

# Emittance Tuning for the Future Circular e<sup>+</sup>/e<sup>-</sup> Collider (FCC-ee)

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The Future Circular e<sup>+</sup>/e<sup>-</sup> Collider (FCC-ee) presents an appreciable challenge for emittance tuning due to the strong final focusing and chromaticity correction, required to achieve high luminosities in the order of  $10^{36}\text{cm}^{-2}\text{s}^{-1}$ . The proposed 100 km e<sup>+</sup>/e<sup>-</sup> circular collider, FCC-ee is being designed to undertake precision studies and rare decay observations in the range of 90 to 350 GeV center of mass energy, with luminosities ranging from  $1.7 \times 10^{35}\text{cm}^{-2}\text{s}^{-1}$  to  $230 \times 10^{35}\text{cm}^{-2}\text{s}^{-1}$ . In order to reach these luminosities, large beta values are encountered preceding to the IP, increasing the susceptible to misalignments and field errors. In this paper, we describe a comprehensive correction strategy used for the low emittance tuning. The strategy includes programs that have been developed to optimise the lattice based on Dispersion Free Steering, linear coupling compensation based on Resonant Driving Terms and beta beat correction utilising response matrices.

*European Physical Society Conference on High Energy Physics - EPS-HEP2019 -  
10-17 July, 2019  
Ghent, Belgium*

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## 1. INTRODUCTION

The e+/e- Future Circular Collider (FCC-ee) will be implemented in stages and operate at four energies: Z-pole (45.6 GeV), WW threshold (80 GeV) and ZH production peak (120 GeV) and the ttbar production threshold (182.5 GeV), with two Interaction Points (IPs) [1]. Table 1 summarizes some of the key accelerator parameters over the four energies. A complete list of parameters can be found in the Conceptual Design Report (CDR) [2].

Table 1: Baseline Beam Parameters of the Four Operational Energies for FCC-ee [2]

Parameters	Z-pole	WW	H(ZH)	t $\bar{t}$
Beam Energy [GeV]	45.6	80	120	182.5
$\epsilon_x$ [nm·rad]	0.27	0.28	0.63	1.45
$\epsilon_y$ [pm·rad]	1	1	1.3	2.7
$\beta_x^*$ [mm]	0.15	0.2	0.3	1
$\beta_y^*$ [mm]	0.8	1	1	1.6
Number of bunches	16640	1300	328	33
$\mathcal{L}$ [ $10^{34}\text{cm}^{-2}\text{s}^{-1}$ ]	230	32	8	1.5

More recently, the FCC study has investigated the possibility of 4 IPs. Figure 1 shows the beta functions around the ring for the 4 IP option of the ttbar lattice (182.5 GeV). As can be seen from this figure, the value of  $\beta_y$  remains above 1000 m over many magnetic elements around the IP. Where the beta function is high, the machine is particularly sensitive to magnet misalignments and field errors.

Typically, the smaller the value of the beta function at the IP, the larger the chromaticity, and the stronger the chromaticity correction required [3]. The strong sextupoles required for local chromaticity correction [4] pose two challenges. Firstly, the strong sextupoles introduce non-linearities that are not easily accounted for by the inherently linear correction techniques outlined in the following sections. Misalignments of quadrupoles and dipoles can cause the beam to be steered off-centre through a sextupole, which introduces a skew quadrupole field, leading to vertical dispersion and greater coupling. Secondly, any residual beta-beating presented in the corrected lattice can result in growth of the horizontal emittance. A degradation of the linear transfer matrix between paired sextupoles gives rise to residual nonlinear aberrations. Secondly, any residual beta-beating presented in the corrected lattice can result in growth of the horizontal emittance. The beta-beating can result in the phase advance between the sextupoles not being well preserved and the aberrations not cancelled. For the reasons listed above, FCC-ee poses a unique challenge when it comes to emittance tuning.

## 2. Correction Methods

Reducing the x-y coupling and residual vertical dispersion around the ring are key to minimizing the vertical emittance and reaching high luminosities. The correction methods implemented include MICADO orbit correction, Dispersion Free Steering (DFS), coupling correction through

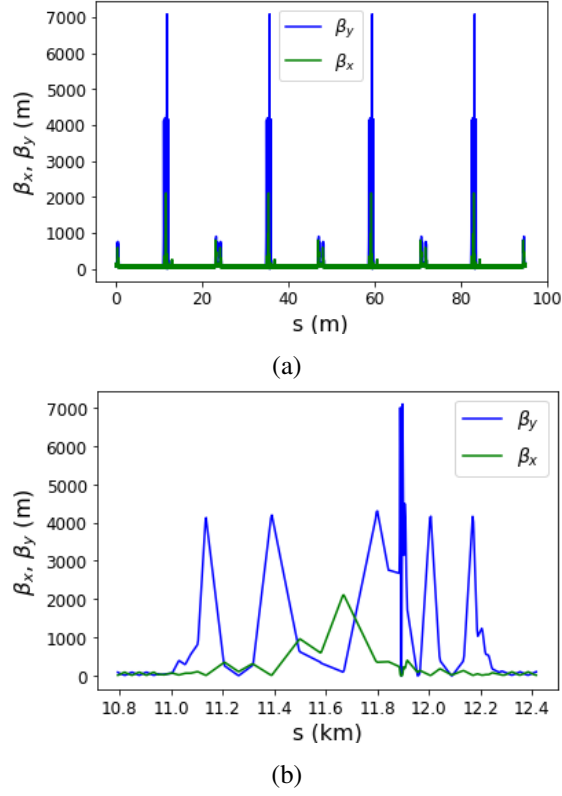


Figure 1: Beta functions near the IP of the 182.5 GeV lattice.

consideration of the Resonant Driving Terms (RDTs) and beta-beat correction. Corrector magnets and Beam Position Monitors (BPMs) are installed at every quadrupole magnet, tallying 1636 in the horizontal plane and 1640 in the vertical plane, around the 100 km ring. One skew quadrupole and one trim quadrupole are installed at every sextupole magnet for coupling and beta-beat correction. The following subsections very briefly outline the main components of the correction techniques used.

## 2.1 Dispersion-Free Steering

DFS aims to correct the orbit and dispersion simultaneously. The method is based upon response matrices relating the orbit,  $y$ , and dispersion,  $D_y$ , to the corrector kick,  $\theta$ ,

$$\begin{pmatrix} (1-\alpha)\vec{y} \\ \alpha\vec{D}_y \end{pmatrix} = \begin{pmatrix} (1-\alpha)\mathbf{A} \\ \alpha\mathbf{B} \end{pmatrix} \vec{\theta}$$

where  $\mathbf{A}$  and  $\mathbf{B}$  are the response matrices of the orbit and the dispersion due to a corrector kick respectively, and  $\alpha$  is a weighting factor, which can shift the emphasis to or from correcting the vertical orbit or the vertical dispersion.

## 2.2 Coupling Correction

Coupling can be introduced by misaligned sextupoles and rolled quadrupoles. Coupling correction can be applied through consideration of the response of the two coupling Resonant Driving

Terms (RDTs)  $f_{1001}$  and  $f_{1010}$ , and the correction applied through skew quadrupoles installed at every sextupole magnet. The analytical form of these two RDT terms are:

$$f_{1010}^{1001} = \frac{\sum_w J_e \sqrt{\beta_x^w \beta_y^w} e^{i(\Delta\phi_{w,x} \pm \Delta_{w,y})}}{4(1 - e^{2\pi i i(Q_u \pm Q_v)})} \quad (2.1)$$

where  $J$  is the vector of the skew strength,  $\beta_x^w$  and  $\beta_y^w$  are the horizontal and vertical beta functions at the location of the skew strength, and  $\Delta\phi_{w,x}$  and  $\Delta_{w,y}$  are the phase advances between the observation point and the skew component in the  $x$  and  $y$  plane respectively.

Combined coupling and dispersion correction can be applied using a large response matrix,  $\mathbf{M}$  containing the RDTs and vertical dispersion,  $D_y$ . The response matrix measures the response to a skew quadrupole field,  $\vec{\mathbf{J}}$ , and the system can be inverted via SVD, is [5]:

$$\begin{pmatrix} \vec{f}_{1001} \\ \vec{f}_{1010} \\ D_y \end{pmatrix} = -\mathbf{M} \vec{\mathbf{J}}.$$

### 2.3 Beta-beating Correction

Beta-beating can compromise the value of  $\beta^*$ , reducing the achievable luminosity. Beta-beating can also distort the optics model, affecting the efficiency of the correction schemes [6]. For the beta-beating correction, a SVD response-matrix method is used in two stages. Firstly, a response matrix representing the change in phase advance between the sextupoles where the trim quadrupoles are installed is used [7]. It has been previously shown that correction of the phase advance is as effective as correcting the actual beta function [8]. In the second stage, a response matrix of the beta functions values are measured and the correction applied with a weighted SVD. This weighted SVD approach places additional emphasis to the quadrupoles near the IP, where the beta function is large. For the results presented here, we used weighting factor of 10 for the BPMs where the  $\beta_y$  was above 2000 m and a weighting factor of 1 was applied to every other BPM [6].

### 2.4 Correction Strategy

A correction strategy has been devised and implemented to minimize the final vertical emittance, through minimizing the residual vertical dispersion, coupling and beta-beating. The first step of the algorithm is to set all sextupole strengths to zero. Throughout the correction algorithm, the sextupole strengths are increased by 10 % at a time. Once at 100 % of the sextupole's design value, additional coupling and dispersion correction are applied to ensure a small final vertical emittance can be achieved.

All of the corrections, aside from orbit correction, were applied through python scripts called as macros in MAD-X [9]. To begin with, the beam energy was set to 1 GeV, the RF turned off, and energy loss from synchrotron radiation was not included. This allows for faster computation and is considered a valid approach for a fully tapered machine [2, 10]. At the final stage of the algorithm, synchrotron radiation is turned on for the emittance calculation, which is based upon the Chao formalism for equilibrium emittance [11].

The following correction strategy was implemented:

1. Sextupoles are turned off, and an orbit correction performed with MICADO in MAD-X [9].
2. Coupling correction is performed, followed by rematching of the tune, followed by beat-beating correction.
3. DFS ( $D_y$  correction) is performed followed by coupling correction (which is needed due to the change in the corrector strengths brought about by DFS).
4. Sextupoles are then set to 10% of their design strength, and the following applied
  - (a) orbit corrections
  - (b) beta-beating correction
  - (c) coupling correction
  - (d) beta-beating correction
  - (e) coupling + dispersion correction
  - (f) sextupole strengths are increased by 10% and Step 4 repeated until the design sextupole strength is reached.
5. Final correction of coupling, dispersion and beat-beating is applied in this final step until the vertical dispersion and beta-beating are less than a set tolerance.

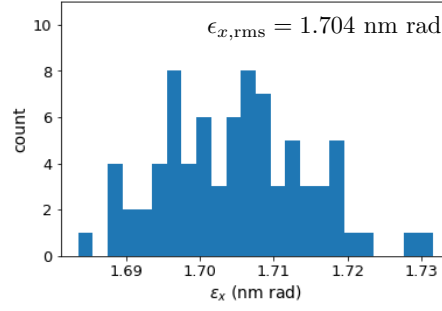
Throughout the correction algorithm, the tunes, orbit and the maximum beta value are continuously monitored. If the maximum orbit deviation in  $x$  or  $y$  becomes larger than 1 mm, or if either tune moves  $\pm 0.1$  from the nominal tune, then the sequence is redirected to orbit correction or tune re-matching as appropriate. Similarly, if the maximum beta value (which is used as a proxy to indicate the level of beta-beating) becomes  $\pm 5\%$  of the design value, then the correction scheme is redirected to beta-beating correction [12]. These measures ensure that the orbit remains under control and minimizes the risk of running into a resonance.

Table 2: Standard Deviation of the Misalignment and Roll Errors Introduced into the Lattice (BPM misalignments positioned relative to quadrupoles)

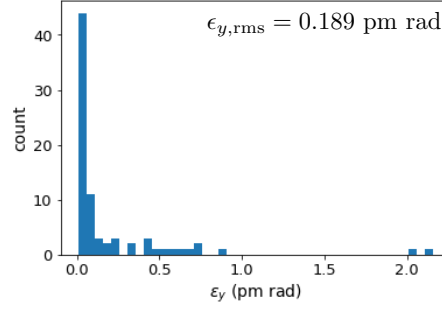
	$\sigma_x$ ( $\mu\text{m}$ )	$\sigma_y$ ( $\mu\text{m}$ )	$\sigma_\theta$ ( $\mu\text{rad}$ )
quadrupoles	100	100	100
sextupoles	100	100	100
dipoles	100	100	100
BPM	40	40	100

### 3. Corrected Lattices

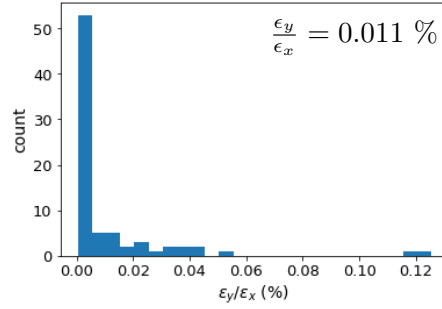
One hundred random seeds were used to introduce misalignments and roll angles to all magnet types, to create 100 different lattices, upon which the correction algorithm was tested. The standard deviation of the misalignments and roll angles are summarized in Table 2. These errors were randomly distributed via a Gaussian Distribution truncated at 2.5 sigma. In addition to misaligning magnets, BPM roll errors were introduced through a rotation of of the coordinate system before and after each BPM.



(a)



(b)



(c)

Figure 2: Distribution of (a) horizontal emittances, (b) vertical emittances, and (c) coupling ratios of the corrected t $\bar{t}$ bar lattices. The errors introduced are summarized in Table 2.

After applying the correction scheme, the final emittances can be greatly reduced. Figure 2 shows the distributions of horizontal and vertical emittances, and coupling ratios for the corrected t $\bar{t}$ bar (182.5 GeV) 4IP lattices. The mean value of the vertical emittances achieved is 0.189 pm-rad, and the mean coupling ratio is  $\varepsilon_y/\varepsilon_x = 0.011\%$ .

The scale of the vertical dispersion before correction is extremely large, with the Root Mean Square (RMS) value before correction being 60.03 m. After applying the series of correction methods, the final RMS vertical dispersion at full sextupole strength is 0.114 mm. Figure 3a shows the vertical dispersion for one seed before correction with sextupoles turned off. This can be compared to Fig. 3b, which shows the final vertical dispersion after correction strategy has been applied with sextupoles at their design value.

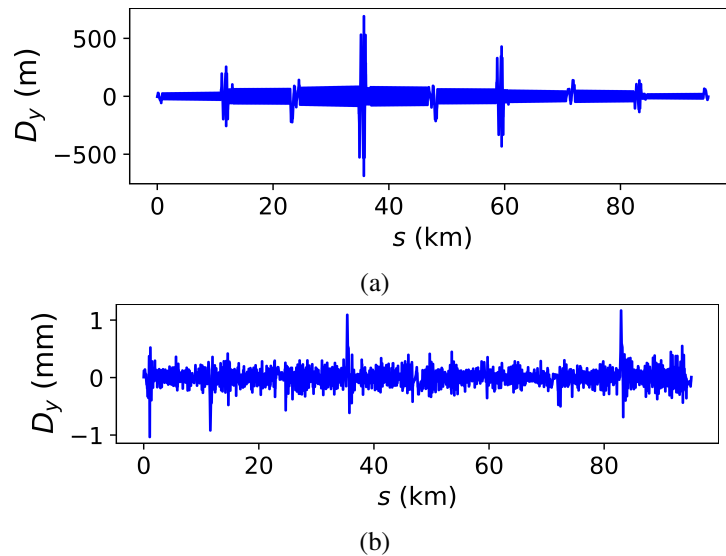


Figure 3: Vertical dispersion around the ring for one seed. a) Vertical dispersion after errors introduced, with sextupoles turned off. b) Vertical dispersion after errors introduced and correction strategy applied, with sextupoles at full strength.

#### 4. Conclusion

FCC-ee presents unique challenges when it comes to emittance tuning. The large ring size, the small vertical emittance and the low coupling ratio makes the FCC-ee design particularly susceptible to misalignment and field errors. For magnet misalignments with a standard deviation of  $100\ \mu\text{m}$  in the transverse planes and a roll angle error of  $100\ \mu\text{rad}$  for all magnet types, and with relative BPM errors misalignment with a standard deviation of  $40\ \mu\text{m}$  and BPM roll angles of  $100\ \mu\text{rad}$ , the average final vertical emittance achieved for the ttbar lattice, after correction is  $0.189\ \text{pm}\cdot\text{rad}$  and the average coupling ratio  $\varepsilon_y/\varepsilon_x = 0.011\ \%$ .

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