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# $B \to \pi \ell \nu$ form factors and $|V_{ub}|$ with Möbius domain wall fermions

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We report on a calculation of form factors for the semileptonic decay of $B$ meson to pion on $2+1$-flavour lattices with lattice spacings from 0.080 fm down to 0.044 fm. Using the Möbius domain wall fermion action for both sea and valence quarks, we simulate pions with masses down to 225 MeV. By utilizing a range of heavy quark masses up to 2.44 times the mass of the charm quark we extrapolate to the physical $b$ quark mass. We discuss the dependence of the form factors on the pion mass, heavy quark mass, lattice spacing and the momentum-transfer. We extract the CKM matrix element $|V_{ub}|$ through a simultaneous fit with the $B \to \pi \ell \nu$ differential branching fractions provided by the Belle and BaBar collaborations after a chiral-continuum and physical $b$ quark extrapolations of our lattice data.

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PoS(LATTICE2019)143

## 1. Introduction

The semileptonic process $B \to \pi\ell\nu$ may be used to extract the element $|V_{ub}|$ of the Cabibbo–Kobayashi–Maskawa matrix. Here we report on our lattice QCD study of this decay, which forms a part of a larger series of studies of heavy quark processes, including other exclusive decays like $B \to D^{(*)}$ [1] and inclusive decays [2]. We use the Möbius domain-wall fermion action [3] for all quarks, which has the advantage of including all relativistic effects for the heavy quarks, but necessitates extrapolating to physical $m_b$ from lower heavy quark masses, $m_h$. Preliminary results have been reported in [4].

The CKM matrix element can be related to the (experimental) differential decay rate by

$$\frac{\mathrm{d}\Gamma(B \to \pi\ell\nu)}{\mathrm{d}q^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}|k_\pi|^3|f_+(q^2)|^2. \tag{1.1}$$

Thus calculating the form factor $f_+(q^2)$ from lattice QCD allows us to extract $|V_{ub}|$. Here $k_\pi$ is the pion four momentum in the $B$ meson rest frame and $q^\mu = p_B^\mu - k_\pi^\mu$ is the momentum transfer. $p_B$ denotes the four momentum of the $B$ meson.

## 2. Form factors

For a pseudoscalar to pseudoscalar decay, the vector matrix element can be written as

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[(p_B + k_\pi)^\mu - \frac{M_B^2 - M_\pi^2}{q^2}q^2\right] + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2}q^\mu. \tag{2.1}$$

We have two form factors, $f_+(q^2)$ and $f_0(q^2)$. These are both calculable in lattice QCD, even though $f_0(q^2)$ is not accessible experimentally as it is suppressed by the small lepton mass.

Useful parametrisation in the context of Heavy Quark Effective Theory (HQET) [5] is

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = 2\sqrt{M_B}\left[f_1(v\cdot k_\pi)v^\mu + f_2(v\cdot k_\pi)\frac{k_\pi^\mu}{v\cdot k_\pi}\right], \tag{2.2}$$

where $v^\mu = p_B^\mu/M_B$ is the heavy quark velocity and $E_\pi = v\cdot k_\pi = M_B^2 + M_\pi^2 - q^2/(2M_B)$. The HQET form factors $f_1(v\cdot k_\pi)$ and $f_2(v\cdot k_\pi)$ stay finite in the limit of infinitely heavy $b$ quark. Corrections of the form $1/m_h$ are expected for finite $m_h$.

These two sets of form factors are not independent, and $f_+(q^2)$ and $f_0(q^2)$ can be written in terms of $f_1(v\cdot k_\pi)$ and $f_2(v\cdot k_\pi)$ as

$$\begin{aligned} f_+(q^2) =& \sqrt{M_B}\left[\frac{f_2(v\cdot k_\pi)}{v\cdot k_\pi} + \frac{f_1(v\cdot k_\pi)}{M_B}\right] \\ f_0(q^2) =& \frac{2}{\sqrt{M_B}}\frac{M_B^2}{(M_B^2 - M_\pi^2)}\left[f_1(v\cdot k_\pi) + f_2(v\cdot k_\pi) - \frac{v\cdot k_\pi}{M_B}\left(f_1(v\cdot k_\pi) + \frac{M_\pi^2}{(v\cdot k_\pi)^2}f_2(v\cdot k_\pi)\right)\right]. \end{aligned} \tag{2.3}$$

## 3. Lattice setup

We use gauge ensembles generated with $2+1$ flavour Möbius domain-wall fermions, and the gauge action is tree-level Symanzik improved. Lattice spacings included in this calculation

PoS(LATTICE2019)143

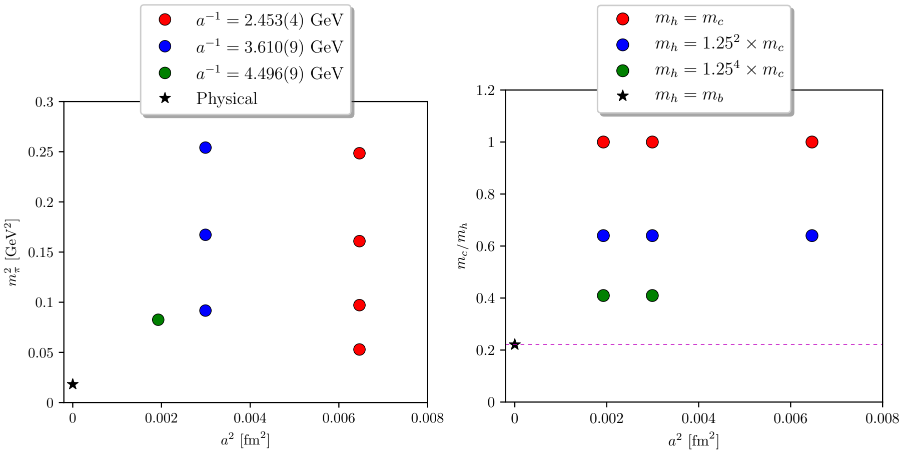


**Figure 1:** On the left: The light sea quark masses used for each lattice spacing. The light valence quark masses are the same as the sea quark masses. On the right: The heavy valence quark masses used for each lattice spacing.

are approximately 0.080 fm, 0.055 fm and 0.044 fm, corresponding to $\beta = 4.17$, $\beta = 4.35$ and $\beta = 4.47$, respectively. Pion masses range from 500 MeV down to 225 MeV, where we use a larger volume at $\beta = 4.17$ for the lightest pion such that we maintain $M_\pi L > 4$. Heavy quark masses are chosen to be $m_c$, $1.25^2 m_c$ and $1.25^4 m_c$, ensuring that $am_h < 0.7$ to avoid large discretization effects from the heavy quark mass. See Fig. 1 for an illustration of the light and heavy quark masses used in this study. The plot on the left shows the sea light quark masses used for each lattice spacing. The valence light quark masses are the same as the sea quark masses. The plot on the right shows the valence heavy quark masses used for each lattice spacing. Note that the ensembles and correlators used in the study of the $D \to \pi\ell\nu$ process in [6] are a subset of the data used in this study.

The $B$ meson is kept at rest in our calculations while we give pion momenta $p = (0,0,0)$, $(0,0,1)$, $(0,1,1)$, $(1,1,1)$ in units of $2\pi/L$. We calculate correlators from all permutations of a given momentum and average these to improve our signal.

We also need to renormalize our vector currents. We have a light quark and a heavy quark at the current insertion, and we calculate the renormalization factor as $Z_V = \sqrt{Z_{V,hh} Z_{V,ll}}$. The heavy-heavy renormalisation factor $Z_{V,hh}$ is calculated by demanding that the vector matrix element $\langle B_s | V | B_s \rangle$ for the heavy current gives 1. The renormalization factors for the light current, $Z_{V,ll}$, are from [7]. For the lightest heavy quark masses, i.e. when $am_h = am_c$, we find that it is sufficient to renormalize our currents using results from the massless coordinate space current correlators as described in [7].

## 4. Extrapolations

Choosing the temporal ($\mu = 0$) or spatial ($\mu = 1,2,3$) vector current in equation (2.2) naturally gives the combinations $f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$ of the form factors. Therefore we use

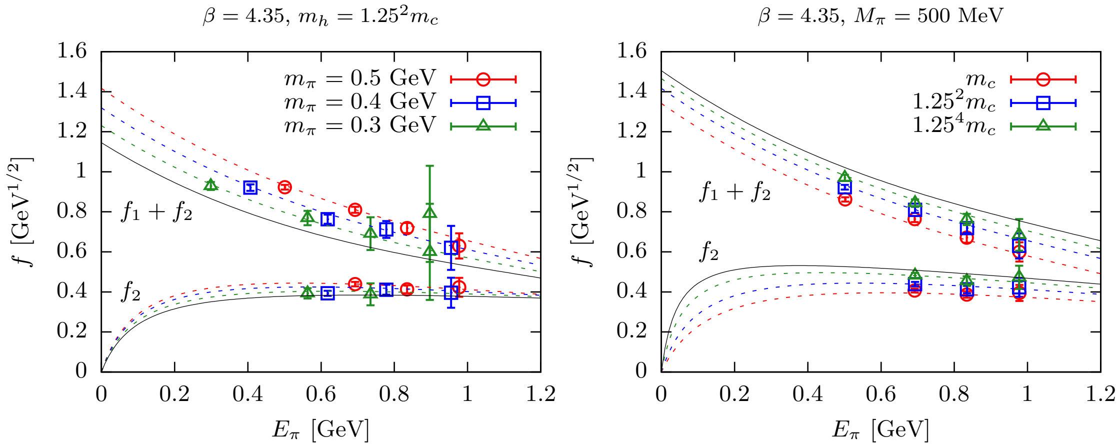


**Figure 2:** On the left: Extrapolation to physical pion mass. Red squares, blue circles and green triangles show the lattice data points at different light quark masses ($M_\pi = 500$, 400 and 300 MeV) at fixed lattice spacing, when the heavy quark mass is kept fixed. The corresponding dotted lines show the fits at those values of the parameters, and the solid black line shows the continuum and physical pion mass extrapolation at fixed heavy quark mass. On the right: Extrapolation to physical $B$ mass. Red squares, blue circles and green triangles show the lattice data points at different heavy quark masses at fixed lattice spacing when the light quark mass is kept fixed. The corresponding dotted lines show the fits at those values of the parameters, and the solid black line shows the continuum extrapolation and extrapolation to physical $B$ mass at fixed pion mass. In practice, we do all extrapolations (to continuum and to physical pion and $B$ meson masses) simultaneously.

the fit functions

$$f_1(v\cdot k_\pi) + f_2(v\cdot k_\pi) = C_0\left(1 + \sum_{n=1}^{3} C_n E_\pi^n + C_4 M_\pi^2 + \chi_{\log} + C_5 E_\pi M_\pi^2 + \frac{C_6}{m_h}\right)(1 + C_7 a^2), \qquad (4.1)$$

$$f_2(v\cdot k_\pi) = D_0 \frac{E_\pi}{E_\pi + \Delta_B}\left(1 + D_1 E_\pi + D_2 M_\pi^2 + \chi_{\log} + D_3 E_\pi M_\pi^2 + \frac{D_4}{m_h}\right)(1 + D_5 a^2)$$

to extrapolate our lattice data to the physical limit: to continuum ($a \to 0$), and to physical pion and $B$ meson masses. The extrapolation to physical pion mass is guided by the $M_\pi^2$ and $E_\pi M_\pi^2$ terms and the chiral logs $\chi_{\log}$. The heavy quark mass dependence is taken to be of the form $1/m_h$, where we use $m_h = M_{\eta_h}/2$ as a proxy for the heavy quark mass. ($M_{\eta_h}$ is the mass of the pseudoscalar heavy-heavy meson $\eta_h$, i.e. $\eta_b$ at physical heavy quark mass.) Form factor $f_2(v \cdot k_\pi)$ is expected to have a pole $(E_\pi + \Delta_B)^{-1}$ with $\Delta_B = M_{B^*} - M_B$. Discretisation effects are covered by the $a^2$ terms.

The extrapolations are illustrated in Fig. 2. In the fit we do all extrapolations in one step, but we have examined each extrapolation individually by changing only one of the masses, $m_l$ or $m_h$, or the lattice spacing $a$, while keeping the other parameters fixed. The extrapolations are seen to be smooth, and they affect the form factors in different directions: the pion extrapolation approaches the physical form factors from above, whereas the extrapolation towards the $b$ quark mass approaches the physical form factors from below. The smallest pion mass used in this study is $\sim 225$ MeV and the largest $B$ meson mass is $\sim 3.4$ GeV, so the extrapolations are sizeable but well under control. The three lattice spacings give good control over the continuum extrapolation.

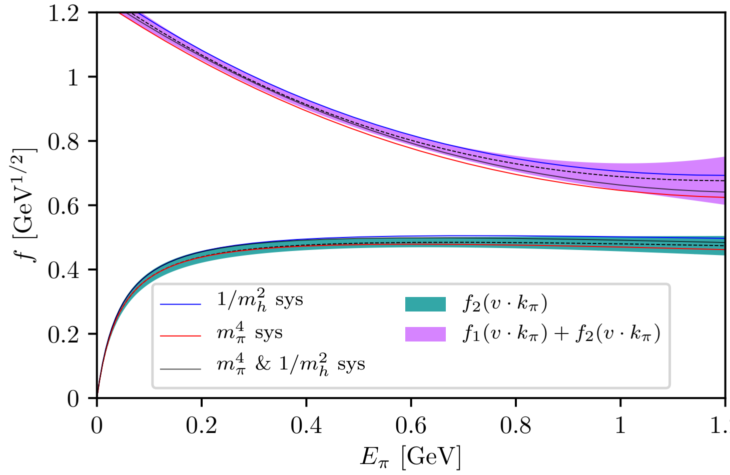


**Figure 3:** Form factors $f_1(E_\pi) + f_2(E_\pi)$ and $f_2(E_\pi)$ in the continuum and physical limit. The dashed black line shows the original fit and the error bands show the statistical uncertainty. Blue, red and grey solid lines show the central values of the fits used in estimating systematic uncertainties by including higher order terms $1/m_h^2$, $M_\pi^4$, and both effects combined, respectively.

We estimate the systematic effects in our result by re-doing the fit with higher order terms added to the fit functions. Adding higher powers of $E_\pi$ changes the form factors $f_2(v \cdot k_\pi)$ and $f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi)$ by 2-3%. The effect of including $M_\pi^4$ terms is roughly $-5\%$, whereas including terms proportional to $1/m_h^2$ increases the value of the form factors by 5%. This is illustrated in Fig. 3. All in all, the systematic effects are estimated to be roughly of the same size as the statistical uncertainty.

## 5. $z$-expansion

Experimental results of the differential decay rate for $B \to \pi \ell \nu$ are available from both BaBar and Belle [8, 9, 10, 11]. Combining these experimental results with our lattice calculation we can extract a value for the CKM matrix element $|V_{ub}|$. To do this we pick synthetic data points from the lattice calculation of the form factors and fit them together with the experimental data using the so-called $z$-expansion:

$$f_0(z) = \sum_{n=0}^{N_z-1} a_n z^n, \tag{5.1}$$

$$f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N_z-1} b_n \left[ z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right],$$

where

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (M_B + M_\pi)^2. \tag{5.2}$$

By choosing $t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$ we have a mapping between $q^2$ and $z$ where the whole kinematic range is now $-0.3 < z < 0.3$. This is the advantage: $|z|$ is small and we can use the expansion in powers of $z$ given in equation (5.1). We also utilize the kinematic constraint

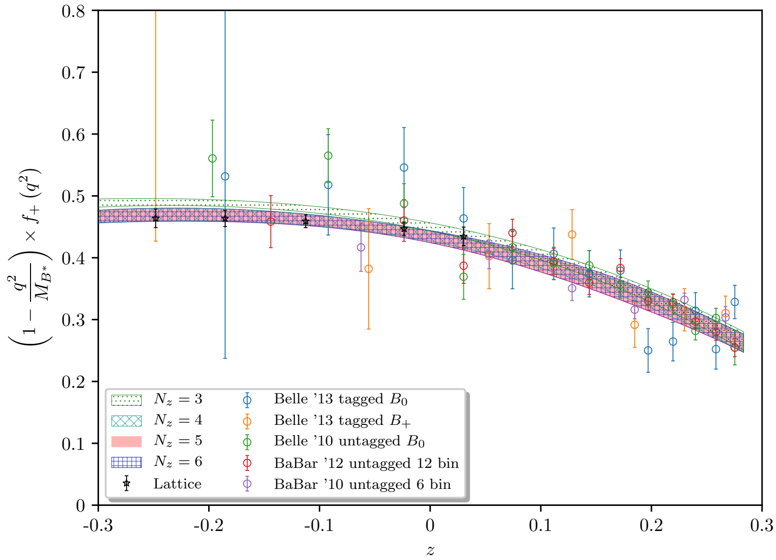


**Figure 4:** z-expansion and combined fit to lattice and experimental data. The fit result for the form factor $(1 - q^2/M_{B^*}^2) f_+(q^2)$ in z-space with experimental and (synthetic) lattice data points.

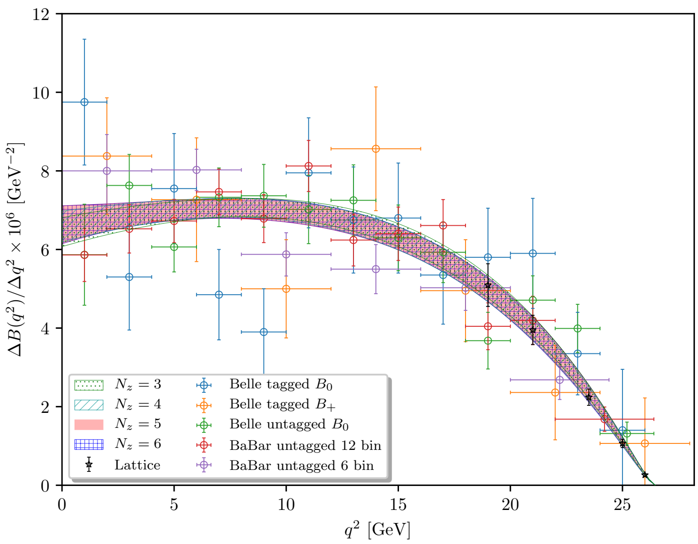


**Figure 5:** z-expansion and combined fit to lattice and experimental data. The same fit and data points as in Fig. 4, but now in $q^2$ space. Here we present the data points and fit as branching fraction per $q^2$ bin instead of the form factor.

$f_+(q^2 = 0) = f_0(q^2 = 0)$. $N_z = 5$ gives a good $\chi^2$, and adding higher order terms does not change the result of the fit.

Equation (1.1) gives the relation between the differential decay rates and the form factors, and $|V_{ub}|$ is included as a free fit parameter. Figures 4 and 5 summarize the results of the fit. The shape of the form factor from experiment and lattice is in good agreement. Note that lattice results and experimental results are highly complementary: lattice QCD results are available and most precise in the high $q^2$ region, whereas experimental results are most precise in the low $q^2$ region.

## 6. Conclusions

Our results are still preliminary, as the combined fit to lattice and experimental data does not contain all correlations, and full systematic errors are not included yet. This preliminary analysis gives $|V_{ub}| = 3.45(14) \times 10^{-3}$. The systematic uncertainties are likely to be of the same size as the statistical uncertainties.

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