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Nucleon charges and form factors using clover and HISQ ensembles

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We present high statistics ($\mathcal{O}(2\times 10^5)$ measurements) preliminary results on (i) the isovector charges, $g^{u-d}_{A,S,T}$, and form factors, $G^{u-d}_E(Q^2)$, $G^{u-d}_M(Q^2)$, $G^{u-d}_A(Q^2)$, $\widetilde{G}^{u-d}_P(Q^2)$, $G^{u-d}_P(Q^2)$, on six $2+1$-flavor Wilson-clover ensembles generated by the JLab/W&M/LANL/MIT collaboration with lattice parameters given in Table 1. Examples of the impact of using different estimates of the excited state spectra are given for the clover-on-clover data, and as discussed in [1], the biggest difference on including the lower energy (close to $N\pi$ and $N\pi\pi$) states is in the axial channel. (ii) Flavor diagonal axial, tensor and scalar charges, $g^{u,d,s}_{A,S,T}$, are calculated with the clover-on-HISQ formulation using nine 2+1+1-flavor HISQ ensembles generated by the MILC collaboration [2] with lattice parameters given in Table 2. Once finished, the calculations of $g^{u,d,s}_{A,T}$ will update the results given in Refs. [3, 4]. The estimates for $g^{u,d,s}_S$ and $\sigma_{N\pi}$ are new. Overall, a large part of the focus is on understanding the excited state contamination (ESC), and the results discussed provide a partial status report on developing defensible analyses strategies that include contributions of possible low-lying excited states to individual nucleon matrix elements.

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1. Isovector charges with 2+1-flavor clover fermions

Examples of ESC in the vector charge, g_V^{u-d}, and form factors G_E and G_M are illustrated in Fig. 1. g_V^{u-d} does not vary monotonically with source-sink separation τ, however it is constant to within 1–2%. So we take the average of the central points with the largest τ (the plateau method). With this choice the identity $Z_V g_V^{u-d} = 1$ is satisfied to within 3% with the Z_V calculated in Ref. [5].

Data for the charges, $g_{A,S,T}^{u-d}$, show significant ESC as discussed in [5, 6]. As described in Ref. [1], the key parameter controlling ESC is the energy, E_1, of the first excited state. Its value, obtained from a 4-state fit to the 2-point function using emperical Bayesian priors [7], is much larger than that of non-interating $N\pi$ or $N\pi\pi$-states, especially for physical M_π ensembles. Using the energy of the $N\pi$ state as a prior for E_1 in a 3-state fit gives a much lower output value for E_1 but with an equally good χ^2/DOF, indicating a flat direction in the parameter space. Note that with a small E_1, even E_0 is slightly smaller. We have, therefore, analyzed the ESC using multiple strategies and, here, compare two for $g_{A,S,T}^{u-d}$ based on 3^*-state fits (3-state truncation of the spectral decomposition of the 3-point functions with $\langle 2'|\mathcal{O}|2\rangle = 0$). The standard $\{4,3^*\}$ and $\{3^{N\pi},3^*\}$. In $\{4,3^*\}$, the spectrum is taken from the standard 4-state fit [8]. In $\{3^{N\pi},3^*\}$, the energy $E_1^{N\pi}$ of the lowest possible state, $N(\mathbf{1})\pi(-\mathbf{1})$, is used as a prior for E_1 in a 3-state fit and the resulting outputs, ground-state amplitude A_0 and energies E_0, E_1 and E_2, are used as inputs in fits to the 3-point functions. The data and fits for $\{4,3^*\}$ and $\{3^{N\pi},3^*\}$ are compared in Fig. 2 for the $a091m170L$ ensemble, where one expects the largest effect as it has the smallest $M_\pi \sim 170$ MeV and $Q^2_{\rm min}$, with $Q^2 = \vec{p}^2 - (E_N - M_N)^2$ being the Euclidean 4-momentum squared transferred.

The value of g_A^{u-d} is sensitive to input E_1 used in the ESC fits, however, different fits are not distinguished by χ^2/DOF, again indicating a flat direction. Renormalized charges in the $\overline{MS}$ scheme at 2 GeV, $g_{A,S,T}^{u-d}|_R = Z_{A,S,T}^{u-d} g_{A,S,T}^{u-d}$, are obtained using $Z_{A,S,T}^{u-d}$ from Ref. [5]. Their chiral-continuum (CC) extrapolation is done using the ansatz $f(a,M_\pi) = c_1 + c_2 a + c_3 M_\pi^2$ (see Fig. 3), and the results at $M_\pi = 135$ MeV and $a = 0$ are given in Table 3. The difference in g_A^{u-d} is a measure of the systematic uncertainty associated with ESC fits. Data for $g_{S,T}^{u-d}$ from the two strategies, shown in Fig. 3 and the extrapolated values in Table 3, are consistent within 1σ and the ESC fits do not prefer the low $E_1^{N\pi}$.

2. Form factors

We pointed out in Ref. [1] that the large violation of the PCAC relation between axial and

Ensemble ID	a (fm)	M_π (MeV)	$L^3 \times T$	$M_\pi L$	$N_{\rm conf}$	$N_{\rm meas}^{\rm HP}$	$N_{\rm meas}^{\rm LP}$	τ
$a127m285$	0.127(2)	285(3)	$32^3 \times 96$	5.85	2002	8008	256,256	{8, 10, 12, 14}
$a094m270$	0.094(1)	270(3)	$32^3 \times 64$	4.11	2469	7407	237,024	{8, 10, 12, 14, 16}
$a094m270L$	0.094(1)	269(3)	$48^3 \times 128$	6.16	1854	7416	237,312	{8, 10, 12, 14, 16, 18}
$a091m170$	0.091(1)	170(2)	$48^3 \times 96$	3.7	2754	11016	352,512	{8, 10, 12, 14, 16}
$a091m170L$	0.091(1)	170(2)	$64^3 \times 128$	5.08	1825	9125	292,000	{8, 10, 12, 14, 16}
$a073m270$	0.0728(8)	272(3)	$48^3 \times 128$	4.8	2454	9816	314,112	{11, 13, 15, 17, 19}

Table 1: Lattice parameters of $2+1$-flavor clover ensembles generated by the JLab/W&M/LANL/MIT collaboration. $N_{\rm meas}^{\rm LP}$ low-precision and $N_{\rm meas}^{\rm HP}$ high-precision measurements of 2- and 3-point functions are made using the bias corrected truncated solver method (see Ref. [5] for details.). τ gives the source-sink separations studied. Statistics on $a094m270L$, $a091m170$ and $a091m170L$ ensembles are being increased.

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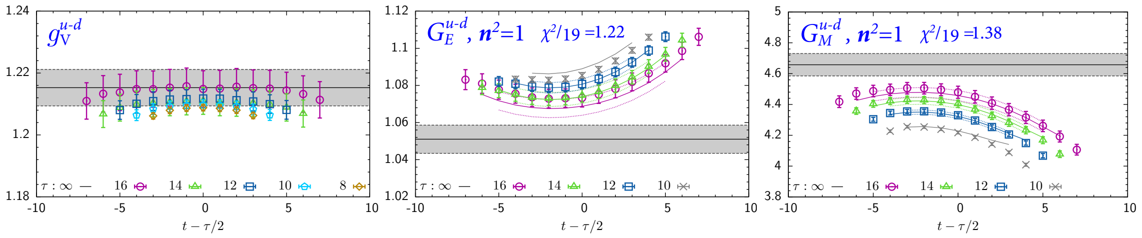

Figure 1: Example of ESC in unrenormalized isovector vector charge g_V^{u-d} and form factors $G_{E,M}^{u-d}(\vec{p}^2)$ at $\vec{p}^2 = (2\pi/L)^2\vec{n}^2$ with $\vec{n}^2 = 1$ on $a09m170L$ clover lattices. For g_V^{u-d}, the $\tau \to \infty$ value (grey band) is the average of the 5 middle data points with $\tau = 16$. Fits to $G_{E,M}^{u-d}$ use the $\{4,3^*\}$ strategy.

pseudoscalar form factors observed in [7] is due to lower energy $N\pi$ excited states that are not exposed by the $\{4,3^*\}$ analysis. Including them addressed the PCAC relation [1]. We, therefore explore 2 fit strategies here. The top 5 panels in Fig. 4 show renormalized G_E^{u-d}, G_M^{u-d}, axial (G_A^{u-d}), induced pseudoscalar ($\widetilde{G}_P^{u-d}$) and pseudoscalar (G_P^{u-d}) form factors analyzed using the standard $\{4,3^*\}$ strategy. The bottom 5 panels are with (i) 2-state simultaneous fit to all V_μ channels for G_E and G_M with E_1 left free, and (ii) the S_{A4} strategy defined in [1] for the axial channels.

The G_E and G_M data show better collapse onto a single curve (indicating no significant a, M_π, volume dependence) plotted versus Q^2/M_N^2, and the agreement with the Kelly curve is better compared to the clover-on-HISQ data discussed in Ref. [8]. The main difference between the two strategies is in the errors: the errors from the simultaneous fits are larger, especially at the larger Q^2.

In the axial channels, data with S_{A4} satisfies PCAC with most of the change occuring in $\widetilde{G}_P^{u-d}$ and G_P^{u-d} as discussed in [1]. Note that data for G_A^{u-d}, $\widetilde{G}_P^{u-d}$ and G_P^{u-d}, shown in Fig. 4, will move up or down depending on the value of g_A, which, as shown in Tab. 3, has unresolved systematics. Thus, resolving the ESC in g_A is essential before comparing/using $G_A(Q^2)$ in phenomenology.

Ensemble ID	a (fm)	M_π (MeV)	$M_\pi L$	$L^3 \times T$	$N^l_{\rm conf}$	$N^l_{\rm src}$	$N^s_{\rm conf}$	$N^s_{\rm src}$	$N_{\rm LP}/N_{\rm HP}$
$a15m310$	0.1510(20)	320(5)	3.93	$16^3 \times 48$	1917	2000	1919	2000	50
$a12m310$	0.1207(11)	310(3)	4.55	$24^3 \times 64$	1013	5000	1013	1500	30
$a12m220$	0.1184(10)	228(2)	4.38	$32^3 \times 64$	958	11000	958	4000	30
$a09m310$	0.0888(8)	313(3)	4.51	$32^3 \times 96$	1081	4000	1081	2000	30
$a09m220$	0.0872(7)	226(2)	4.79	$48^3 \times 96$	712	8000	847	10000	30/50
$a09m130$	0.0871(6)	138(1)	3.90	$64^3 \times 96$	1270	10000	877	10000	50
$a06m310$	0.0582(4)	320(2)	3.90	$48^3 \times 144$	830	4000	956	10000	50
$a06m220$	0.0578(4)	235(2)	4.41	$64^3 \times 144$	593	10000	554	10000	50
$a06m135$	0.0570(1)	136(1)	3.7	$96^3 \times 192$	553	500	553	500	50

Table 2: Parameters of the $2+1+1$-flavor HISQ ensembles used for the calculation of disconnected contributions (update of work in Refs. [3, 4]). $N^{l,s}_{\rm conf}$ gives the number of gauge configurations analyzed for light (l) and strange (s) flavors. $N^{l,s}_{\rm src}$ the number of random sources used per configurations, and $N_{\rm LP}/N_{\rm HP}$ the ratio of low- to high-precision meausurements. Results for the connected contributions are taken from Ref. [6].

Charge	$\{4,3^*\}$	$\{3^{N\pi},3^*\}$
$g_A^{u-d}\vert_R$	1.20(5) [0.26]	1.26(5) [0.24]
$g_S^{u-d}\vert_R$	1.08(10) [0.32]	1.09(14) [0.49]
$g_T^{u-d}\vert_R$	0.95(5) [0.05]	0.94(6) [0.35]

Table 3: $g_{A,S,T}^{u-d}|_R$ in $\overline{MS}$ scheme at 2 GeV calculated in 2 ways to remove ESC, and [χ^2/DOF] of CC fits.

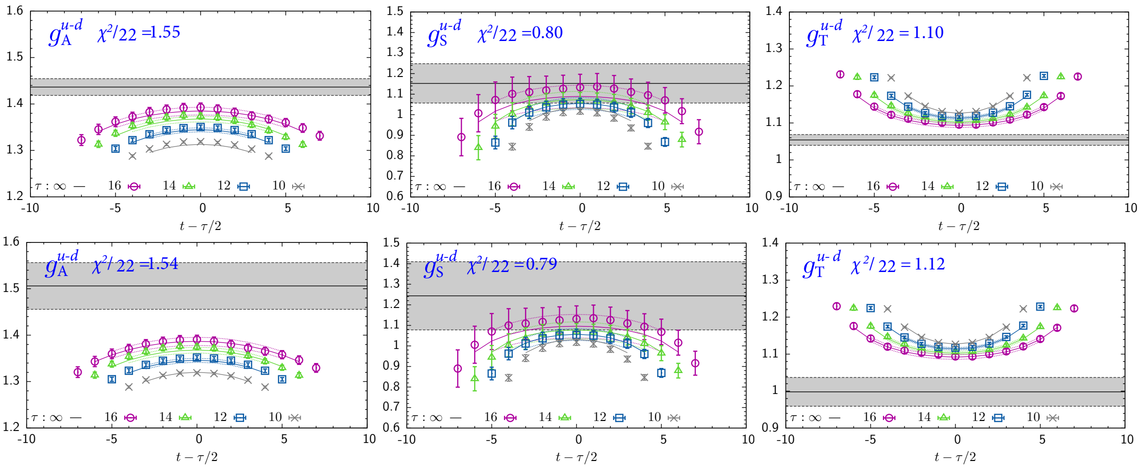

Figure 2: Data and ESC fits for unrenormalized charges $g^{u-d}_{A,S,T}$ on $a09m170L$ clover lattices using the $\{4,3^*\}$ fit (top 3 panels) and the $\{3^{N\pi},3^*\}$ fit (bottom 3 panels). Values of τ and [χ^2/DOF] are given in the legend.

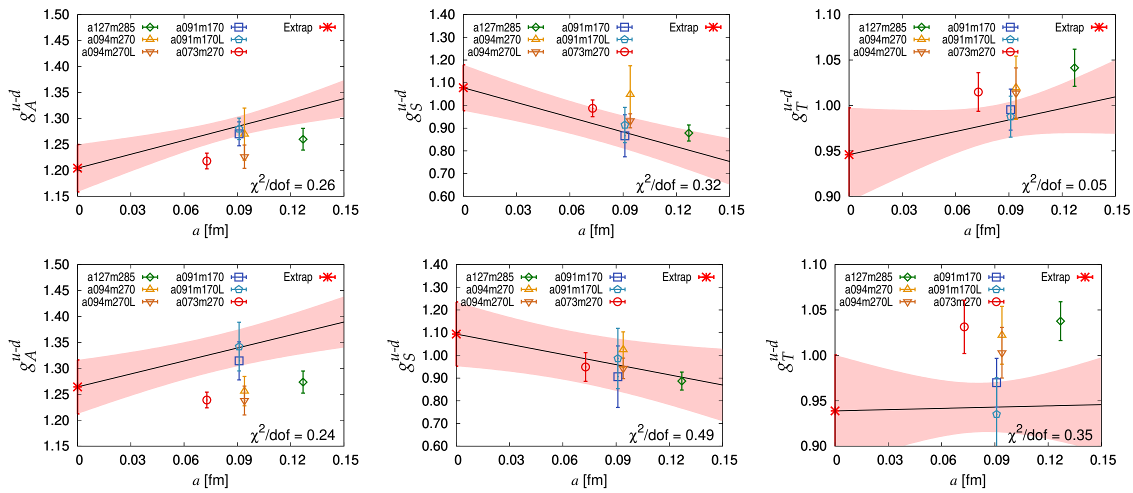

Figure 3: Chiral-continuum extrapolation of the renormalized (in $\overline{MS}$ at 2 GeV) isovector charges using the ansatz $f(a,M_\pi) = c_1 + c_2 a + c_3 M_\pi^2$. Results with $\{4,3^*\}$ ($\{3^{N\pi},3^*\}$) strategy are shown in the top (bottom) 3 panels. In each pannel, the pink band shows the result of the simultaneous fit plotted versus the lattice spacing a with M_π set to 135 MeV. The value in the continuum limit, $a = 0$, is marked with a red star.

3. Flavor diagonal charges on $2+1+1$-flavor HISQ lattices

The flavor diagonal charges presented here are obtained using the same ESC strategy as discussed in [3, 4]. Alternate analyses taking into account possible lower excited states are in progress. The connected and disconnected contributions shown in Fig. 5 are analyzed separately to construct the renormalized charges $g^f_{A,S,T}|_R = Z^{ff'}_{A,S,T}(g^{f',\text{conn}}_{A,S,T} + g^{f',\text{disc}}_{A,S,T})$, where f, f' are quark flavors. The connected contribution, $g^{f,\text{conn}}_{A,S,T}$, are taken from Ref. [6]. Here, we update $g^{f,\text{disc}}_{\Gamma}$ using the larger data set shown in Table 2, and present new results on the connected and disconnected contributions (right two panels in Fig. 5) for the renormalization matrix $Z^{ff'}_{A,S,T}$ in the 3-flavor theory using the RI-sMOM scheme. The matching between the lattice RI-sMOM and continuum $\overline{MS}$ schemes, and the running to 2 GeV are done using 2-loop perturbation theory. Additionally, we give our first preliminary data for the scalar charges.

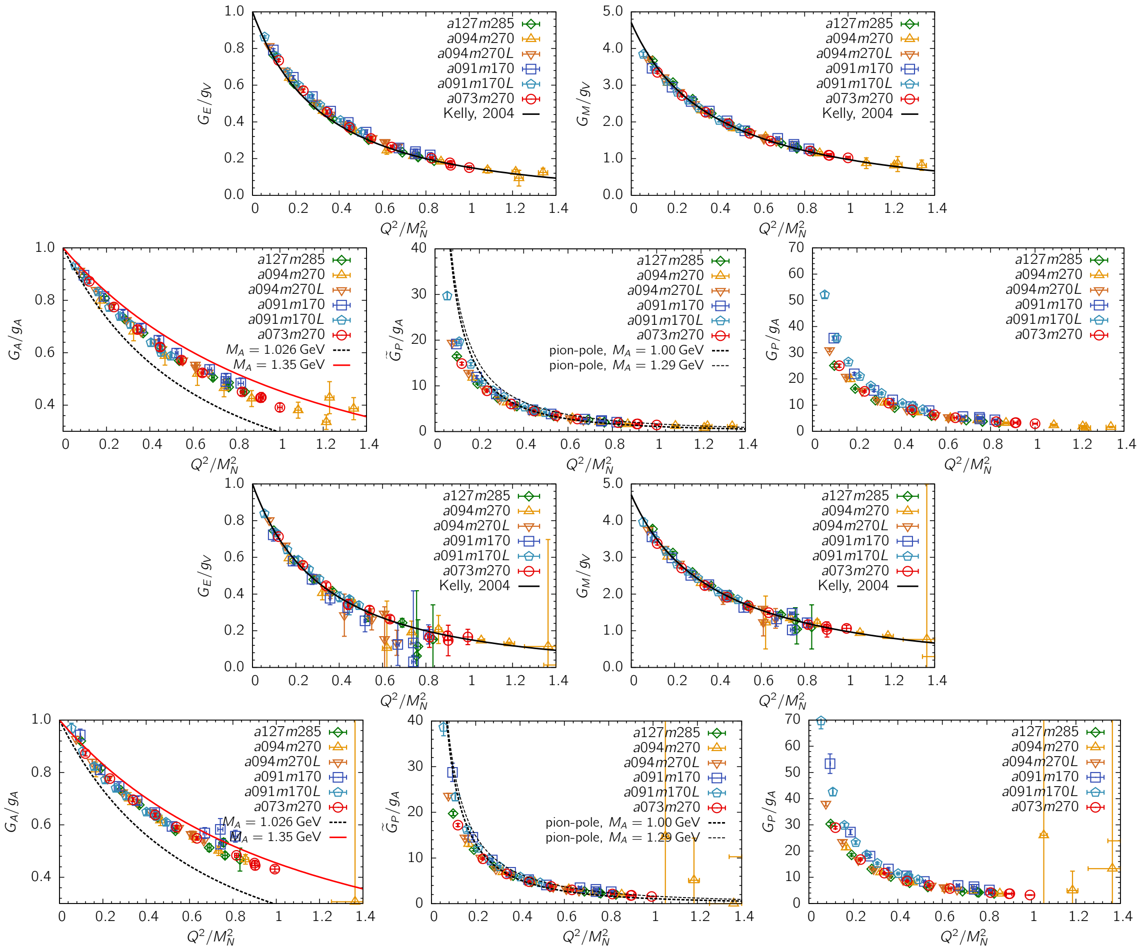

Figure 4: Renormalized isovector form factors (G_E^{u-d}, G_M^{u-d}, G_A^{u-d}, $\widetilde{G}_P^{u-d}$ and G_P^{u-d}) versus Q^2/M_N^2. Top (bottom) 5 panels show data with standard $\{4,3^*\}$ (new) strategy. The value of g_A is taken from $\{4,3^*\}$ fit.

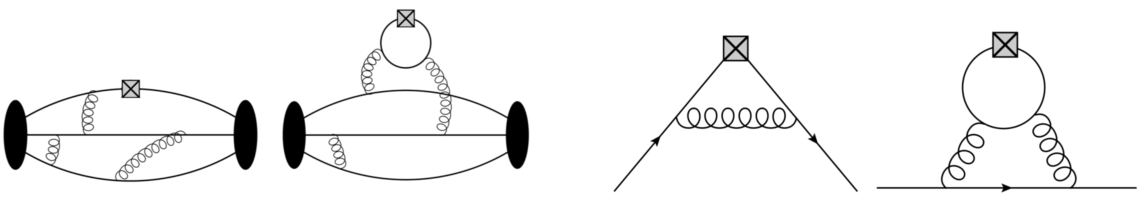

Figure 5: Connected and disconnected diagrams for (i) the 3-point functions that give nucleon charges (left 2 panels) and (ii) the renormalization of the flavor diagonal quark bilinear operators in 3-flavor theory (right 2).

The new data for Z factors in Table 4 show that the difference between the isovector $(u-d)$ and isoscalar $(u+d)$ renormalization constants for the axial and tensor operators is small for all 4 values of a. This validates the approximation $g_{A,T}^{u+d}|_R = Z_{A,T}^{u+d,u+d} g_{A,T}^{u+d} + Z_{A,T}^{u+d,s} g_{A,T}^{s} \approx Z_{A,T}^{u-d} g_{A,T}^{u+d}$ made in Refs. [3, 4] for our clover-on-HISQ calculations. Also, the off-diagonal mixing $Z_{A,T}^{u+d,s} g_{A,T}^{s}$ is tiny since $Z_{A,T}^{u+d,s} \lesssim 0.1$ and the purely disconnected contributions are even smaller: $g_A^s \approx 0.05$ and $g_T^s \approx 0.002$. With only 8 data points, the chiral-continuum extrapolations of the disconnected contributions $g_{A,T}^{l,s,\text{disc}}|_R$ are carried out using the simple ansatz $g(a,M_\pi) = c_1 + c_2 a + c_3 M_\pi^2$ and the data and fits are shown in Fig 6. Combining the disconnected contributions with the connected

a	Z_A^{u-d}/Z_V^{u-d}	$Z_A^{u+d,u+d}/Z_V^{u-d}$	a	Z_T^{u-d}/Z_V^{u-d}	$Z_T^{u+d,u+d}/Z_V^{u-d}$
0.15	1.080(13)	1.0856(11)	0.15	1.032(18)	1.031(19)
0.12	1.061(11)	1.073(11)	0.12	1.0538(76)	1.0541(68)
0.09	1.0380(40)	1.0484(32)	0.09	1.0795(34)	1.0796(34)
0.06	1.0227(19)	1.0383(28)	0.06	1.0956(70)	1.0959(70)

Table 4: Renormalization factors Z_A and Z_T for $u-d$ (isovector) and $u+d$ (isoscalar) operators in $\overline{MS}$ scheme at 2 GeV on HISQ lattices. Renormalizing using ratios with Z_V^{u-d} is intended to cancel some of the statistical and systematic uncertainties as discussed in Ref. [6]. Errors quoted are the larger of the two: half the difference between RI-MOM and RI-sMOM results or the largest statistical error.

contributions $g_{A,T}^{l,\text{conn}}|_R$ presented in Ref. [6], our preliminary updated flavor diagonal charges are

$$g_A^u|_R = 0.790(23)(30) \qquad g_A^d|_R = -0.425(15)(30) \qquad g_A^s|_R = -0.053(7) \tag{3.1}$$

$$g_T^u|_R = 0.783(27)(10) \qquad g_T^d|_R = -0.205(10)(10) \qquad g_T^s|_R = -0.0022(12)\,, \tag{3.2}$$

where the second is a systematic error assigned to the chiral-continuum extrapolation [6].

There remain issues regarding the systematics in the calculation of the matrix element of the scalar operator that are still being investigated: the values for the the renormalization constants, Z_S, show significant differences between the RI-MOM and RI-sMOM schemes. For example, there are $5 \sim 30\%$ differences in Z_S^{u-d}, and $5 \sim 10\%$ differences in $Z_S^{u+d,u+d}$ with the differences increasing as the lattice spacing becomes larger. For the mixing matrix element $Z_S^{s,u+d}$, the RI-MOM scheme gives $-0.04 \sim -0.1$, which is much larger than the RI-sMOM scheme result of $-0.003 \sim -0.02$. In fact, in the calculation of strangeness, $g_S^s|_R = Z_S^{s,s} g_S^s + Z_S^{s,u+d} g_S^{u+d}$, the larger value of mixing $Z_S^{s,u+d}$ in RI-MOM scheme gives a negative value for $g_S^s|_R$! For the time being, we use the RI-sMOM scheme, in which case the corresponding mixing term $Z_S^{s,u+d} g_S^{u+d}$ gives about $6 \sim 20\%$ correction to the diagonal term $Z_S^{s,s} g_S^s$.

The renormalized strangeness $g_S^s|_R$, from the clover-on-HISQ calculation, is plotted versus a and M_π^2 in Fig. 7, along with the nucleon sigma term $\sigma_{\pi N} = m_l g_S^{u+d}$ that is independent of the renormalization scheme. We have used $am_l = \frac{1}{2}(\kappa^{-1} - \kappa_{\text{crit}}^{-1})$ for the definition of the bare quark mass and $g_S^{u+d} = g_S^{u+d,\text{conn}} + g_S^{u+d,\text{disc}}$ for the unrenormalized isoscalar scalar charge. The data show a significant a dependence in $g_S^s|_R$ while the large linear dependence of $\sigma_{\pi N}$ on M_π^2 comes from the quark mass in the definition of $\sigma_{\pi N}$. The dependence of $g_S^{u+d}|_R$ on a and of $g_S^s|_R$ on M_π^2 is not clear.

Conclusions: We have presented the status of ongoing calculations of nucleon matrix elements and are performing a more detailed analysis of the excited state contamination in the extraction of **all** nucleon matrix elements. The analysis of flavor diagonal scalar charges, $g_S^{u,d,s}$ is new, however, a complete understanding of all the systematics is still under investigation.

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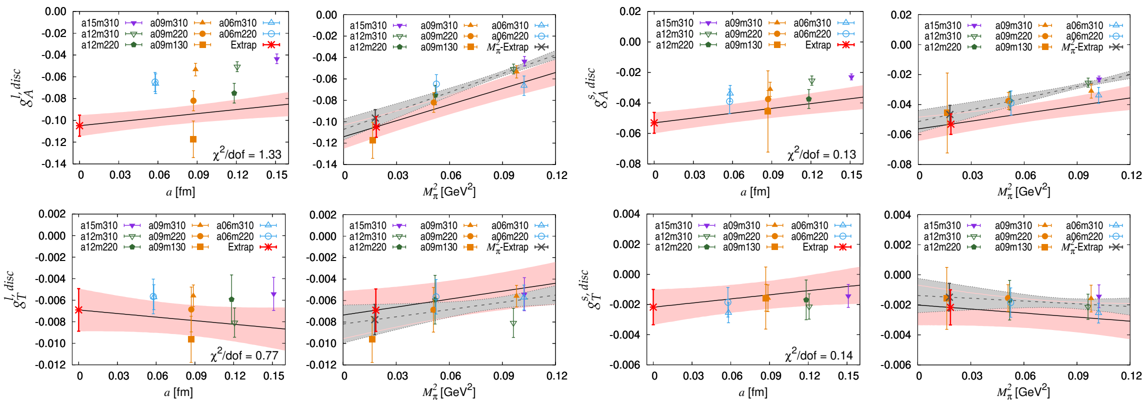

Figure 6: The extrapolation of the disconnected contributions of the renormalized (in $\overline{MS}$ at 2 GeV) flavor diagonal charges $g_A^{l(s),\mathrm{disc}}|_R$ (top row) and $g_T^{l(s),\mathrm{disc}}|_R$ (bottom row) using the chiral-continuum fit ansatz $g(a, M_\pi) = c_1 + c_2 a + c_3 M_\pi^2$. The parameters for the eight clover-on-HISQ ensembles are given in Table 2.

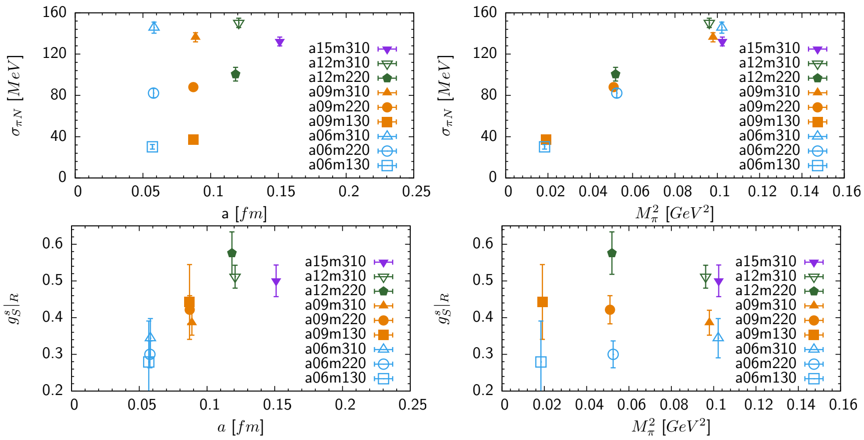

Figure 7: (Top) The nucleon sigma term $\sigma_{N\pi}$ plotted versus a and M_π^2. (Bottom) The nucleon strangeness $g_s^s|_R$ renormalized in $\overline{MS}$ scheme at 2 GeV versus a and M_π^2.

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