

Polyakov Loop Susceptibility and Correlators in the Chiral Limit

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In quenched QCD the Polyakov loop is an order parameter of the deconfinement transition, but with decreasing quark mass, the peak in the Polyakov loop susceptibility becomes less pronounced, and it loses its interpretation as an indicator for deconfinement. For this $N_f = 2 + 1$ HISQ study, we fix the strange quark mass m_s at its physical value and investigate the dependence of the Polyakov loop on the light quark mass m_l in the range $m_s/m_l = 27 - 160$, following m_l toward the chiral limit. In particular we will look how the inflection point and susceptibility behave as we decrease m_l , to see whether one finds any indication of a crossover, and therefore whether the Polyakov loop is sensitive to the chiral phase transition. Preliminary results show no signal of a crossover from the real part of the Polyakov loop in the vicinity of the chiral crossover. Closely related is an investigation of Polyakov loop correlations and the Debye mass in this limit. Preliminary results suggest little or no dependence on m_l .

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1. The Polyakov loop

For lattice QCD in a finite volume $N_\sigma^3 \times N_\tau$, the thermal Wilson line $L_{\vec{x}}$, the Polyakov loop $P_{\vec{x}}$, and its spatial average P are given by

$$L_{\vec{x}} \equiv \prod_\tau U_4(\vec{x}, \tau), \quad P_{\vec{x}} \equiv \frac{1}{3} \text{tr} L_{\vec{x}}, \quad \text{and} \quad P \equiv \frac{1}{N_\sigma^3} \sum_{\vec{x}} P_{\vec{x}}, \quad (1.1)$$

respectively. In general we define a susceptibility χ_O for an intensive observable O as

$$\chi_O = N_\sigma^3 \left(\langle O^2 \rangle - \langle O \rangle^2 \right). \quad (1.2)$$

The Polyakov loop susceptibility and the susceptibility of the real part of the Polyakov loop are then $\chi_{|P|}$ and $\chi_{\text{Re } P}$. At low temperatures with static quarks in the infinite volume limit, the Polyakov loop expectation value $\langle |P| \rangle$ is zero due to the global \mathbb{Z}_3 symmetry of the gauge action. At higher temperatures this center symmetry is spontaneously broken, and $\langle |P| \rangle$ acquires a nonzero value, signalling a finite static quark-antiquark free energy at large separations, and hence deconfinement. At finite N_σ , $\langle |P| \rangle$ as a function of temperature has an inflection point, and the slope at this point diverges in the infinite volume limit. The susceptibility as a function of temperature, meanwhile, exhibits a pronounced peak at finite N_σ whose height diverges in the infinite volume limit as $\chi_{|P|}^{\max} \sim N_\sigma^3$, reflecting the first order nature of the deconfinement phase transition in pure SU(3) gauge theory.

At finite quark mass, the Polyakov loop is no longer a strict order parameter; the finite quark mass breaks the \mathbb{Z}_3 symmetry explicitly, and $\langle |P| \rangle$ and $\langle \text{Re } P \rangle$ are never zero. Nevertheless at larger-than-physical quark mass, some remnants of critical behavior seem to remain, and in particular, past studies have found inflection points for $\langle |P| \rangle$ and the chiral condensate to appear at similar temperatures. For example a 2008 study [1] using improved staggered fermions of $N_f = 2 + 1$ flavors, with light quark masses corresponding to about a 50% larger-than-physical pion mass, performed on coarse lattices with $N_\tau = 4$ and 6, found inflection points in both $\langle |P| \rangle$ and the chiral order parameter to lie around the same temperature.

While these earlier studies of Polyakov loop expectation values seemed to clearly indicate that the inflection point of the Polyakov loop is related to the chiral transition, only the latter has a clear-cut interpretation as a phase transition in the chiral limit. This ‘‘coincidence’’ of inflection points in the temperature dependence of $\langle |P| \rangle$ and the chiral condensate is often taken as evidence for the coincidence of a chiral and deconfinement transition. To what extent deconfinement or the melting of bound states in general can be associated with properties of the Polyakov loop at finite quark mass is, however, an open question. In fact, studies with improved fermion actions in general showed that the QCD transition, which is a pseudo-critical crossover transition at non-zero values of the quark masses, tends to become a *smoother* transition when going closer to the continuum limit and using more highly improved fermion actions such as HISQ or stout actions [3]. Indeed a potential hint that this behavior weakens was already given in Ref. [2], which we show in Figure 1, where ones sees that $\chi_{|P|}^{\max}$ decreases with decreasing quark mass for fixed N_τ . Moreover there are studies with more highly improved actions that do not find the inflection point of $\langle |P| \rangle$ to coincide with the chiral inflection point [4, 5], challenging the notion of using the Polyakov loop as an

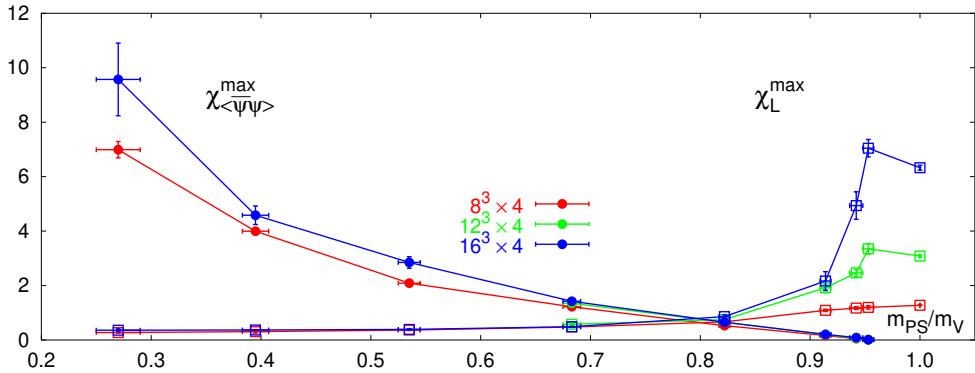


Figure 1: Dependence of peak heights of Polyakov loop and chiral susceptibilities on $N_t = 4$ lattices with $N_\sigma = 8, 12$, and 16 as a function of the pseudo-scalar mass in units of the vector meson mass, which is another way of stating quark mass. This calculation used a standard staggered fermion discretization scheme. Figure taken from Ref. [2].

observable for deconfinement of light degrees of freedom. In actuality, then, it is not clear to what extent properties of the Polyakov loop can provide a reasonable criterion for deconfinement.

Therefore one aim of the present study is to investigate whether an indication of deconfinement from the Polyakov loop, if any, weakens as we lower the light quark mass m_l . We employ an improved staggered action (HISQ) that leads to greatly reduced taste violations, thus providing a better approach to the continuum limit. We analyze the Polyakov loop and Polyakov loop correlation functions on lattices with temporal extent $N_t = 8$ and 12 , which in other thermodynamics calculations with HISQ have been shown to provide results close to the continuum limit [6].

2. Debye screening

The Polyakov loop relates to $F_{q\bar{q}}$, the color-averaged free energy of a static quark-antiquark pair in equilibrium at temperature T , by

$$\exp[-F_{q\bar{q}}(r, T)/T] = \left\langle P_{\vec{x}} P_{\vec{y}}^\dagger \right\rangle \xrightarrow{r \rightarrow \infty} |\langle P \rangle|^2 = \langle \text{Re } P \rangle^2, \quad rT = |\vec{x} - \vec{y}|/aN_t, \quad (2.1)$$

because the expectation value of the Polyakov loop lies in the real \mathbb{Z}_3 sector for finite quark masses. The gauge-invariant color-averaged Polyakov loop correlator can be decomposed into (in general gauge-dependent) color singlet and color octet contributions [7, 8, 9]. In particular the color singlet

$$F_1(r, T) = -T \log \left\langle \frac{1}{3} \text{tr } L_{\vec{x}} L_{\vec{y}}^\dagger \right\rangle, \quad (2.2)$$

which clearly depends on the gauge. Hence it is important to restrict to a particular gauge before measuring F_1 . In the deconfined phase, the interaction between two charges is screened by the medium. The distance at which in-medium modifications of the quark-antiquark interaction dominate is characterized by the Debye screening radius r_D . Its inverse, the Debye screening mass m_D , can be extracted from the long-distance ($rT \gg 1$) behavior of F_1 as

$$F_1(r, T) \simeq -\frac{4}{3} \frac{\alpha(T)}{r} e^{-rm_D(T)} + F_1(r = \infty, T). \quad (2.3)$$

$N_\sigma^3 \times N_\tau$	m_s/m_l	avg. # TU	$N_\sigma^3 \times N_\tau$	m_s/m_l	avg. # TU
$32^3 \times 8$	27	62 000	$60^3 \times 12$	40	30 000
$40^3 \times 8$	40	52 000		80	17 000
$56^3 \times 8$	80	20 000			
	160	17 000			

Table 1: Summary of parameters used in these proceedings and corresponding statistics, reported in average molecular dynamic time units (TU) per parameter combination.

The Debye mass is known to depend on N_f at high temperatures through a leading order perturbative calculation. At lower temperatures, lattice calculations still show qualitative agreement with perturbation theory [10]. The Debye mass will also depend on m_l ; hence another goal of this study will be to see how m_D changes with m_l .

3. Setup and simulation parameters

We use for our analysis configurations that were generated with the HISQ action in (2+1)-flavor QCD with a physical value of the strange quark mass and light quark masses in the range $m_l = m_s/27$ to $m_s/160$, corresponding to $140 \text{ MeV} \gtrsim m_\pi \gtrsim 58 \text{ MeV}$ [11]. The bare coupling β is taken in the range 6.260-6.850 depending on the lattice, which was chosen so that the temperatures would lie in the vicinity of the chiral pseudo-critical temperature. Measurements of the free energies are made in the Coulomb gauge, with the gauge fixing carried out using the over-relaxation algorithm [12]. The scale has been set using the experimental value $f_K = 156.1/\sqrt{2} \text{ MeV}$ [13] as well as the updated results for the kaon decay constant calculated with the HISQ action, $f_K a(\beta)$, given in Ref. [14]. We renormalize the Polyakov loop using the qq -scheme [15], where the $T = 0$ potential is obtained from a fit of the combined data shown in Fig. 14 of Ref. [6] using the ansatz

$$V_{T=0}(r) = \frac{A}{r} + B + Cr. \quad (3.1)$$

Additive renormalization constants have been calculated at $(r/a)^2 = 4$. Distances up to $(r/a)^2 = 13$ have been tree-level corrected [16]. Error bars are calculated in 32 jackknife bins unless otherwise stated. The lattice sizes, quark masses, and statistics are summarized in Table 1.

4. Results

The real part of the renormalized Polyakov loop is shown for different values of m_s/m_l in the two plots in the top row of Figure 2 for $N_\tau = 8$ (left) and $N_\tau = 12$ (right) lattices. We plot this instead of $\langle |P^{\text{ren}}| \rangle$ because $\langle |P^{\text{ren}}| \rangle$ receives significant finite volume corrections from $\text{Im } P$, and because after explicit breaking of the \mathbb{Z}_3 symmetry by finite quark mass, the system settles in a region where P is centered on the real axis. Vertical lines indicate the positions of chiral pseudo-critical temperatures taken from Ref. [11], which are at roughly 151, 154, and 158 MeV for $N_\tau = 8$, and 149 and 156 MeV for $N_\tau = 12$. In the entire temperature range probed by us $\langle \text{Re } P^{\text{ren}} \rangle$ is convex for both N_τ . Calculations performed with other lattice sizes similarly show no indication that a

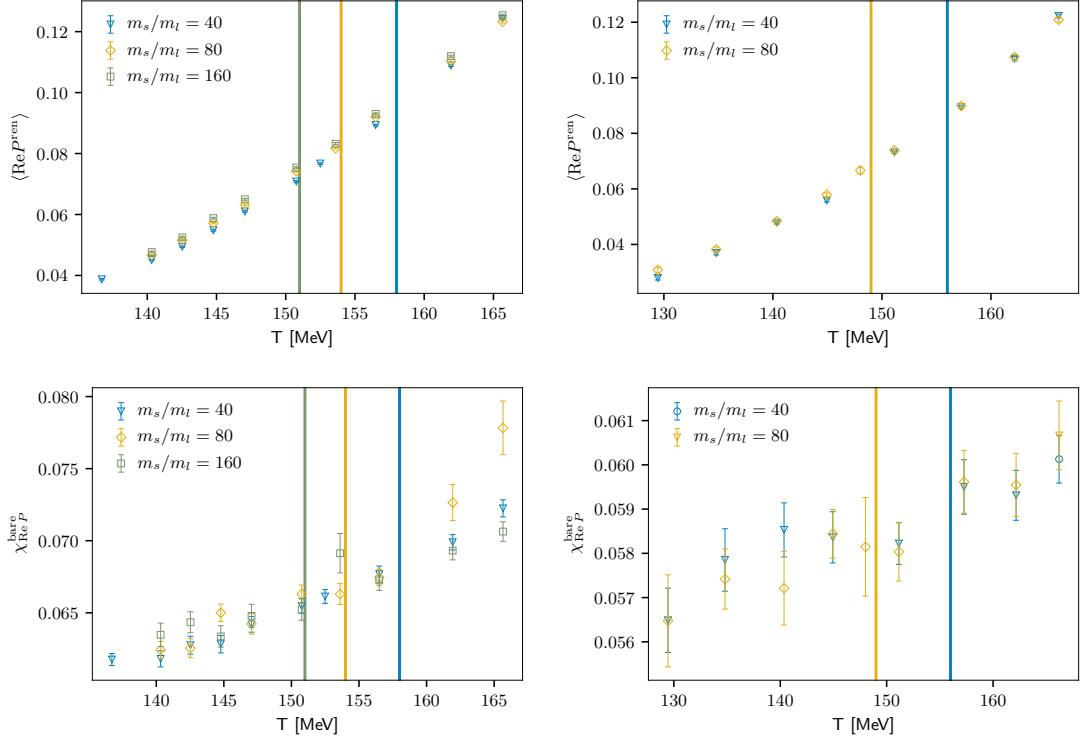


Figure 2: Real part of the renormalized Polyakov loop (top) and the susceptibility of the real part of the bare Polyakov loop (bottom) near the chiral pseudo-critical temperature for $N_\tau = 8$ (left) and $N_\tau = 12$ (right) for different m_s/m_l . The vertical lines indicate the chiral pseudo-critical temperatures corresponding to each quark mass ratio.

crossover signal from the Polyakov loop coincides with the signal from the chiral condensate. An inflection point, which needs to exist as $\langle |P^{\text{ren}}| \rangle$ (or equivalently at $N_\sigma = \infty$, $\langle \text{Re}P \rangle$) will eventually approach unity at high temperature, will therefore only occur at temperatures larger than $1.1 T_{pc}$. For $N_\tau = 8$, decreasing m_l leads to an increase in $\langle \text{Re}P^{\text{ren}} \rangle$ across all probed temperatures, with no indication that the slope changes. Meanwhile for $N_\tau = 12$ there is no m_l dependence within the statistics.

The two plots in the bottom row of Figure 2 show $\chi_{\text{Re}P}^{\text{bare}}$, the susceptibility of the real part of the bare Polyakov loop, for different m_l for $N_\tau = 8$ (left) and $N_\tau = 12$ (right) for the same configurations. We see no peak in this range, which is consistent with what we find for $\langle \text{Re}P^{\text{ren}} \rangle$. There is no sensitivity to m_l for either N_τ in this range of temperatures. Our preliminary results thus show no overlap of a signal from the Polyakov loop with the chiral condensate. Higher statistics and more results at higher temperatures are needed to clearly see the effect of m_l on $\chi_{\text{Re}P}^{\text{max}}$.

In Figure 3 we plot the renormalized singlet free energy against the Polyakov loop separation r in physical units at $N_\tau = 8$ for different m_s/m_l . The left plot shows results at $\beta = 6.285$, which corresponds to a temperature of about 141 MeV, while the right plot has $\beta = 6.445$, which is at about 166 MeV. The data are given at our largest available N_σ for each β and m_l combination, which we find are large enough to suppress finite volume effects. For both plots one can see by eye that the long-distance results for all m_l agree within statistical uncertainty. This suggests that

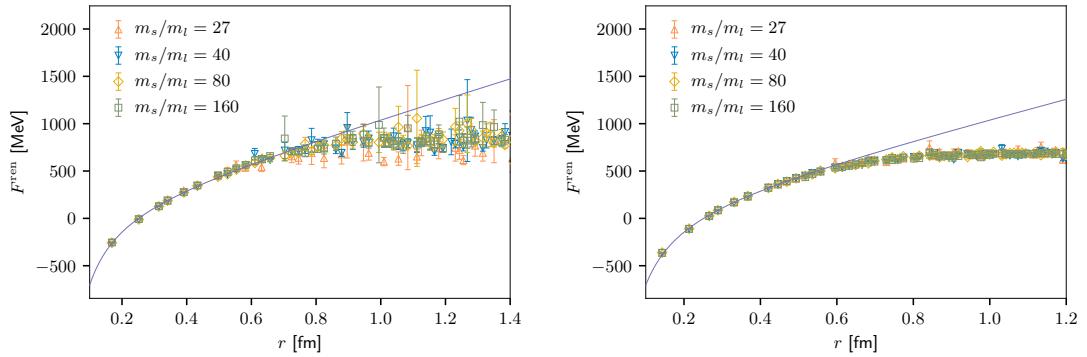


Figure 3: Dependence of F_1 on m_l for $N_\tau = 8$ and our largest N_σ for $\beta = 6.285$ or $T = 141$ [MeV] (left) and $\beta = 6.445$ or $T = 166$ [MeV] (right). The purple line indicates the zero temperature potential.

the Debye screening masses, eventually extracted from these correlation functions, show little or no dependence on the light quark masses.

Extraction of m_D has not yet yielded any precise results, as F_1 tends to be rather noisy away from short distances, which has made carrying out a long-distance fit difficult. We plan to try smoothing our configurations using the gradient flow [17], which mostly affects only short-distance physics, and could therefore improve our long-distance signal without spoiling it.

5. Summary and Outlook

We presented here some first results for our research on indicators of hadron melting toward the chiral limit. The Polyakov loop does not exhibit any indication of a crossover near the chiral pseudo-critical temperature at lower-than-physical quark mass, consistent with the results of more recent HISQ studies at physical m_l . More results at other m_l are forthcoming, which will allow us to examine the behavior of $\chi_{\text{Re}P}^{\max}$ and the slope of $\langle \text{Re}P \rangle$ in the chiral limit. Further results at other N_τ will also be analyzed, allowing for a continuum limit extrapolation. We are working toward a determination of m_D , however statistical noise in the free energy at large distances make the calculation difficult. Our current data suggest m_D will have no or little dependence on m_l in the range investigated. Smoothing using the gradient flow may be attempted in the future.

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