$ for $\mu < \nu$ and k_N is an integer defined modulo $\hat{L}$ and coprime with it.

The main formula underlying the reduction mechanism is given by

$$\lim_{N\longrightarrow\infty} \lim_{V\longrightarrow\infty} \frac{1}{N} \langle \mathrm{Tr}(U(\gamma))\rangle = \lim_{N\longrightarrow\infty} \frac{z(\gamma)}{N} \langle \mathrm{Tr}(V(\gamma))\rangle_{\rm TEK} \equiv W(\gamma) \tag{2.2}$$

where the left-hand side is the standard Wilson loop expectation value at infinite volume and infinite N: $W(\gamma)$. The second expression is the expectation value of the equivalent loop in the TEK model, where $V(\gamma)$ is the corresponding ordered product of the position independent V_μ matrices and $z(\gamma)$ a path dependent element of Z(N) (to be specified later). The equality holds at every value of the coupling b, if the same action (Wilson action in our case) is taken for the infinite volume and the reduced model.

The validity of Eq. 2.2 for our flux choices has been verified in various ways. First by checking the validity of the assumptions entering the non-perturbative proof of Eguchi and Kawai. The result can also be proven analytically to all orders of perturbation theory [5]. Finally, by direct lattice computations [9] of both sides of the equation for various lattice sizes and values of N.

For theoretical and practical purposes it is important to understand the nature of the finite N corrections to Eq. 2.2. The perturbative proof [5] gives us information about this. At large b the reduced model is driven to a mimimum of the TEK action. This is given by matrices $V_\mu = \Gamma_\mu$, where the new matrices, called twist-eaters, satisfy $\Gamma_\mu \Gamma_\nu = z_{\nu\mu} \Gamma_\nu \Gamma_\mu$. Indeed, for a given closed lattice path γ, the product of the twist-eaters along the path $\Gamma(\gamma)$ is just the unit matrix times the $z^*(\gamma)$ factor mentioned earlier. Perturbing around these minima we generate a perturbative expansion where the planar diagrams coincide with those obtained for a lattice volume of size $\hat{L}^4$. This explains the volume independencein the large N limit, and shows that these finite N corrections adopt the form of finite volume corrections. Non-planar diagram suppression is different than in ordinary infinite volume large N theory and depends on the choice of k_N. Indeed, the choice of k_N has a strong effect on the validity of the reduction program and here we will stick to the prescription given in Ref. [6].

Adding a few flavours of quarks in the fundamental representation can be readily done in the standard lattice gauge theory. In the large N limit, quarks do not affect the gauge field probability distribution. Quark and meson propagators are computed in the background field of the pure gauge configurations. The reduced model is not constructed on the basis of a discretized lattice gauge theory with quarks reduced to a single point. This would lead to problems, as TBC become singular for quarks in center-sensitive representations. The philosophy is quite different. We can in principle allow the quarks to live in an infinite lattice, but they propagate in the background field obtained from the V_μ. The situation is similar to that encountered in condensed matter theory in which electrons propagate in an infinite solid, but the background field produced by the ions has the periodicity of the lattice. The infinite lattice gives rise to a continuous Bloch momentum, and the energy levels depend on them giving rise to bands. The actual construction is long enough to make it impossible to review here. We refer the reader to our previous papers in which the method is explained [10]. We should just mention that the methodology can be implemented for different types of lattice fermions. In this work we have used Wilson fermions as before, but also included results with twisted mass.

To conclude this section let us comment briefly about systematic errors. Equation 2.2 holds at infinite N, while our simulations have been done at rather large, but finite, values of N. A good deal of this finiteness is equivalent to working on a finite volume of size $\hat{L}^4$. Thus, the corresponding physical period of the torus is given by $l = \hat{L}a(b)$, where $a(b)$ is the lattice spacing in physical units. The values that we use in this work are given in units of the string tension, as determined from our previous work [1]. If we want our results not to be affected, this finite effective size should be kept much larger than the relevant physical scales of the problem. No doubt that one of these scales is Λ_{QCD}. This forces $\hat{L} = \sqrt{N}$ to be large enough and limits the maximum value of b as well. This is the standard restriction of LGT, with $\sqrt{N}$ replacing the lattice length L. When approaching the chiral limit the pion mass vanishes and the pion propagator gets affected by the finite volume. Hence, we have also avoided coming too close to this limit. All our results have been for values of b and N for which the effective lattice size is kept within reasonably safe limits.

A final comment about the comparison of our method and that based on extrapolation from a more standard lattice approach, as used in the Refs. [11, 12]. In some sense the methods can be seen as complementary. Our method goes directly to the large N limit, but does not produce the correct finite N corrections, so a combination of both results could stabilize the extrapolation and provide better determined $1/N^2$ corrections. Some of the problems appearing at small N such as chiral logs, are absent in our method. Ultimately, all methodologies should agree on the results. In any case, we think that all works performed at finite lattices should consider the use of TBC, which would reduce considerably the finite size errors at almost no cost.

3. Results

This work reports the calculation of the lowest lying meson masses using our procedure based on the TEK model. The gauge field is generated with the TEK probability distribution using the overrelaxation technique explained in Ref. [13]. The results presented here were obtained with $N = 289$, implying $\hat{L} = 17$. In order to check and study the continuum limit we simulated three values of b, 0.355, 0.36 and 0.365. For these values, the lattice spacing measured in string tension

PoS(LATTICE2019)113

units is given by 0.241, 0.206 and 0.178 respectively, with errors of order 2 %. This corresponds to effective lattice sizes of 4.10, 3.50 and 3.03. Our results are based on 800 configurations per value of b. We have also generated results for b=0.37, but some observables might start to exhibit finite N effects. Our complete results including b=0.37 and various values of N will be presented in our future publication [2].

The methodology to extract masses is based on measuring exponential decay of the correlation functions of operators with the right quantum numbers. The basic operators are those obtained by inserting a Clifford algebra matrix Γ inside a local $\bar{\Psi}(x)\Gamma\Psi(x)$ meson operator. This gives the J^{PC} quantum numbers 0^{-+}, 1^{--}, 0^{++} 1^{++} and 1^{+-}. We will refer to these states with the names given in standard QCD (π, ρ, a_0, a_1 and b_1). For each possible matrix we consider a whole family of operators (up to 12) obtained by smearing the gauge field and the fermion field. We then use a variational technique to generate an optimal operator which couples maximally to the lowest mass state with the corresponding quantum numbers.

As mentioned earlier, most of our results are obtained for Wilson fermions. We used between 5 and 7 different values of the hopping parameter for each b. For the pseudoscalar and vector cases, which we label π and ρ, we also studied the twisted mass Dirac operator for 4 values of μ. The use of twisted mass is specially important for the determination of the decay constants and operator renormalization Z coefficients, which will not be presented here. In summary the main results of our investigation are the following:

1. Good exponential fall-off of the meson correlators is observed, allowing a fairly precise determination of the masses. Examples are shown in Fig. 1.

2. The pseudoscalar propagator behaves as expected for a spontaneously chiral symmetry breaking phase. This means that the mass of the lightest state squared vanishes linearly when the hopping κ parameter tends to a certain value κ_c. For the twisted mass case the vanishing is linear with the parameter μ as shown in Fig. 2.

3. The masses of the other states depend linearly on m_{PCAC}. In Fig. 3 we display the results for the ρ and a_0 states. In all cases the masses in string tension units are consistent for the two largest values of b. A simultaneous linear fit to these two values gives the slopes and intercepts given in Table 1. Both statistical and systematic errors are given. The intercepts correspond to the masses of these mesons in the chiral limit. Our results are compared with the values of Ref. [11].

4. Our data shows good scaling behaviour within our statistical errors. This can be easily appreciated from Fig. 4 which displays the masses in the chiral limit obtained by fitting independently the results for each value of b. The horizontal bands give our estimates for the masses in the continuum limit.

5. We have performed a full analysis of systematic errors. We address the reader to our future publication giving full details of the methodology, tests and results.

4. Conclusions and Outlook

Our results show that the idea of volume reduction can be used succesfully to determine the physical properties of large N gauge theories. We want to stress that this can be extended to theories with dynamical fermions. This includes theories with quarks in the adjoint representation, encompassing very interesting theories ranging from SUSY Yang-Mills to theories that are expected to lay

PoS(LATTICE2019)113

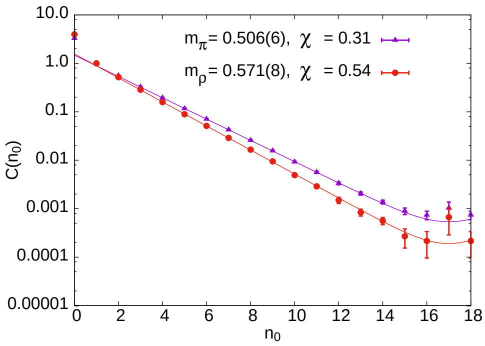

Fig. 1: Example of pion and ρ correlators for the optimal operator.

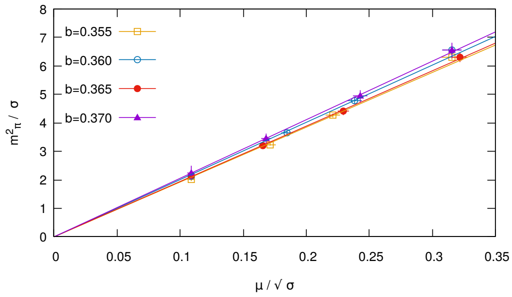

Fig. 2: m_π^2 versus μ for twisted mass fermions.

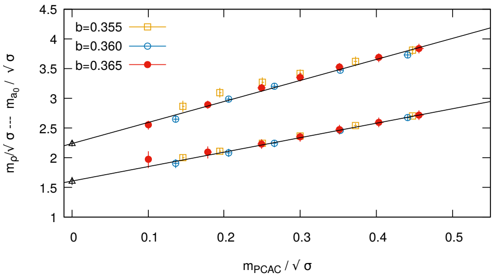

Fig. 3: Mass of ρ (lower) and a_0 mesons (upper) in units of the string tension versus $m_{\rm PCAC}$.

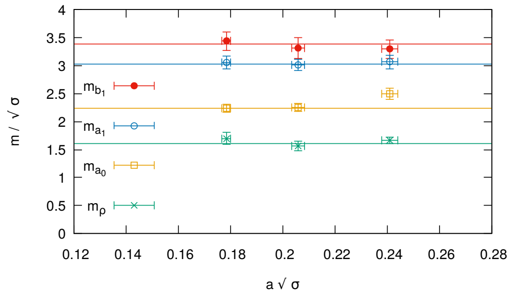

Fig. 4: Masses in the chiral limit.

	slope	mass/$\sqrt{\sigma}$	Ref. [11]
ρ	2.43(12)	1.61(7)(5)	1.538(7)
a_0	3.53(22)	2.24(5)(4)	2.40(4)
a_1	2.32(15)	2.99(8)(2)	2.86(2)
b_1	2.22(17)	3.20(12)(18)	2.90(2)

Table 1: Summary of meson masses.

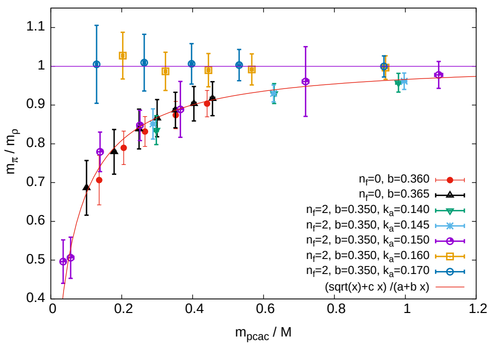

Fig. 5: The ratio of masses of π and ρ mesons made of fundamental quarks of mass $m_{\rm PCAC}$ for 2 flavours of adjoint quarks with various quark masses.

within the conformal window. In Fig. 5 we show results for the ratio of pion to rho masses (made of quarks in the fundamental) for the theory with $N_f = 2$ light quarks in the adjoint, in deep contrast with the chiral symmetry breaking of the $N_f = 0$ theory. Furthermore, theories with quarks in the fundamental representation in the Veneziano limit also seem to be accesible with our methods.

Acknowledgments

A.G-A and M. G.P. acknowledge financial support from the MINECO/FEDER grant FPA2015-68541-P, the MINECO Centro de Excelencia Severo Ochoa Program SEV-2016- 0597 and the EU H2020-MSCA-ITN-2018-813942 (EuroPLEx). M.O. is supported by the Japanese MEXT grant No. 17K05417. This research used computational resources of the SX-ACE system provided by Osaka University through the HPCI System Research Project (Project ID:hp170003, hp180002).

References

[1] A. González-Arroyo and M. Okawa, *The string tension from smeared Wilson loops at large N*, *Phys. Lett.* **B718** (2013) 1524–1528, [1206.0049].

[2] M. García Pérez, A. González-Arroyo and M. Okawa, *In preparation*.

[3] T. Eguchi and H. Kawai, *Reduction of Dynamical Degrees of Freedom in The Large N Gauge Theory*, *Phys. Rev. Lett.* **48** (1982) 1063.

[4] A. González-Arroyo and M. Okawa, *A Twisted Model for Large N Lattice Gauge Theory*, *Phys. Lett.* **120B** (1983) 174–178.

[5] A. González-Arroyo and M. Okawa, *The Twisted Eguchi-Kawai Model: A Reduced Model for Large N Lattice Gauge Theory*, *Phys. Rev.* **D27** (1983) 2397.

[6] A. González-Arroyo and M. Okawa, *Large N reduction with the Twisted Eguchi-Kawai model*, *JHEP* **07** (2010) 043, [1005.1981].

[7] J. Groeneveld, J. Jurkiewicz and C. P. Korthals Altes, *Twist as a Probe for Phase Structure*, *Phys. Scripta* **23** (1981) 1022.

[8] G. Bhanot, U. Heller and H. Neuberger, *The Quenched Eguchi-Kawai Model*, *Phys.Lett.* **B113** (1982) 47.

[9] A. González-Arroyo and M. Okawa, *Testing volume independence of SU(N) pure gauge theories at large N*, *JHEP* **12** (2014) 106, [1410.6405].

[10] A. González-Arroyo and M. Okawa, *Large N meson masses from a matrix model*, *Phys. Lett.* **B755** (2016) 132–137, [1510.05428].

[11] G. S. Bali, F. Bursa, L. Castagnini, S. Collins, L. Del Debbio, B. Lucini, and M. Panero, *Mesons in large-N QCD*, *JHEP* **06** (2013) 071, [1304.4437].

[12] T. DeGrand and Y. Liu, *Lattice study of large N_c QCD*, *Phys. Rev.* **D94** (2016) 034506, [1606.01277].

[13] M. G. Pérez, A. González-Arroyo, L. Keegan, M. Okawa and A. Ramos, *A comparison of updating algorithms for large N reduced models*, *JHEP* **06** (2015) 193, [1505.05784].