PROCEEDINGS OF SCIENCE

Axial U(1) symmetry and mesonic correlators at high temperature in $N_f = 2$ lattice QCD

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We investigate the high-temperature phase of QCD using lattice QCD simulations with $N_f = 2$ dynamical Möbius domain-wall fermions. On generated configurations, we study the axial $U(1)$ symmetry, overlap-Dirac spectra, screening masses from mesonic correlators, and topological susceptibility. We find that some of the observables are quite sensitive to lattice artifacts due to a small violation of the chiral symmetry. For those observables, we reweight the Möbius domain-wall fermion determinant by that of the overlap fermion. We also check the volume dependence of observables. Our data near the chiral limit indicates a strong suppression of the axial $U(1)$ anomaly at temperatures ≥ 220 MeV.

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PoS(LATTICE2019)178

1. Introduction

In the high-temperature region of quantum chromodynamics (QCD), one of open questions is the fate of the $U(1)_A$ symmetry. In the low-temperature phase, the $U(1)_A$ symmetry is known to be broken by a quantum anomaly which is related to topological excitations of gluon fields, e.g, instantons. In the high-temperature region with restored chiral symmetry (in other words, above the critical temperature, $T > T_c$), the restoration or violation of the $U(1)_A$ symmetry is still a long-standing problem not only in theoretical approaches [1, 2, 3] but also in lattice QCD simulations at $N_f = 2$ [4, 5, 6, 7, 8] and $N_f = 2+1$ [9, 10, 11, 12, 13, 14].

In older studies, lattice simulations reported a sizable $U(1)_A$ symmetry breaking above the critical temperature. However, many studies applied the staggered-type fermions, where chiral symmetry is explicitly broken, and it was difficult to precisely measure how much of $U(1)_A$ symmetry breaking is due to lattice artifacts. Recently, chiral fermions were employed to simulate lattice QCD at high temperature [4, 5, 7, 9, 10, 12, 13] (in Refs. [12, 13], only for valence quark sector). JLQCD Collaboration studied with $N_f = 2$ chiral fermions [4, 7]. In Ref. [4], we generated the gauge ensembles with dynamical overlap fermions and applied a topology fixed approach at the $Q = 0$ sector. In Ref. [7], gauge ensembles are generated with the Möbius domain-wall (MDW) fermions [15, 16], and a overlap/domain-wall reweighting technique [17, 7] was applied, where observables measured on MDW fermion ensembles are reweighted to those on overlap fermion ensembles. A disappearance of the $U(1)_A$ anomaly (at around $1.2T_c$) was also reported in simulations with $N_f = 2$ non-chiral fermions by other groups [6, 8]. In Ref. [14], they found that the $U(1)_A$ symmetry is good at $1.3T_c$ but not near T_c.

In these proceedings, we report on our recent results of the observables at $T = 220$ MeV such as the Dirac spectrum, $U(1)_A$ susceptibility, screening masses from mesonic correlators, and

PoS(LATTICE2019)178

Table 1: Numerical parameters of lattice simulations. $L^3 \times L_t$ and m are the lattice size and quark mass, respectively. $\bar{\Delta}^{\mathrm{ov}}_{\pi-\delta}$ and χ_t are our results of the $U(1)_A$ susceptibility and topological susceptibility from the fermionic definition, respectively.

$L^3 \times L_t$	am	$\bar{\Delta}^{\mathrm{ov}}_{\pi-\delta} a^2$ on OV	$\chi_t a^4$
$24^3 \times 12$	0.001	$1.5(0.6) \times 10^{-6}$	≈ 0
$24^3 \times 12$	0.0025	$3.6(1.3) \times 10^{-5}$	$5.0(3.7) \times 10^{-8}$
$24^3 \times 12$	0.00375	0.00017(7)	$2.3(0.7) \times 10^{-7}$
$24^3 \times 12$	0.005	0.00091(42)	$9.0(2.0) \times 10^{-7}$
$24^3 \times 12$	0.01	0.00389(92)	$1.7(0.2) \times 10^{-6}$
$32^3 \times 12$	0.001	$1.8(1.4) \times 10^{-5}$	$8.8\ (8.8) \times 10^{-12}$
$32^3 \times 12$	0.0025	0.00017(6)	$3.5(3.0) \times 10^{-8}$
$32^3 \times 12$	0.00375	0.00026(8)	$7.9(3.0) \times 10^{-8}$
$32^3 \times 12$	0.005	0.00291(188)	$9.3(1.9) \times 10^{-7}$
$32^3 \times 12$	0.01	0.01358(263)	$2.9(0.4) \times 10^{-6}$
$40^3 \times 12$	0.005	0.00785(178)	$5.4(0.6) \times 10^{-7}$
$40^3 \times 12$	0.01	0.01162(140)	$2.0(0.2) \times 10^{-6}$
$48^3 \times 12$	0.001	$2.2(0.9)\times 10^{-6}$	$4.2(4.3) \times 10^{-16}$
$48^3 \times 12$	0.0025	0.00012(4)	$4.9(4.4) \times 10^{-9}$
$48^3 \times 12$	0.00375	0.00032(12)	$1.5(0.7) \times 10^{-7}$
$48^3 \times 12$	0.005	0.00135(63)	$2.9(1.1) \times 10^{-7}$

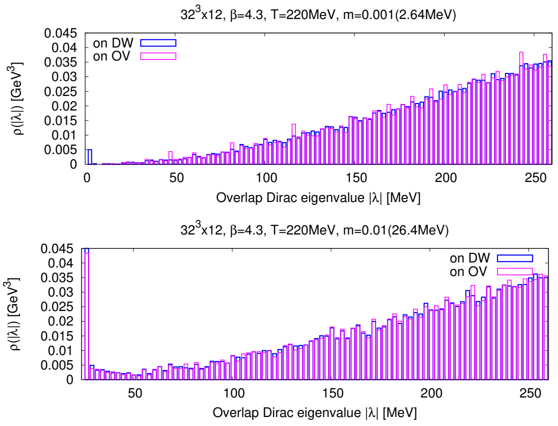

Figure 1: Spectral density $\rho(|\lambda|)$ for overlap-Dirac eigenvalues λ at $T = 220$ MeV. Upper panel: $m = 2.64$ MeV. Lower panel: $m = 26.4$ MeV.

topological susceptibility in $N_f = 2$ lattice QCD simulations. The simulation parameters are summarized in Table 1. Our gauge ensembles are generated with the tree-level Symanzik improved gauge action and dynamical MDW fermions. We use the gauge coupling $\beta = 4.30$ and the lattice spacing $1/a = 2.64$ GeV ($a \sim 0.075$ fm), which is finer than that of configurations used in the previous works [4, 7]. We simulate lattice volumes $L = 24, 32, 40, 48$, and the length of the fifth dimension in the MDW fermion formulation is $L_s = 16$. The physical quark mass (as the average of up and down quark masses) is estimated to be $am = 0.0014(2)$ (3.7(5) MeV). Some of our results were already reported in previous proceedings [18, 19, 20, 21].

2. Overlap Dirac spectrum

In Fig. 1, we plot spectral density of overlap Dirac eigenvalues, $\rho(\lambda) = (1/V)\langle\sum_{\lambda'}\delta(\lambda-\lambda')\rangle$ for two typical ensembles. The blue and magenta bins denote the spectra on the MDW fermions ensembles (DW) and reweighted overlap fermion ensembles (OV), respectively. At $m = 2.64$ MeV for the OV ensembles, we find a suppression of both low eigenmodes and chiral zero modes. The suppression of the low eigenmodes is related to the $U(1)_A$ symmetry restoration in the light quark mass region. The disappearance of the chiral zero modes is related to the suppression of the topological susceptibility. At $m = 26.4$ MeV, low eigenmodes are enhanced, which is related to the $U(1)_A$ symmetry breaking.

3. $U(1)_A$ susceptibility

The $U(1)_A$ susceptibility $\Delta_{\pi-\delta}$ is an order parameter of the $U(1)_A$ symmetry breaking. This is defined from a spacetime integral of the difference between two-point correlators of isovector-pseudoscalar ($\pi^a \equiv i\bar{\psi}\tau^a\gamma_5\psi$) and isovector-scalar ($\delta^a \equiv \bar{\psi}\tau^a\psi$) operators:

$$\Delta_{\pi-\delta} \equiv \chi_\pi - \chi_\delta \equiv \int d^4x \langle \pi^a(x)\pi^a(0) - \delta^a(x)\delta^a(0)\rangle, \tag{3.1}$$

where a is an isospin index in $N_f = 2$ QCD. The $U(1)_A$ susceptibility in the lattice theory is defined by a summation of low-lying eigenvalues of the overlap Dirac operator, $\lambda_i^{(\mathrm{ov},m)}$ [22]:

$$\Delta^{\mathrm{ov}}_{\pi-\delta} = \frac{1}{V(1-m^2)^2}\left\langle \sum_i \frac{2m^2(1-\lambda_i^{(\mathrm{ov},m)2})^2}{\lambda_i^{(\mathrm{ov},m)4}} \right\rangle, \tag{3.2}$$

PoS(LATTICE2019)178

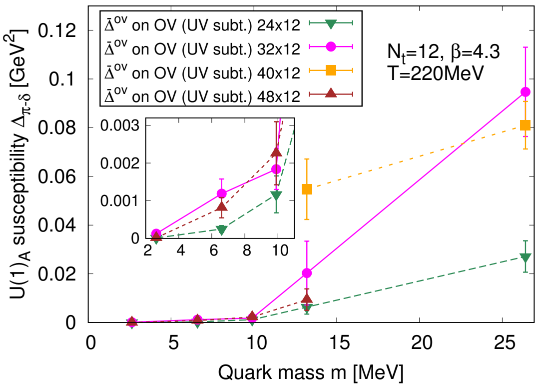

Figure 2: $U(1)_A$ susceptibilities, $\bar{\Delta}^{\text{ov}}_{\pi-\delta}$ (3.2), from the eigenvalue density of the overlap-Dirac operators at $T = 220$ MeV.

where we set the lattice spacing $a = 1$. This summation is truncated at the lowest 40 eigenvalues.[1]

In Fig. 2, we show the $U(1)_A$ susceptibility at $T = 220$ MeV. In the light quark mass region, we find strong suppression of the $\Delta^{\text{ov}}_{\pi-\delta}$. For example, at the lowest quark mass and $L = 32$, the ratio of $\Delta^{\text{ov}}_{\pi-\delta}$ to temperature is $\sqrt{\Delta^{\text{ov}}_{\pi-\delta}}/T \approx 5\%$. The volume dependence is small for $L = 24$–48. The data at different volumes are consistent except for the heaviest quark mass at $L = 24$, whose aspect ratio against temperature is $L/L_t = 2$.

4. Screening mass difference from spatial mesonic correlators

The screening mass is defined by the exponential decay of spatial correlators, which may be used to measure a violation of $U(1)_A$ symmetry. We investigate the difference between the effective screening masses

$$\Delta m_{scr}(z) = |m^{PS}_{scr}(z) - m^{S}_{scr}(z)|, \tag{4.1}$$

where $m^{PS}_{scr}(z)$ and $m^{S}_{scr}(z)$ are the effective screening masses at a spatial coordinate z for isovector-pseudoscalar ($\pi^a \equiv i\bar{\psi}\tau^a\gamma_5\psi$) and isovector-scalar ($\delta^a \equiv \bar{\psi}\tau^a\psi$) operators, respectively.

In Fig. 3, we show the difference between the effective screening masses measured by the MDW operator (without reweighting), where the horizontal axis is a dimensionless spatial distance ($zT = (n_z a/N_t a) = n_z/N_t$). For the screening masses with light quark mass, we find a small value of $\Delta m_{scr}(zT)$, which indicate the restoration of the $U(1)_A$ symmetry and it is consistent with the results of the $U(1)_A$ susceptibility $\bar{\Delta}^{\text{ov}}_{\pi-\delta}$. For heavy quark masses, the mass difference becomes large, which implies the $U(1)_A$ symmetry breaking.

5. Topological susceptibility

The topological susceptibility χ_t is defined as a gauge ensemble average of the topological charge Q_t:

$$\chi_t = \frac{\langle Q_t^2 \rangle}{V}, \tag{5.1}$$

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[1] From this definition, we further apply two types of subtractions: a subtraction of the contributions from chiral zero modes and an ultraviolet divergence (or lattice cutoff). For a justification of the zero mode subtraction, see Ref. [2, 7]. For the parametrization scheme of the lattice cutoff contribution by different valence quark masses, see Ref. [20, 21].

PoS(LATTICE2019)178

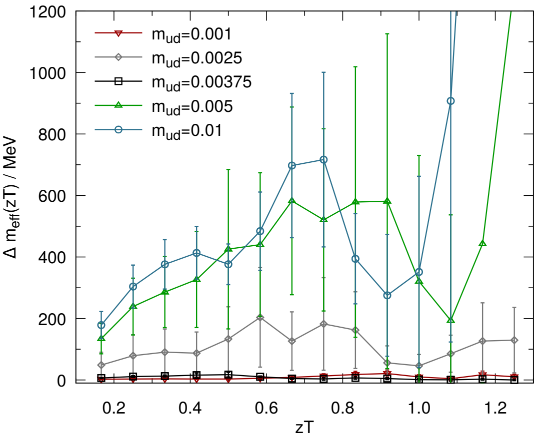

Figure 3: Difference between effective screening masses (4.1) from spatial mesonic collerators for $U(1)_A$ partners at $T = 220$ MeV and $L = 32$. The horizontal axis is defined as a dimensionless spatial distance $zT = (n_z a/N_t a) = n_z/N_t$.

For the topological charge Q_t, we employ two definitions. As a fermionic definition, Q_t is defined through the index theorem for the overlap Dirac operator:

$$Q_t = n_+ - n_-, \tag{5.2}$$

where $n_\pm$ are the numbers of chiral zero modes with positive or negative chirality, respectively. As a gluonic definition, Q_t is defined as a summation over spacetime x at a flow time t :

$$Q_t(t) = \frac{1}{32\pi^2} \sum_x \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr} F_{\mu\nu}(x,t) F_{\rho\sigma}(x,t), \tag{5.3}$$

where $F_{\mu\nu}(x,t)$ is the clover-type discretization of the field strength tensor [23].[2]

In Fig. 4, we plot the topological susceptibility χ_t at $T = 220$ MeV. We show the results from the fermionic definition (5.2) on the OV ensembles and the gluonic definition (5.3) on the MDW ensembles, respectively. In the light quark mass region, χ_t is strongly suppressed with both the definitions. Furthermore, the volume dependence between $L = 24$ and 48 is small. In the heavy quark mass region, the value of χ_t becomes nonzero, which is in agreement with the peak structure of the Dirac spectra in the lower panel of Fig. 1.

6. Summary and discussion

In these proceedings, we studied the high-temperature phase of QCD at $T = 220$ MeV by using $N_f = 2$ lattice QCD simulations with dynamical MDW fermions. We found small values of the $U(1)_A$ susceptibility (3.2) and the difference of mesonic screening masses (4.1) in light quark mass region, $m \lesssim 10$ MeV, which indicates the $U(1)_A$ symmetry restoration in the chiral limit ($m \to 0$). Furthermore, we found strong suppression of the topological susceptibility in the light-quark mass region. The mesonic and baryonic correlators at higher temperature were already reported in Refs. [24, 25, 26].

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[2] This definition is usually not an integer, but we find a well-discretized distribution of $Q_t(t)$ at $t = 5$.

PoS(LATTICE2019)178

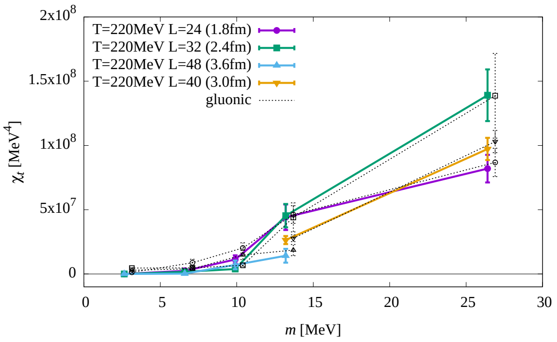

Figure 4: Topological susceptibilities χ_t at $T = 220$ MeV. Colored points: χ_t from the fermionic definition (5.2) on reweighted OV ensembles. Uncolored points: χ_t from the gluonic defnition (5.3) on MDW ensembles.

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PoS(LATTICE2019)178