

Perturbations for Slotheon Field Dark Energy and its Evolution

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We consider in this work a Slotheon scalar field for Dark Energy. A Slotheon field is inspired by extra dimensional models of Dvali, Gabadadze and Porrati (DGP) related to brane world. In the decoupling limit of this model, the DGP theory in Minkowski space-time is described by a scalar field that obeys a shift symmetry. When extended to curved space time this scalar field is termed as Slotheon field. In this work we calculate general relativistic perturbations by deriving the Dark Energy and matter perturbation equations and solve them numerically. The results are then compared with similar quantities derived from Λ CDM model. It appears that the Slotheon field model for Dark Energy is more akin to Dark energy in Λ CDM model than that in standard Quintessence model. A comparison has also been made between the matter power spectra obtained from Slotheon model and Λ CDM model.

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1. Introduction

The source of Dark Energy that constitutes about 68.5% of the total content of the Universe and is thought to have caused the late time acceleration of the Universe still remains an enigma. Although the cosmological constant Λ is popularly used to interpret Dark Energy but it is associated with cosmic coincidence problem [1]. Also, if the cosmological constant is attributed to the vacuum energy then a fine tuning of the order of 10^{121} is required to be made. The other favourite interpretation of Dark Energy is the quintessence Dark Energy [2] where, unlike the cosmological constant the Dark Energy is considered to be varying with time and generally formulated by considering a scalar field ϕ with the potential $V(\phi)$ whereby the change of ϕ is very slow as time progresses.

In this work we also consider a scalar field driven varying Dark Energy that causes the late time acceleration of the Universe but in this case the origin of the scalar field is from an extra dimensional model. In particular we consider a scalar field arising out of the Dvali, Gabadadze and Porrati (DGP) [3] extra dimensional model that originates from the braneworld. The basic concept of this model is related to the understanding that the localised gravity in four dimensions (brane) weakens at large distances. As the Planck's mass $M_{\text{pl}} \rightarrow \infty$ and when the crossover scale r_c for transition from four dimensions to five dimensions also tends to infinity ($r_c \rightarrow \infty$), the theory can be described by a scalar field π [4, 5] with a shift symmetry $\pi \rightarrow \pi + a + b_\mu x^\mu$, where a and b_μ represent a constant and a constant vector respectively. The scalar field (in Minkowski space) which describes this theory in curved spacetime is termed as Slotheon scalar field. In this work the Dark Energy and their perturbations as well as the late time acceleration are addressed with this Slotheon field model.

The inhomogeneities in Dark Energy are expressed in terms of the Dark Energy perturbations. The perturbations are important to obtain the matter power spectrum. We calculate these perturbations as also their evolutions for the Slotheon Dark Energy models and obtain the power spectrum.

In section 2 we discuss the formalism of the background evolutions and the evolutions of perturbations for Slotheon field Dark Energy. In section 3 the results are discussed. Finally a summary is given in section 4.

2. Background Evolution and Perturbation of Slotheon Field Dark Energy

The Lagrangian density of the Slotheon field π is written in terms of the covariant derivative, $\pi_{;\mu}$ of π , the Einstein's tensor $G_{\mu\nu}$, the potential $V(\pi)$ and by adding the Einstein-Hilbert term. Note that this field π moves slower than the ordinary scalar in scalar field theory and hence the nomenclature. The action for the Slotheon field is given as [6]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \left(M_{\text{pl}}^2 R - \left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2} \right) \pi_{;\mu} \pi_{;\nu} \right) - V(\pi) \right] + S_m \left[\psi_m; e^{2\beta\pi/M_{\text{pl}}} g_{\mu\nu} \right], \quad (2.1)$$

where β is a dimensionless coupling with matter field and field π . By varying the above action the equations of motion for the field π are obtained as

$$M_{\text{pl}}^2 G_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(\pi)}, \quad (2.2)$$

$$\square\pi + \frac{1}{M^2} \left[\frac{R}{2} \square\pi - R^{\mu\nu} \pi_{;\mu\nu} \right] - V_\pi = -\frac{\beta}{M_{\text{pl}}} T^{(m)}, \quad (2.3)$$

where the radiation component of the energy momentum tensor $T_{\mu\nu}$ is not included. The above symbols above have their usual significance. The energy momentum tensor for field π , $T_{\mu\nu}(\pi)$ is given by

$$\begin{aligned} T_{\mu\nu}^{(\pi)} = & \pi_{;\mu}\pi_{;\nu} - \frac{1}{2}g_{\mu\nu}(\nabla\pi)^2 - g_{\mu\nu}V(\pi) + \frac{1}{M^2} \left(\frac{1}{2}\pi_{;\mu}\pi_{;\nu}R - 2\pi_{;\alpha}\pi_{;(\mu}R_{\nu)}^{\alpha} \right. \\ & + \frac{1}{2}\pi_{;\alpha}\pi^{;\alpha}G_{\mu\nu} - \pi^{;\alpha}\pi^{;\beta}R_{\mu\alpha\nu\beta} - \pi_{;\alpha\mu}\pi_{;\nu}^{\alpha} + \pi_{;\mu\nu}\pi_{;\alpha}^{\alpha} \\ & \left. + \frac{1}{2}g_{\mu\nu} \left(\pi_{;\alpha\beta}\pi^{;\alpha\beta} - (\pi_{;\alpha}^{\alpha})^2 + 2\pi_{;\alpha}\pi_{;\beta}R^{\alpha\beta} \right) \right). \end{aligned} \quad (2.4)$$

In the above $R_{\mu\alpha\nu\beta}$ denotes a Riemann curvature tensor. For a spatially flat background, assuming the coupling $\beta = 0$ the equations of motion are now written as

$$3M_{\text{pl}}^2 H^2 = \rho_m + \frac{\dot{\pi}^2}{2} + \frac{9H^2 \dot{\pi}^2}{2M^2} + V(\pi), \quad (2.5)$$

$$M_{\text{pl}}^2 (2\dot{H} + 3H^2) = -\frac{\ddot{\pi}^2}{2} + V(\pi) + (2\dot{H} + 3H^2) \frac{\dot{\pi}^2}{2M^2} + \frac{2H\dot{\pi}\ddot{\pi}}{M^2}, \quad (2.6)$$

$$0 = \ddot{\pi} + 3H\dot{\pi} + \frac{3H^2}{M^2} \left(\ddot{\pi} + 3H\dot{\pi} + \frac{2\dot{H}\dot{\pi}}{H} \right) + V_{\pi}. \quad (2.7)$$

The form of the potential $V(\pi)$ is taken as $V(\pi) = V_0 \exp\left(-\frac{\lambda\pi}{M_{\text{pl}}}\right)$ where λ is a constant.

In order to compute the perturbations for the Slotheon field π we first write the scalar perturbed metric in Newtonian gauge as [7]

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 + 2\Psi)\delta_{ij}dx^i dx^j, \quad (2.8)$$

where Φ and Ψ are the gravitational potential and spatial curvature perturbation respectively and $a(t)$ is the scale factor. For no anisotropic stress $\Phi = -\Psi$ and considering the matter to be a perfect fluid we have $T_{\nu}^{\mu} = (\rho + p)u^{\mu}u_{\nu} + p\delta_{\nu}^{\mu}$. The quantities such as density, pressure, velocity and field etc. can be expressed as the sum of their respective unperturbed components and the perturbations as given in the following equations

$$\rho(t, \vec{x}) = \bar{\rho}(t) + \delta\rho(t, \vec{x}), \quad (2.9)$$

$$p(t, \vec{x}) = \bar{p}(t) + \delta p(t, \vec{x}), \quad (2.10)$$

$$u^{\mu} = \bar{u}^{\mu} + v^{\mu}, \quad (2.11)$$

$$\pi(t, \vec{x}) = \bar{\pi}(t) + \delta\pi(t, \vec{x}) \quad (2.12)$$

and similarly the energy momentum tensor T_{ν}^{μ} is expressed as $T_{\nu}^{\mu} = \bar{T}_{\nu}^{\mu} + \delta T_{\nu}^{\mu}$. For the Slotheon field tensor $T_{\mu\nu}^{\pi}$, the perturbations are obtained as

$$\begin{aligned} \delta\rho_{\pi} = & -\frac{1}{a^3 M^2} \left[-a^3 M^2 \delta\pi V_{\pi} + \dot{\pi} (2\dot{a}\nabla^2(\delta\pi) - \delta\dot{\pi}(a^3 M^2 + 9a\dot{a}^2) \right. \\ & \left. + \dot{\pi}a (a^2 M^2 \Phi - \nabla^2\Phi + 9\dot{a}(a\dot{\Phi} + 2\Phi\dot{a})) \right], \end{aligned} \quad (2.13)$$

$$(\bar{\rho}_{\pi} + \bar{p}_{\pi})v_i = -\frac{\dot{\pi}}{a^2 M^2} \left[-2a\delta\dot{\pi}\dot{a} + \delta\pi(a^2 M^2 + 3\dot{a}^2) + a(a\dot{\Phi} + 3\Phi\dot{a})\dot{\pi} \right] |_{i}, \quad (2.14)$$

$$\begin{aligned} \delta p_\pi = \frac{1}{a^3 M^2} & \left[-a^3 M^2 \delta \pi V_\pi + \dot{\pi} (-2a^2 \delta \dot{\pi} \dot{a} + a(-a^2 M^2 \Phi + a^2 \ddot{\Phi} + \nabla^2 \Phi - \Phi_{ii}) \dot{\pi} \right. \\ & + \dot{a}(\nabla^2(\delta \pi) - \delta \pi_{ii} + 2a(3a\dot{\Phi} + \Phi\dot{a})\dot{\pi}) + a\delta\dot{\pi}(-\dot{a}^2 + a(aM^2 - 2\ddot{a})) \\ & \left. + 4\Phi\dot{\pi}\ddot{a} + a(\nabla^2 \delta \pi - \delta \pi_{ii} - 2a\delta\dot{\pi}\dot{a} + 2a(a\dot{\Phi} + 4\Phi\dot{a})\dot{\pi})\dot{\pi} \right]. \end{aligned} \quad (2.15)$$

In order to obtain the equations of motion for perturbations, we need to solve the Einstein's equation $\delta G_V^\mu = 8\pi G \delta T_V^\mu$ for perturbed quantities. Working in the Fourier space (replacing ∇^2 by $-k^2$) the perturbed equations of motion are written as [8]

$$3H^2\Phi + 3H\dot{\Phi} + \frac{k^2\Phi}{a^2} = -4\pi G \sum_i \delta \rho_i, \quad (2.16)$$

$$k^2(\dot{\Phi} + H\Phi) = 4\pi G a \sum_i (\bar{\rho}_i + \bar{p}_i)\theta_i, \quad (2.17)$$

$$\ddot{\Phi} + 4H\dot{\Phi} + 2\dot{H}\Phi + 3H^2\Phi = 4\pi G \sum_i \delta p_i. \quad (2.18)$$

Note that we have not changed the notations for Φ , $\delta \rho_i$, δp_i etc. for brevity. In the above, the summation is over matter perturbations and the Dark Energy perturbations due to Slotheon field and $k = \frac{2\pi}{\lambda_p}$ where λ_p is the length scale of the perturbation. Also the perturbation $\delta \pi$ of the Slotheon field π is obtained as (using Eqs. 2.1, 2.8)

$$\begin{aligned} \frac{1}{M^2} & \left[H\dot{\pi} \frac{k^2\Phi}{a} + 18\Phi\dot{a}^3\dot{\pi} - 2\dot{a}k^2\delta\pi + 2\dot{a}\dot{\pi}k^2\Phi - 3\dot{a}^3\delta\dot{\pi} + H^2a(-k^2\delta\pi) + \right. \\ & \frac{\dot{\pi}k^2\dot{\Phi}}{a} - a^3M^2V_{\pi\pi}\delta\pi + 2M^2a^3\Phi(V_\pi + 2\dot{\pi}) + 4a^3M^2\dot{\pi}\Phi + \dot{\pi}a^3k^2\dot{\Phi} - \\ & a^3M^2\delta\dot{\pi} + 36a^2\Phi\dot{\pi}H\ddot{a} + 18\Phi a^3H^2\ddot{\pi} - M^2ak^2\delta\pi + 2\dot{\pi}ak^2\Phi - 6a^2H\ddot{a}\delta\pi + \\ & 30a^3H^2\dot{\pi}\Phi + 2\dot{\pi}ak^2\dot{\Phi} - 3a^3H^2\delta\dot{\pi} + 12a^3M^2\Phi H\dot{\pi} + 6\dot{\pi}a^2\ddot{a}\dot{\Phi} - \\ & \left. a^3H(3M^2\delta\dot{\pi} + 6\dot{\pi}\dot{\Phi} - \dot{\pi}k^2\Phi - 6\dot{\pi}\ddot{\Phi}) \right] = 0. \end{aligned} \quad (2.19)$$

3. Numerical Solutions of Perturbation Equations and Results

The linearised perturbation equations (Eqs. 2.16 - 2.19) are now solved numerically along with the background equations (Eqs. 2.5 - 2.7) to obtain the evolutions of the cosmological density parameter (Ω), the Dark Energy equation of state (ω), Dark Energy density fluctuations $\delta_\pi (= \frac{\delta \rho_\pi}{\bar{\rho}_\pi})$ and matter density fluctuations $\delta_m (= \frac{\delta \rho_m}{\bar{\rho}_m})$. To this end certain dimensionless variables defined as $x = \frac{\dot{\pi}}{\sqrt{6HM_{\text{pl}}}}$, $y = \frac{\sqrt{V(\pi)}}{\sqrt{3HM_{\text{pl}}}}$, $\lambda = -M_{\text{pl}} \frac{V_\pi}{V(\pi)}$, $\varepsilon = \frac{H^2}{2M^2}$, $q = \delta\pi / \frac{d\pi}{dN}$ ($N = \ln(a)$ is the number of e-foldings), will be useful. The equations are then written in terms of these dimensionless variables.

Thus we obtain the equation of state and density parameters as

$$\Omega_\pi = y^2 + x^2(1 + 18\varepsilon), \quad (3.1)$$

$$\Omega_m = 1 - y^2 - x^2(1 + 18\varepsilon), \quad (3.2)$$

$$\omega_{\text{eff}} = \frac{p_{\text{total}}}{\rho_{\text{total}}} = -1 - \frac{2\dot{H}}{3H^2} \quad (3.3)$$

$$\omega_\pi = \frac{\omega_{\text{eff}}}{\Omega_\pi}. \quad (3.4)$$

In the above $\Omega = \frac{\bar{\rho}_i}{\rho_c}$ ($i = \pi, m$ etc. and ρ_c is the critical density of the Universe), $p_{\text{total}} = p_m + p_\pi$, $\rho_{\text{total}} = \rho_m + \rho_\pi$ and ω_π is the Slotheon field Dark Energy equation of state (EOS) parameter (ω_{eff} being the effective equation of state).

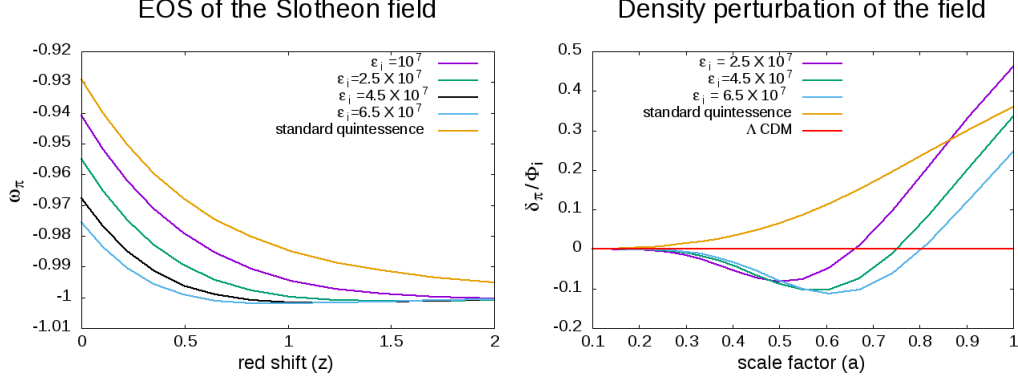


Figure 1: (a) Variations of Slotheon Dark Energy equation of state parameters (b) Evolutions of Slotheon field density fluctuations.

In Fig. 1(a) the variations of ω_π with z are shown while in Fig. 1(b) we plot the evolutions of perturbations of Slotheon field δ_π with a . The Slotheon field EOS is different from standard quintessence for certain chosen initial values ϵ_i of the parameter ϵ but it tends to Λ CDM for higher ϵ_i . It may be noted here that higher values of ϵ_i indicate greater affinity towards the Slotheon field Dark Energy. Similar trends are also observed for the evolution of perturbation for Slotheon field π from Fig. 1(b).

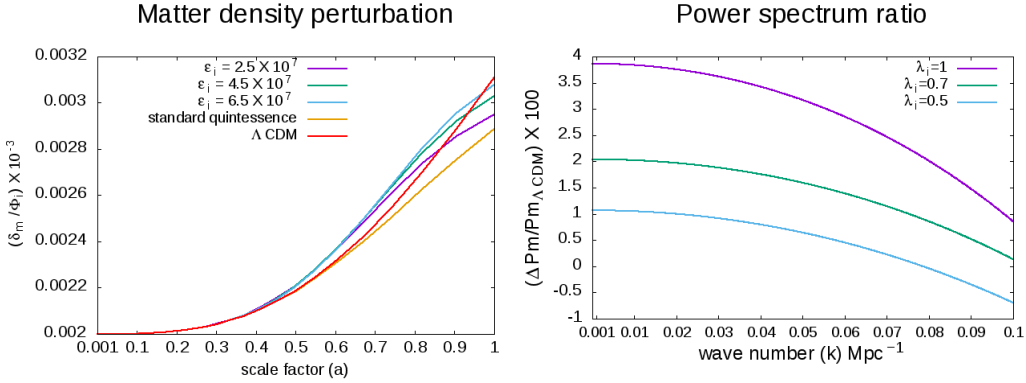


Figure 2: (a) Evolutions of matter density perturbations considering the Slotheon field. (b) Matter power spectrum in presence of Slotheon field. Shown is the difference (in %) of the power spectra for the Λ CDM model and Slotheon field model and their variations.

In Fig. 2(a) we show the variations of matter perturbations δ_m with the scale factor a . Although initially the δ_m calculated with Slotheon field coincides with those obtained considering Λ CDM and standard quintessence models, the former (Slotheon) deviates from both the latter models with the advent of Dark Energy domination, as a increases.

We also compute the matter power spectrum for the Slotheon field and compare it with that of

Λ CDM. The matter power spectrum is defined in terms of matter density fluctuations δ_m as

$$Pm = \langle |\delta_m(k, a)|^2 \rangle . \quad (3.5)$$

In order to compare Pm for Slotheon field with that of Λ CDM model we define a quantity X as

$$X = \frac{Pm_{\Lambda\text{CDM}} - Pm_{\text{slotheon}}}{Pm_{\Lambda\text{CDM}}} \times 100 = \frac{\Delta Pm}{Pm_{\Lambda\text{CDM}}} \times 100 . \quad (3.6)$$

In Fig. 2(b) we plot the variations of X with wave number k for three different initial values λ_i of the parameter λ . It appears from Fig. 2(b) that the percentage difference is more for smaller wave number but it reduces as the value of k increases. Also X decreases with the decrease of λ_i . This can be explained from the fact that as λ_i decreases the potential $V(\pi)$ tends to be more flat and thereby approaches Λ CDM model.

4. Summary

Within the framework of Slotheon Dark Energy model (inspired by Dvali, Gabadadze and Porrati (DGP) model related to extra dimension) we explore the Dark Energy perturbations and growths with redshift and compare them with those for Λ CDM model and standard quintessence model. We have also computed the matter power spectrum for Slotheon scalar field model and comparisons have been made with the power spectrum when Λ CDM model is considered. we find that in certain circumstances the Slotheon field model is more akin to the Λ CDM model results. Similar trend is also observed for the case of power spectrum.

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