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# Composite electroweak sectors on the lattice

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In the post-Higgs discovery era, the primary goal of the Large Hadron collider is to discover new physics Beyond the Standard Model. One fundamental question is does new beyond the Standard Model composite dynamics provides the origin of the Higgs field and potential. After reviewing the main motivations to consider composite models based on a new strongly interacting sector, we summarise the efforts of the lattice community to investigate the viability of models featuring a composite Higgs sector. We argue that first principle calculations are necessary in view of the fast improvements in accuracy of experimental measurements in the Higgs sector. We stress the importance for lattice calculations to provide a testing benchmark for non perturbative mechanisms. It is highlighted that the rich phenomenology of non-abelian gauge theories raises a number of questions that can be explored using lattice calculations. First principle results therefore provide crucial insights in the theory landscape that could guide the next generation of Composite Higgs models.

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## 1. Introduction

The success of the Standard Model (SM) of particle physics to describe a huge amount of experimental results is undeniable. The discovery of the Higgs boson in 2012 by the ATLAS and CMS experiments[1, 2], after an endeavour that started half a century ago, marks the beginning of a new era where finding deviations from the Standard Model is crucial to answer fundamental questions about our Universe. A profusion of approaches to the physics Beyond the Standard Model (BSM) must be explored and scrutinised in view of the latest experimental results. Bearing this in mind the Higgs discovery is undoubtedly a major step in our understanding of the interactions at the fundamental level, and confirms our effective understanding of the origin of mass at the electroweak scale. Since 2012, the combined measurements of the Higgs' mass have been improved to the subpercent accuracy and read $m_H = 124.97 \pm 0.24(\mathrm{Total} \pm 0.16(\mathrm{Stat.only})$ GeV[3]. The on-going efforts to determine quantum numbers of the Higgs greatly favoured the ones of the Standard Model, *i.e* a CP-even scalar particle[4, 5]. A lot of attention has also been given to the tests investigating the coupling of the Higgs boson to the fermions and gauge bosons. Parametrising the strength of the coupling between the Higgs and vector bosons (V) or fermions (F), using the ratio $\kappa_{i=V,F} = \frac{g_{Hi}}{g_{Hi}^{\mathrm{SM}}}$, the Higgs coupling to gauge bosons is measured with a $10-15\%$ error while the Higgs coupling to the third generation of fermions is currently measured with a 20 to 30% error[6] (68% CL). These results are summarised in Fig. 1. Investigations of the Higgs potential remain extremely challenging at the LHC, but a first bound on the Higgs cubic self-coupling has been derived by considering the coupling contribution at EW contributions at NLO[7]. So far, all the current experimental results suggest a very Standard Model like Higgs boson.

The experimental results put severe constraints on new physics scenarios and in particular, in the context of composite Higgs models reviewed here, the precision of the Higgs mass determination is such that it should not be ignored as a probe to rule out scenario involving a non-standard Higgs. Until now, the coupling measurements $\kappa_{V,F}$ provide less stringent constraints. While the experimental results are constantly pushing the limit of the predictive power of the Standard Model, numerous experimental evidence and theoretical puzzles that calls for BSM physics are still lacking an explanation.

It is well-known that a number of theoretical and phenomenological facts question our understanding of the interactions at the fundamental level and calls for New Physics beyond the Standard Model. From the theoretical point of view the naturalness problem, the hierarchy problem or the strong CP problem have attracted a lot of attention. From the phenomenological point of view, the Dark Matter density, the neutrino masses, or the origin of the Matter-Antimatter asymmetry, are examples of limitations of the Standard Model. Over the years a large number of mechanisms and theories have been proposed to explain some or several of these issues and are under investigations.

Composite Models based on gauge theories with a number of fermions in various irreducible representations of the gauge group, are particularly interesting because they are known to generate scales dynamically and evade the naturalness problem. The fact that they exhibit a non trivial non perturbative dynamics at low energy is also source of a rich phenomenology that can be exploited to build extensions of the Standard Model. Composite Models face the challenge that robust quantitative predictions require to resort to expensive lattice simulations to ultimately compare with experimental data. Lattice calculations can furthermore provide inputs to guide model builders, to



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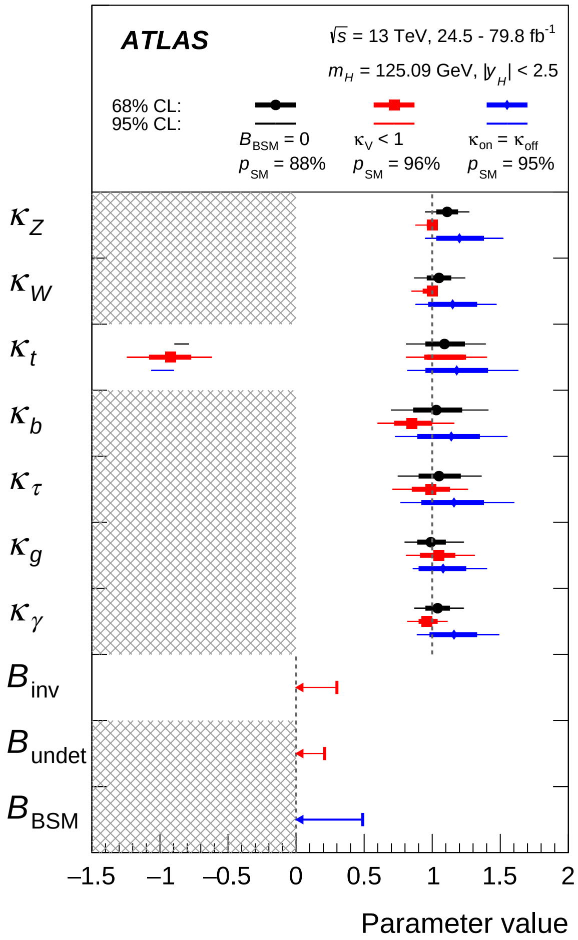


**Figure 1:** *Best-fit values and uncertainties for Higgs boson coupling modifiers per particle type with effective photon and gluon couplings assuming no BSM contributions (black). More details can be found in [6].*

test mechanisms, and to suggest new experimental signatures that are sensitive to the underlying dynamics. Over the years many scenarios relying on a new strong dynamics BSM composite models have been proposed and received considerable attention. From technicolor models, to walking technicolor and to Pseudo-Nambu Goldstone Composite Higgs, the idea of confining new fundamental degrees of freedom into the known particles is among the most intriguing possibilities to address the flaws of the Standard Model.

Although compositeness can be used in other context, for instance in the context of Dark Matter, we limit ourselves to models related to the dynamical breaking of the electroweak symmetry.

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## 2. Model building: Pseudo-Nambu Goldstone Composite Higgs and Pseudo-Dilaton Higgs

Broadly speaking, the underlying theories that are used to design models featuring a composite Higgs are non abelian gauge theories parametrised by a a gauge group $G$, and a number $N_f$ of Dirac massless fermions in a representation $R$. Depending on the number of fermions and on the gauge group, the large distance behaviour of the theory is expected to change drastically. While for $N_f$ large enough, it is known that asymptotic freedom is lost [8], for small enough $N_c$ and $N_f$, for instance in QCD, asymptotic freedom, confinement and spontaneous chiral symmetry breaking are expected to occur. In between this two regimes, it is expected that a conformal window exists, where the $\beta$-function exhibit an infrared fixed point (IRFP) for the gauge coupling $g^2$. In that regime, the theory is conformal at long distances, and all the states of the theory are therefore massless. We refer the reader to Fig. 2 for a summary of the perturbative expectations[9]. The determination of the boundaries of the conformal window is however a non-perturbative question, crucial for model building, that can be studied using lattice field theory.. A brief update on the latest lattice calculations that address that issue will be discussed at a later stage. Just below, the critical number of flavour for which the theory becomes conformal, the system might develop interesting features for model building, among which the possibility that the lightest scalar bound state becomes a pseudo-nambu Goldstone boson associated to the spontaneous breaking of dilation symmetry, which mass is controlled by the distance to the conformal window. Lattice studies are underway to determine if this is occurring or not, but a number of challenges render the conclusions unclear at this stage.

In this section, we focus on the model building point of view. We will assume that we have at our disposal a theory featuring the following properties. First, the theory features asymptotic freedom, confinement and spontaneuous chiral symmetry breaking. Second, the mass of the scalar state is close to the spontaneous symmetry breaking scale $F_{\rm PS}$ in the chiral limit. This last assumption is related to to the possibility that the scalar state is a pseudo-nambu Goldstone boson associated with restoration of the conformal symmetry when $N_f$ is increased.

In the following, pseudo Nambu Goldstone Bosons (pNGB) will refer to the degrees of freedom associated with the spontaneous symmetry breaking of chiral symmetry, while pseudo-Dilaton will refer to the light scalar state associated to the closeness of the lower bound of the conformal window. The pNGB Composite Higgs and pseudo-Dilaton Composite Higgs scenarios share common features that we start by discussing. The scale of New Physics (NP) will be referred as $\Lambda_{\rm NP}$. The Electroweak (EW) scale will be denoted $\Lambda_{\rm EW}$. Models of composite Higgs can be summarised as follow. At the New Physics scale, assume that the physics is described by the effective Lagrangian of the Standard Model without a Higgs sector and with massless fermions augmented by the Lagrangian of a carefully chosen Strongly Coupled Theory $\mathscr{L}_{\rm SCT}$ which confinement scale is given by $\Lambda_{\rm NP}$. At the energy $\Lambda_{\rm EW} \ll \Lambda_{\rm NP}$, the new strongly interacting sector is replaced by its low energy effective field theory (EFT), exactly like QCD is described by chiral perturbative theory at low energy, so that the effective Lagrangian is the one of the Standard Model with massless fermions plus corrections suppressed by powers of $1/\Lambda_{\rm NP}$. The origin of the mass of the weak bosons and of the Higgs mass will differ in the two scenarios and will be discussed below. Fermion masses pose a problem common to all these scenario that we will discuss later on. Note that the



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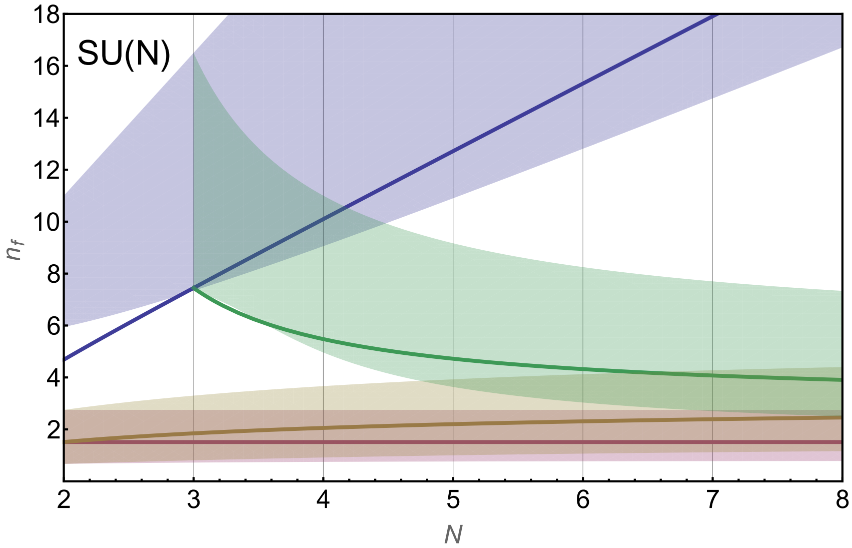


**Figure 2:** *Pertubative (4-loops $\bar{MS}$) calculation of the conformal window for $SU(N)$ groups in the fundamental representation (upper light-blue), two-index antisymmetric (next to the highest light-green), two-index symmetric (third window from the top light-brown) and finally the adjoint representation (bottom light-pink)[9].*

Lagrangian at the scale $\Lambda_{\rm EW}$ is dictated by the chiral symmetry breaking pattern and the quantum numbers of the bound states of the theory. This explains why many effective models can be studied without even specifying an underlying strongly interacting sector at $\Lambda_{\rm NP}$. When a gauge theory $\mathscr{L}_{\rm SCT}$ is known in terms of its underlying fermions content, the model is said to have a UV completion. The choice of the quantum number of the underlying fermions is crucial to engineer a realisation such a scenario.

We now turn our attention to the differences between the two scenarios namely PNGB Composite Higgs and the pseudo-dilaton Composite Higgs scenarios.

In the PNGB Composite Higgs case, the underlying gauge theory $\mathscr{L}_{\rm SCT}$ is assumed to have a global symmetry group $G_F$ spontaneously broken down to a subgroup $H_F$. To preserve the custodial symmetry $G_{\rm cust} = SU(2)_L \times SU(2)_R$ of the Standard Model, the quotient group $G_F/H_F$ must be such that $H_F \supseteq G_{\rm cust}$ and the quantum numbers of the underlying fermions should be chosen so that at least one of the Goldstone bosons have the quantum numbers of the Higgs particle. The Higgs particle being identified with a Nambu-Goldstone boson is naturally light [10, 11, 12, 13, 14, 15].

The effective theory at the scale $\Lambda_{\rm EW}$ describes massless SM fermions. The Higgs mass is generated by the EW interaction exactly like the pions would acquire a mass if the electromagnetic interaction is switched on in massless QCD. A vacuum expectation for the Higgs field is not generated, and electroweak symmetry remains unbroken. To trigger EW symmetry breaking the interactions with the SM fermions must be taken into account at the scale $\Lambda_{\rm EW}$. Depending on the full Higgs potential, the Higgs potential might trigger electroweak symmetry breaking (EWSB). We assume the potential to be such that EWSB occurs, and we will denote $V$ the minimum of the potential. Then, setting the scale so that $F_{\rm PS} \sin V/F_{\rm PS} = v_{\rm EW} = 246{\rm GeV}$ where $F_{\rm PS}$ is the pseu-



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doscalar decay constant of the strongly interacting sector, the mass of the vector bosons is by construction the Standard Model one at tree-level. Furthermore it can be shown that $\kappa_{V,F} = 1 + \mathscr{O}(\xi)$ where $\xi = (v_{\rm EW}/F_{\rm PS})^2$ which therefore guarantees small deviations from the SM couplings if the so-called vacuum misalignement $\xi$ is small.

In the near-conformal framework, the scale is set by $F_{\rm PS} = v_{\rm EW} = 246$GeV and the EW quantum numbers are chosen so that $m_\sigma = m_H$ and so that Goldstone bosons become the longitudinal degrees of freedom of the vector bosons (like in a technicolor scenario). While in the case of the pNGB scenario, the low energy effective theory can be systematically written, the question of the existence of an effective theory describing a light scalar is very challenging and has stemmed considerable efforts by the community.

The question of the fermion mass generation, and in a first step, of the heaviest fermion, the top quark, is a long-standing issue. Several mechanisms have been proposed. Typically, they require a new sector at an even higher energy $\Lambda_{\rm UV}$, this new sector typically generates new effective operators at the scale $\Lambda_{\rm NP}$. Two classes of models have been proposed:

- models that effectively generate operator of the form $\frac{1}{\Lambda^2_{\rm UV}} q\bar{q} O_{\rm SCT}$ , where $q$ stands for the a Standard Model quark field and $O_{\rm SCT}$ a bilinear operator of the new fermions fields at $\Lambda_{\rm NP}$. Typically, nothing prevent effective operators that generate Flavour Changing Neutral Current which are tightly constrained by the experiments. Introducing fermion masses into these models is therefore a challenge.

- models that effectively generate operator of the form $\frac{1}{\Lambda^{\dim(O)-1}_{\rm UV}} \bar{q}^a O^a_{\rm SCT}$, where $a$ stands for a $SU(3)_c$ colour indices. This scenario is referred to as Partial compositeness mechanism, introduced in [16], and requires that the strongly interacting sector at $\Lambda_{\rm NP}$ to have QCD charged bound states. Candidate UV completions for partial compositeness scenarios have been extensively studied, first by [17, 18]. Other studies have followed [19, 20]

Fermion mass generation in composite scenario is a challenge for theorists and lattice simulations start be used to test fermion mass generation mechanisms and to guide the model building community.

$$
\begin{array}{lcl}
\Lambda_{\rm NP}: & & \mathcal{L}^{(\rm SM-Higgs)}_{\rm massless} + \mathcal{L}_{\rm SCT} \\
\Big\downarrow & & \\
\Lambda_{\rm EW}: & & \mathcal{L}^{(\rm SM)}_{\rm massless} + \mathcal{O}\left(\dfrac{1}{\Lambda_{\rm NP}}\right)
\end{array}
$$

**Figure 3:** *A sketch of composite Higgs scenarios with massless fermions (at LO).*



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## 3. Lattice investigations of near-conformal scenarios

Many lattice groups are accumulating evidence that near-conformal dynamics gives rise to a light scalar state. The focus has been on models based on $SU(2)$ and $SU(3)$ gauge groups. The lattice calculations of the lightest scalar singlet are demanding because of the costly disconnected contributions associated with the scalar bilinear interpolating field.

The interpretation of lattice calculation is difficult because the prediction of the spectrum in the chiral limit depends strongly of the choice of the effective theory describing the low-lying states. For instance, chiral perturbation theory based on the chiral lagrangian is expected to receive large contributions from the scalar state. Models and effective theories have been designed and consistency checks are now being performed to conclude about their ability to describe the lattice data.

At finite fermion mass, several groups observe signs of chiral symmetry breaking and of a scalar states that have a mass very similar or even below the mass of the Goldstone bosons. This justifies, a posteriori, the use of two-point functions to obtain the mass of the state without having to consider a full-fledged finite volume calculation of the two Goldstone bosons scattering process. In order to improve our understanding of near-conformal gauge theories, further investigations both at the numerical level by reducing the systematics and at the theoretical level by progressing in our understanding of the effective description are required.

To date, the theories that observe candidates for light scalar states are: $SU(3)$ with $N_f = 8$ fundamental fermions, as illustrated in Fig. 4, $SU(3)$ with $N_f = 2$ antisymmetric (or sextet) fermions and also theories are likely to be conformal $SU(2)$ with $N_f = 1$ or 2 adjoint fermions.

### 3.1 Models and Effective theories of near-conformal dynamics

A number of models and effective theories have been proposed to describe theories close to the lower bound of the conformal window which include the scalar field as a degree of freedom[22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. The goal of this section is not to review them but to provide an up-to-date list of the low energy descriptions, and to highlight their main features. The discussion of the latest tests of these low energy description using lattice data is postponed to the next section.

**Bound state model:** *H*oldom and Koniuk consider a Hamlitonian-based bound state model where the pseudo-scalar, scalar, vector and axial-vectors are included[22]. They study the relation between the spectrum and the relevant form factors.

**Chiral perturbation theory with a flavor-singlet scalar:** the central idea is to augment the chiral Lagrangian by an iso-singlet scalar in the most general way[32, 33]. The corresponding low energy description does not rely on the closeness from the lower bound of the conformal window, but it thought to be able to capture a variety of underlying dynamics where the iso-singlet scalar plays a role.

**Generalised Linear sigma model** based on an approximate infrared conformal invariance, the scalar potential breaks chiral symmetry spontaneously[27].



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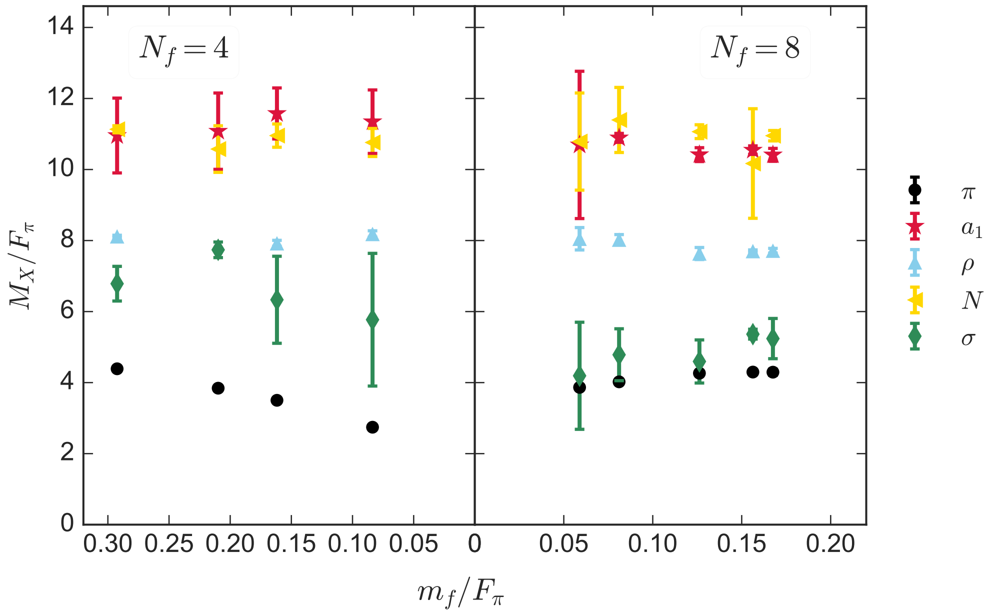


**Figure 4:** *Comparison of the spectroscopy of* $SU(3)$ *gauge theory for* $N_f = 4$*(left) and* $N_f = 8$ *(right) fundamental fermions with. Hadron masses (vertical axis) and the fundamental fermion mass (horizontal axis) are both shown in units of the pion decay constant* $F_\pi$*; the chiral limit* $m_f = 0$ *is at the center of the plot for both theories. The major qualitative difference between the two values of* $N_f$ *is the degeneracy of the light scalar* $\sigma$ *with the pions at* $N_f = 8$. *[21]*

**Effective theory of a pseudo-Nambu-Goldstone boson associated to the spontaneous breaking of the conformal invariance:** several authors build on the idea that the scalar state is associated to the spontaneous breaking of the dilation symmetry. There is a long history of effective description of the dilaton Lagrangian depending on the focus of the authors [23, 24, 34, 25]. *Appelquist, Ingoldby and Piai* add Nambu Goldstone bosons associated to the spontaneous breaking of chiral symmetry to build their effective lagrangian[26]. *Golterman and Shamir* derive an effective theory by introducing a systematic expansion in terms of a parameter controlling the distance to the lower bound of the conformal window in their Dilation-pion low-energy effective theory[25]. The authors also derived a systematic expansion in which the fermion mass is not small relative to the confinement scale. In the large-mass regime of the Dilaton low-energy theory, they have shown that at leading order hyper-scaling relations are expected [28].

**Complex conformal field theory:** inspired by the work of *Gorbenko et al.*[35, 36], and in particular of their analysis of the two-dimensional $Q-$state Potts model with $Q > 4$, *K*uti argues that the slow running of the coupling constant in near conformal theory might be related to the presence of two fixed points at complex coupling, referred as complex CFTs[31]. He suggests an other avenue to explore other types of low energy descriptions.



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### 3.2 Testing low-energy descriptions using lattice results

Several theories are being extrapolated using the low energy descriptions discussed in the previous section. The latest findings will be reviewed here.

The Lattice Strong Dynamics Collaboration presented new preliminary results including pseudo-scalar decay constant and mass, the mass of the scalar state and scattering length of the Goldstone bosons for the $SU(3)$ gauge theory with $N_f = 8$ fundamental fermions[37]. The results suggest that the linear Sigma Model EFT describes the data well. Expressions for scattering lengths of GBs and the scalar decay constant have been derived and additional lattice calculations are underway.

The LSD data have also been confronted to a large mass regime of the Dilaton-EFT [28]. The LSD data are well described by the hyper-scaling scaling predicted under the assumption that the fermion mass is not small compared to the confinment scale [29]. The authors have now shown that the EFT describes the taste-splitting pattern in the discretisation of the theory with staggered fermions. The observed pattern is indeed very different from the one observed in staggered lattice QCD [38]. This fact brings additional evidence that the assumptions that underpin the Dilaton EFT, and in particular its large-mass regime, are predictive.

The latest result regarding simulations of two flavours in the symmetric (sextet) representation of the SU(3) gauge group, the LatHC collaboration, presented results at two lattice spacings[31] and tested the dilaton hypothesis with two typical dilaton potentials. Infinite volume extrapolation of the pseudo-scalar mass and decay constant have been performed and the authors concluded that the dilaton decay constant over pseudo-scalar decay constant and the ratio of the dilaton mass over the GBs mass are very sensitive to the mass of the scalar state. More recently the authors also used Random Matrix Theory in the $\varepsilon$-regime to test further the consistency of their dilatonic Lagrangian. In Fig. 5, the chiral condensate obtained from the GMOR relation and from Random Matrix Theory in the $\varepsilon$-regime are compared and show good agreement.

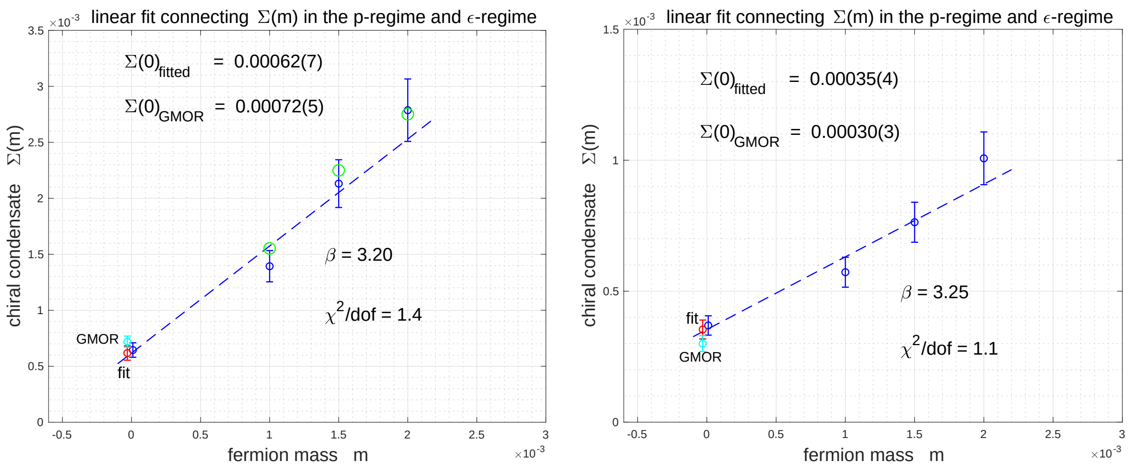


**Figure 5:** *Comparison of the chiral condensate $\Sigma(m)$ obtained from simulations in the $\varepsilon$-regime and by the GMOR relation in the p-regime for two sextet (Dirac) fermions of $SU(3)$ gauge theory. More details can be found in [31].*



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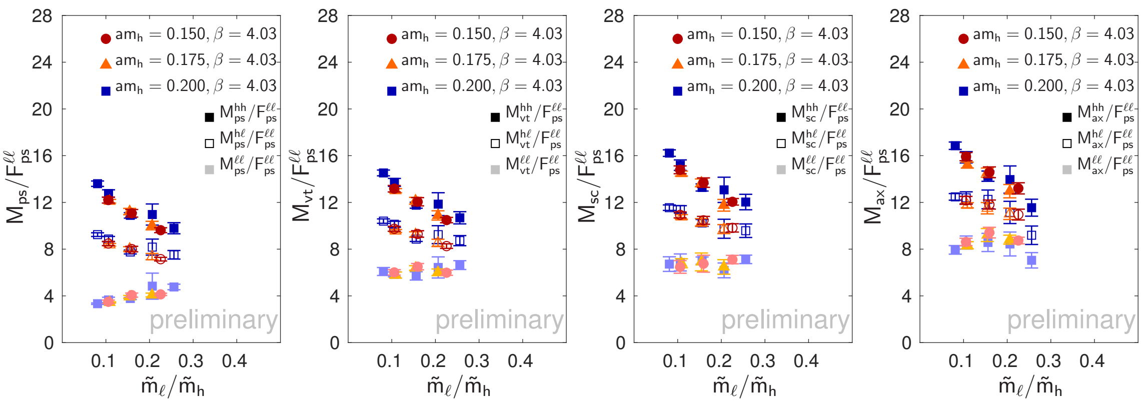


**Figure 6:** *Low-lying connected meson spectrum obtained from light-light, heavy-light, or heavy-heavy two- point correlator functions as a function of the ratio of light over heavy flavour mass to highlight hyperscaling[45].*

### 3.3 Mass-split models: $SU(3)$ with 4 light and 6 heavy fundamental flavours

Motivated by the experimental fact that the Higgs boson is light and that no other heavier resonances have been observed, describing a composite Higgs boson requires a system with a large separation of scales. An alternative approach to near-conformal theory is based on mass-split models which feature a number of light and heavy flavours. Those theories are chirally broken in the IR but conformal in the ultraviolet (UV). Such a class of model features interesting properties from the model building point of view but fails to explain the origin of the two well separated mass scales in a first place.

Mass-split systems can be used to design models involving a light dilaton or Composite Higgs models based solely on the spontaneous breaking of chiral symmetry scenarios [39, 40, 41, 17]. In the UV, the number of flavours is chosen to lie in the Conformal Window and the theory is therefore driven by the conformal fixed point. In the IR, the heavy flavours decouple and the system therefore exhibits spontaneous chiral symmetry breaking.

It has been shown that in such mass-split system, dimensionless ratios exhibit two important features[42, 43, 44]. First, dimensionless ratios of physical observables exhibit hyper-scaling : they are function of the ratio $m_{;}/m_h$ . Such ratios are therefore independent of the mass of the heavy flavours. Second, the authors argue that the range of slowly evolving ("walking") coupling increases when the mass of the heavy flavours is reduced. The walking region can therefore be tuned at the price of introducing explicitly two scales.

Mass-split systems have been investigated previously with 4 light and 8 heavy fundamental flavours of $SU(3)$ and it seems that the lightest scalar is the lightest massive particle in the chiral limit[43].

Results for 4 light and 6 heavy flavours of $SU(3)$ have been presented[45]. They show that the connected meson spectrum exhibits hyperscaling. The calculations of the scalar meson have not been performed yet.



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## 4. Lattice investigations of Pseudo-Nambu Goldstone Boson Higgs scenarios

We review here the questions addressed by lattice collaborations to test PNGBs scenarios in view of the latest experimental results and to provide first principle predictions to guide phenomenologists.

The first information that lattice collaborations can provide regarding a UV completion is its spectrum in isolation of the Standard Model. A related question is to determine if the spectrum is similar to the one observed in QCD or on the contrary rather different. Identifying common features of different UV completion allow to understand better what could be the experimental signatures of such models.

Other low energy properties of UV completion in isolation can also provide insights: form factors, properties of the candidate Higgs boson, the Higgs potential itself, and scattering properties of the PNGBs are as many possible interesting observables that can bring essential information in the search for experimental signatures of a composite electroweak sector at the LHC.

A third class of problems that is addressed is to use lattice calculations as a laboratory to test mechanisms for the generation of the top quark mass or to pave the way for a calculation of the Higgs potential.

### 4.1 $SU(2) = Sp(2)$ gauge theory with $N_f = 2$ fundamental fermions

The most minimalistic known UV completion of PNGBs Higgs boson scenario is based on $SU(2)$ gauge theory with two fermions in the fundamental representation. The theory is far from the conformal window and is therefore QCD-like. Because the fundamental representation of $SU(2)$ is pseudo-real, the flavour symmetry of the classical massless theory is upgraded to $SU(4)$. Classically, the mass term breaks the flavour symmetry down to $Sp(4)$. A number of publications have confirmed non-perturbatively that the chiral breaking pattern is $SU(4) \longrightarrow Sp(4)$ leading to five Goldstone Bosons. The EW embedding has been proposed in [46], and the model is phenomenologically viable [47]. The fact that this theory is used as UV completion by model builders makes it ideal to explore the underlying dynamics in details. For the first time, a full non-perturbative calculation of the vector meson width has been presented[48]. The phase-shift is calculated using the energies of two-pions in finite volume and are related to infinite-volume scattering amplitudes $S(E) = e^{2i\delta(E)}$ via rigorous Lüscher's formalism [49, 50]. The resonance parameters $(g_{\rho\pi\pi}, M_\rho)$ are obtained from the phase shift $\delta(p)$ using the parametrisation:

$$\frac{p_*^3 \cot\delta}{E_{CM}} = \frac{6\pi}{g_{\rho\pi\pi}^2}\left(M_\rho^2 - E_{CM}^2\right) \tag{4.1}$$

as illustrated in Fig. 7.

The calculation shows that the preliminary value of the coupling constant that control the vector meson width is $g_{\rho\pi\pi} \sim 11(2)$, a value somewhat larger than in QCD where $g_{\rho\pi\pi} \sim 6$. This result does not include chiral or continuum extrapolations needed to extract the physical value of $g_{\rho\pi\pi}$. The number can be compared to the phenomenological relation obtained using vector meson dominance by Kawarabayashi & Suzuki [51] and by Riazuddin & Fayyazuddin[52] (KSRF) stating that $g_{\rm VPP}^{\rm KSRF} = m_V/\sqrt{2}F_{\rm PS}$. Using numberes from[53], the KSRF estimate of the coupling read $g_{\rm VPP}^{\rm KSRF} = 9(2)$. The preliminary result, obtained at finite quark mass and finite lattice spacing,



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suggests that the KSRF turns out to be satisfied for $SU(2)$ gauge theory with $N_f = 2$ fundamental fermions. The consequences of such a finding in terms of constraints provided by the LHC remain to be investigated.

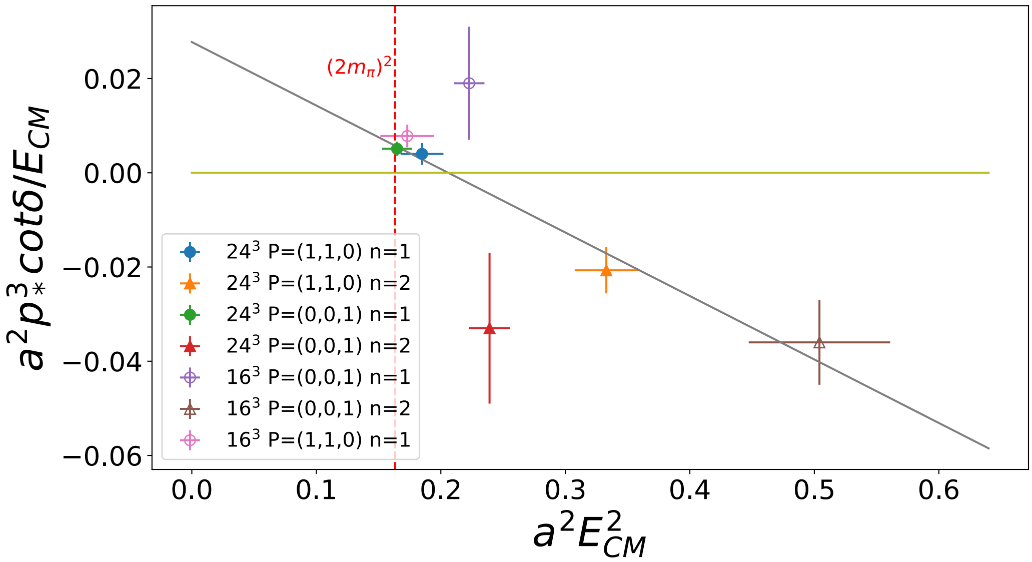


**Figure 7:** *Plot of* $a^2p^{*3}\cot\delta/E_{\mathrm{CM}}$ *as a function of the squared centre-of-mass energy. Points with error bars correspond to energy levels from different ensembles and/or total momenta P. A linear fit allows to determine the resonance parameters.*

### 4.2 $Sp(4)$ gauge theory with $N_f = 2$ fundamental fermions

The latest results on the on-going research programme started in [54] focusing on $Sp(4)$with $N_f = 2$ fundamental fermions have also been presented. The UV completion share the same chiral symmetry breaking pattern $SU(4) \to Sp(4)$ as $SU(2) = Sp(2)$ with two fundamental flavours and therefore shed light on the dependence of the physical observables on the gauge dynamics. One very interesting aspects of the $Sp(4)$ gauge dynamics is that when the two fundamental flavours are supplemented by three antisymmetric fermions, the theory features top-partner candidate and can be shown to be a UV completion of partial composite models. More details concerning this line of research can be found in a recent paper[55]. Finally, the model is also relevant in the context of strongly interacting massive particles (SIMP) as dark matter candidates[56].

Performing unquenched simulations with Wilson fermions, extensive studies of the spectrum (chiral behaviour, discretisation error) have been presented. The low-energy description that explicitly includes the vector and axial-vector states proposed in[54], and based on the idea of hidden local symmetry[57], is used to determine 10 low energy constants using the continuum-extrapolated results of the decay constants and masses in the pseudo-scalar, vector and axial-vector channels. The effective model describes well the continuum extrapolated lattice data of the masses and decay constant squared as a function of the pseudo-scalar mass squared as shown by Fig. 8. The coupling $g_{\mathrm{VPP}}$ appears in the effective Lagrangian, and the value turns out to be compatible with the KSRF



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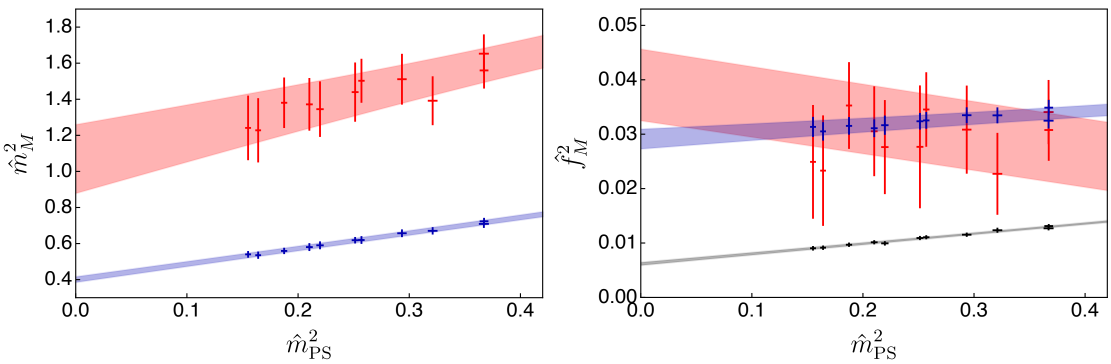


**Figure 8:** *Continuum-extrapolated meson masses and decay constants squared as a function of the pseudo-scalar mass squared. Black, blue and red colours are for PS, V and AV mesons. The global fit results are denoted by shaded bands with their widths representing the statistical errors.*

estimate and close to the value obtained in QCD. The author argue that it provide empirical support for the KSRF relation.

### 4.3 Toward partial compositeness on the lattice

As mentioned earlier, an important issue with models of composite Higgs is to provide mass to the SM fermions and in particular to the top quark. A mechanism used by phenomenologists in the context of Composite Higgs models is referred as partial compositeness. The mechanism provides mass to the top quark by introducing a coupling between the top quark and a top-quark partner at the scale $\mathscr{L}_{\rm SCT}$. This effective operator at the composite scale is generated by an unspecified new sector. A practical requirement for the PNGB UV completion is therefore to have a spin $1/2$ bound state that can be charged under $SU(3)_c$. UV completion that features such top partners have been extensively studied[17, 18]. The authors consider theories involving fermions in mixed representations of the gauge group, and one of the minimal theory that exhibits top partner candidates is based on an $SU(4)$ gauge theory with five Weyl fermions in the antisymmetric representation and three Dirac fermions in the fundamental representation.

One collaboration undertook instead the extensive numerical investigation of $SU(4)$ gauge theory with two Dirac fermions in the antisymmetric representation (denoted $Q$) and two Dirac in the fundamental (denoted $q$). Such a gauge theory is not a UV completion of PNGBs Composite Higgs model, but features spin $1/2$ states ($Qqq$), called chimera baryons, that would correspond to the top partner in the original theory proposed by *Feretti et al*. In terms of number of Weyl fermions, the simulated theory is close to Ferreti's UV completion and might serve as a laboratory to test the mechanism. Any phenomenological conclusion obtained must therefore acknowledge the strong assumption that the predictions do not change by adding one Weyl fermion in the antisymmetric representation, and one Dirac in the fundamental. Several results have been obtained: the masses and decay constants for the pseudo-scalar and vector mesons[58], the baryons masses constituted of four $q$ operators and six $Q$ operators, and of the more interesting $Qqq$ "chimera" baryons[59]. The necessary chiral extrapolations have been performed using the chiral perturbation theory of a two-representation theory derived in [60]. More recently, another collaboration has started to investigate the same gauge theory. While they perform an extensive investigation of the spectrum,



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the focus is more on the algorithmic side of the simulations of gauge theories involving matter content in multiple representations[61].

In summary, while the phenomenological lessons should be taken with caution, the following results are certainly a first interesting step forward in our understanding of simulations with mixed representations and of partial compositeness.

The study of the matrix element relevant in determining the strong sector contribution to the mass of the top quark has been presented in[62] and summarises the recent results obtained in [63]. At the scale $\mathscr{L}_{\mathrm{SCT}}$, the linear mixing between the chimera baryon and the massless top quark is controlled by two coupling constant $G_{L,R}$ generated by the extended sector that can be thought as "Fermi" constants. At at $\Lambda_{\mathrm{EW}}$ scale, the resulting top Yukawa coupling can be written :

$$y_t \approx G_L G_R \frac{Z_L Z_R}{M_B F_{P6}} \tag{4.2}$$

where $M_B F_{P6}$ is a combination of the mass of a chimera baryon and of pseudo-scalar decay constant of the $Q$ fields, already computed in previous studies, and $Z_{L,R}$ are matrix elements of a top partner operator between the vacuum and the top partner state. The results of the renormalised matrix element $Z_{L,R}$ is shown in Fig. 9. The renormalisation is performed perturbatively. The authors argue that imposing $y_t \sim 1$, implies that $\Lambda_{\mathrm{UV}}$ cannot be much larger than $F_6$ as the "model" would require it. We refer to [62] and to [63] for more details.

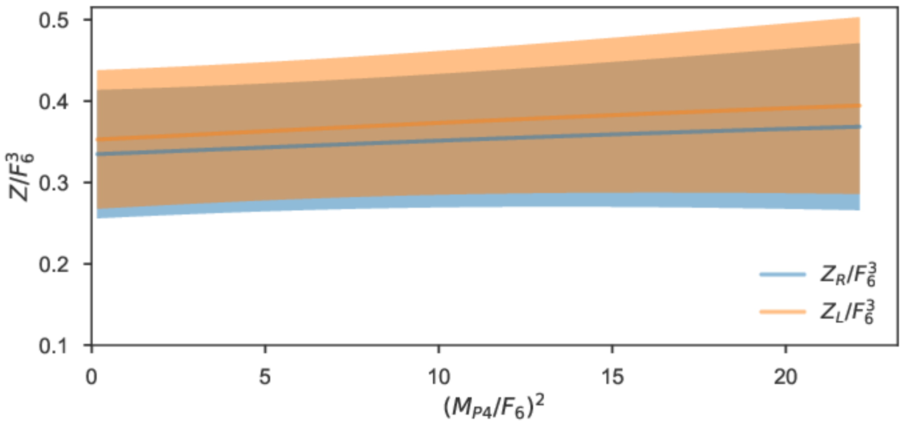


**Figure 9:** *Chimera baryon matrix elements as a function of of the quartet mass.*

## 4.4 Toward lattice constraints of the Higgs potential

The Higgs potential is not guaranteed to trigger EWSB: the potential is induced by loops of gauge bosons and top quarks. The top contribution requires to estimate a 4-point function, as shown in [64] and in [65]. Vector boson contribution is controlled by a low energy constant, similar to the one that controls the pion mass spitting in QCD denoted $C_{LR}$. The low energy constant has been computed for the first time in [66] by using a current-current correlator. The authors take the continuum and chiral limit and find that the value of the low energy constant is

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similar to its QCD counterpart in unit of the pseudo-scalar decay constant. The results do not allow to draw any phenomenological conclusion because the theory is not an UV completion of a PNGBs model but shows that such calculation is possible and pave the way to calculations in more realistic models. Taking the continuum and chiral limits, the authors of [66] find that the $C_{\cdot}LR$ in units of the pseudo-scalar decay constant is roughly of the same size as its QCD counterpart. The results are summarised as a function of the mass of the fermions in the antisymmetric representation in Fig. 10.

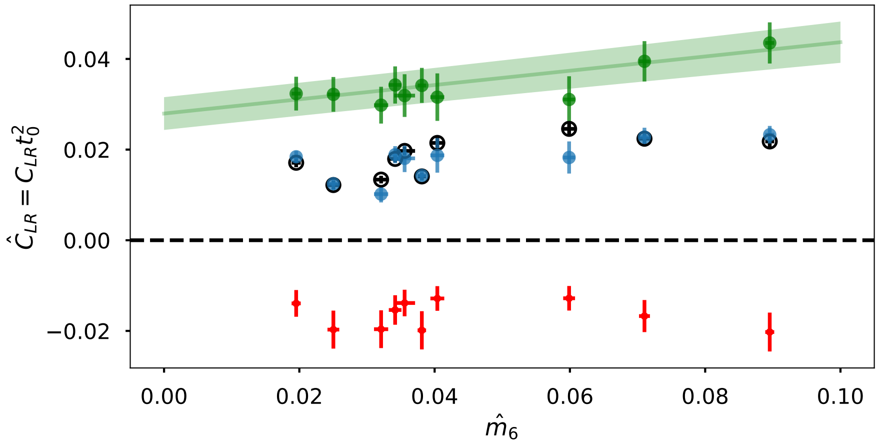


**Figure 10:** *Fit of the $C_{LR}$ data, the continuum prediction is shown by a green band*

## 5. The theory landscape

We summarise in this section the recent developpement that are less directly related to the phenomenology but which are crucial to clarify our understanding of non perturbative gauge dynamics.

### 5.1 Conformal Window and near-conformal $\beta$ functions

A long standing task for the lattice community is to show the existence of a conformal window, and in particular the critical number of flavour marking the onset of the conformal window. The task turns out to be very challenging.

Concerning the gauge group $SU(2)$ with fundamental fermions, extensive investigations suggest no IRFP for $N_f = 2,3,4$, while a IRFP is developed for $N_f = 6,7,8$[67, 68]. For adjoint fermions, evidence for an IRFP have beem found for $N_F = 1,3/2,2$ [69, 70, 71, 72, 73] and see [74, 75] for recent developments.

For the $SU(3)$ gauge group, results are controversial. The latest results for 12 fundamental fermions, using domain-wall fermions, support the existence of an IRFP[76]. while staggered calculations obtained for 12 flavours find no such evidence [77]. More details on the latest results can be found in these proceedings. For the antisymmetric representation, a similar contradiction between lattice results obtained using the staggered discretisation[77] confirms previous published



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results[78] and results obtained using Wilson fermions[79] which find an IRFP. Note that a an independent spectrum study performed using Wilson fermions is compatible with the conformal scenario[80].

### 5.2 Flavour dependence of the ratio $m_V/F_{\mathrm{PS}}$

If a new strongly interacting sector gives rise to a composite Higgs, the most obvious experimental signature would be the observation of new composite particles associated to quantum numbers allowed by the underlying dynamics. It is expected that, for a large class of strongly interacting theories, the lightest of such particles would be the vector meson resonance. It is therefore relevant to determine the mass of such a state in as many theories as possible. An interesting work tackled the issue by studying flavour dependence of the ratio $m_\rho/f_\pi$ below the onset of the conformal windows, i.e in QCD-like theories. The calculation is performed for the $SU(3)$ gauge group with $N_f = 2,3,4,5,6$ fundamental fermions. The mass of the vector meson is obtained from two-point functions in a regime where the $\rho$ is stable. Continuum extrapolations are performed for each number of flavours. A careful analysis of the finite volume effects have been performed and is crucial to obtain the results shown in Fig. 11. The result reads $m_V/F_{\mathrm{PS}} = 7.95(15)$ with no significant $N_f$-dependence. The result relies on extrapolations below the two pion threshold and on the assumption that ignoring the resonant nature of the particle does not affect significantly the prediction. The authors also provide a compilation of that ratio for a number of theories using various gauge groups and fermions in different representations. Once the leading gauge-group dependence is factored out, only a mild gauge group dependence is observed, except for $SU(2)$ gauge theory. Assuming that the KSRF relations holds, such ratio can be used to estimate the value of the coupling constant $g_{\rho\pi\pi}$. The value obtained suggests that $g_{\rho\pi\pi}$ is constant when the number of flavour is changed below the conformal window. Under the assumptions discussed, the study suggests that the vector resonance coupling is not a quantity sensitive to the underlying dynamics. The interested reader is referred to[81] and to a more recent paper [82].

### 5.3 Dynamical generation of particle masses: an alternative to the Higgs mechanism

One collaboration is investigating a novel alternative non-perturbative mechanism for elementary particle mass generation [83, 84]. The framework is based on a conjectured non-perturnative obstruction to the recovery of broken fermionic chiral symmetries which give rise to a dynamically generated fermion mass term. For the first time, the authors provide numerical evidence of the conjectured phenomenon by using lattice simulations[85]. The authors simulate within the simplest 4-dimensional model in which the phenomona is supposed to occur: an $SU(3)$ gauge theory with two fundamental fermions augmented by a colourless scalar doublet, Yukawa terms and a Wilson-like term. Tuning the bare parameters to restore the fermionic chiral symmetry in the effective Lagrangian and performing a continuum extrapolation, they show that the pseudoscalar meson mass is significantly different from zero in the phase where the the exact symmetry acting on fermions and scalars is spontaneously broken. From a phenomenological standpoint, the authors argue the EW interactions can be included without introducing tree-level flavour changing neutral currents. The non-pertubative mechanism would then generate weak bosons mass terms and provide an alternative to the Higgs mechanism. In such an approach, the observed Higgs boson would be a bound state of the new interaction appearing in the vector bosons scattering channel.



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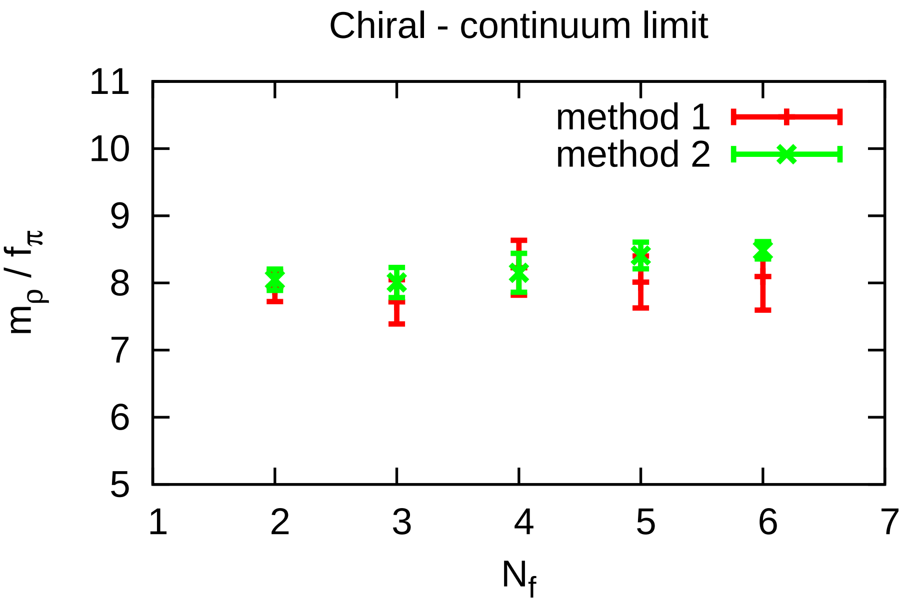


**Figure 11:** *The ratio $m_\rho/f_\pi$ in the chiral-continuum limit for each $N_f$ [82].*

## 6. Conclusions

The fascinating possibility that composite solutions to the puzzles of Beyond the Standard Model physics could benefit from first principle calculations is driving the lattice community to study non pertubative phenomena in various gauge theories.

In the case of near-conformal dynamics, the presence of light scalars is a challenge both from the numerical and theoretical points of view. The studies of PNGBs models is now going beyond spectroscopy and new calculations provide insight in quantities relevant for the experiments.

By exploring the dynamics of such theories, the lattice simulations contribute to deepen our understanding of non-perturbative effects in Quantum Field Theory. There is an impressive amount of results on the behavior of quantum field theory, when the gauge group or the matter content is varied. The results provide quantitative insights which guide the model building community to design more realistic models. The lattice simulations can also test mechanisms relying on non perturbative phenomena in quantum field theory.

Interestingly, the various aspects of the lattice simulations as a tool to address Physics Beyond the Standard Model raise a number of challenges. Often, the investigations question the algorithms developped in the context of Quantum Chromodynamics and their applicability to other theories. In other cases, the problems require new methods to be developped. The interpretation of the lattice results often require a thorough and critical examination in theories that cannot be compared to actual experimental measures.



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