

Precision predictions for $B \rightarrow \rho \tau \nu_\tau$ and $B \rightarrow \omega \tau \nu_\tau$ in the SM and beyond

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We present new precision predictions for semitauonic decays involving ρ and ω final state mesons. These decay channels offer an interesting orthogonal probe to study the existing B anomalies in semitauonic transitions and are accessible with the Belle II experiment. The predictions are based on combining existing light-cone sum-rule calculations for the form factors with measured experimental spectra from the BaBar and Belle collaborations. This allows us to reliably extrapolate the light-lepton form factor predictions to large values of the four-momentum transfer squared, q^2 , and in turn to derive precise predictions for R_ρ and R_ω , the ratio of the total decay rates of $B \rightarrow \rho \tau \nu_\tau$ and $B \rightarrow \omega \tau \nu_\tau$ for τ final states with respect to light leptons in the SM. In addition, we investigate the impact of all four-fermi operators on the semitauonic q^2 spectra and these ratios.

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1. Introduction

The effective Standard Model (SM) Lagrangian describing semileptonic $b \rightarrow u \ell \nu_\ell$ transitions is given by

$$\mathcal{L}_{\text{eff}} = \frac{-4G_F}{\sqrt{2}} V_{ub} (\bar{u} \gamma_\mu P_L b) (\bar{\nu} \gamma^\mu P_L \ell) + \text{h.c.}, \quad (1.1)$$

with Fermi's constant G_F , the CKM matrix element V_{ub} and the projection operator $P_L = (1 - \gamma_5)/2$. For resonant final states the hadronic matrix element for the $b \rightarrow u$ transition can be written as

$$\langle V(p_V) | \bar{u} \gamma^\mu P_L b | B(p_B) \rangle = \sum T_i^\mu F_i(q^2), \quad (1.2)$$

where $q = p_B - p_V$ is the four-momentum transfer in the decay and V denotes any light final state meson (we will discuss $V \in \{\rho, \omega\}$). Further, the T_i denote tensorial structures of the involved 4-momenta and polarizations, and the F_i form factors. The sum runs over all allowed tensorial structures. The decay $B \rightarrow V \ell \nu_\ell$ is shown at parton level and as an effective diagram in Figure 1. In the right diagram the arms are described by the tensorial structures T_i and the blob is described by the form factors F_i . The form factors F_i present in Equation 1.2 cannot be calculated with perturbation theory in the strong coupling constant and have to be determined using non-perturbative methods.

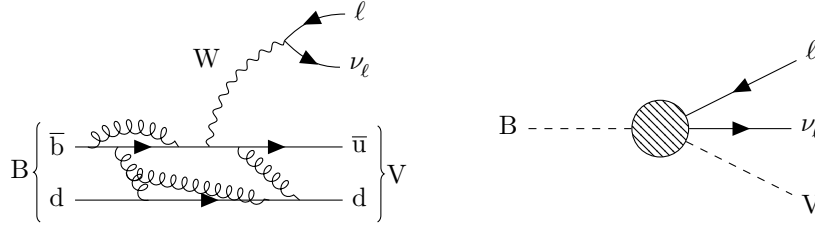


Figure 1: One possible parton level Feynman diagram (a) and the effective Feynman diagram (b). In the effective Feynman diagram, the propagator of the W is integrated out, i.e. the weak interaction is point-like, and the gluon interactions are described by the blob.

The Bourely-Caprini-Lellouch (BCL) parametrization [1] is a model-independent ansatz for the form factors based on a fast converging series expansion of

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

$$\text{with } t_+ = (m_B + m_V)^2,$$

$$\text{and } t_0 = (m_B + m_V)(\sqrt{m_B} - \sqrt{m_V})^2,$$

where m_B is the B meson mass and m_V is the mass of the final state meson. The form factors are expanded as:

$$F_i(q^2) = P_i(q^2) \sum_k \alpha_k^i \left(z(q^2) - z(0) \right)^k,$$

$$\text{with } P_i(q^2) = (1 - q^2/m_R^2)^{-1},$$

where m_R is the mass of first resonance in the spectrum. For the transition into vector-like particles, i.e. $B \rightarrow V \ell \nu_\ell$, there exist a total of 8 independent form factors: $A_P, V, A_0, A_1, A_{12}, T_1, T_2, T_{23}$. The pseudoscalar form factor A_P can be removed using the equations of motion:

$$A_P = -2 \frac{m_M}{m_b + m_u} A_0.$$

For the SM process, which is governed by a V-A current, only the vector (V) and axialvector (A_0, A_1, A_{12}) form factors contribute. Tensor currents can arise in models beyond the SM, and the three tensor (T_1, T_2, T_{23}) form factors can contribute. It is convenient to express the V-A form factors in the helicity basis with the helicity amplitudes [2]:

$$\begin{aligned} H_\pm(q^2) &= \sqrt{\lambda(q^2)} \frac{V(q^2)}{m_B + m_V} \pm (m_B + m_V) A_1(q^2), \\ H_0(q^2) &= \frac{8m_B m_V}{\sqrt{q^2}} A_{12}(q^2), \\ H_s(q^2) &= \frac{\sqrt{\lambda(q^2)}}{\sqrt{q^2}} A_0(q^2). \end{aligned} \quad (1.3)$$

The SM differential rate is then given by

$$\begin{aligned} \frac{d\Gamma}{dq^2} &= |V_{ub}| \frac{G_F^2}{192\pi^3 m_B^3} q^2 \sqrt{\lambda(q^2)} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \\ &\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) \left(H_+^2(q^2) + H_-^2(q^2) + H_0^2(q^2)\right) + \frac{3m_\ell^2}{2q^2} H_s^2(q^2) \right], \end{aligned} \quad (1.4)$$

with the Kaellen function $\lambda(q^2) = \left((m_B + m_V)^2 - q^2\right) \left((m_B - m_V)^2 - q^2\right)$, where m_V denotes the mass of the final state meson. The differential rate can be simplified for light leptons $\ell = e, \mu$ with the zero mass approximation $m_\ell = 0$:

$$\frac{d\Gamma}{dq^2} = V_{ub} \frac{G_F^2}{192\pi^3 m_B^3} q^2 \sqrt{\lambda(q^2)} \left[H_+^2(q^2) + H_-^2(q^2) + H_0^2(q^2) \right]. \quad (1.5)$$

The available theory calculations from light-cone sum-rules (LCSR) [3] for the coefficients of the BCL expansion are only valid in a q^2 region up to about 14 GeV^2 . For the $q^2 > 14 \text{ GeV}^2$ region the prediction solely relies on extrapolation. In the following we show how the precision in the region $q^2 > 14 \text{ GeV}^2$ can be improved by combining the theory prediction with experimental data.

2. Data-Theory Fit

The values of the form factors F_i are evaluated at three distinct points in the spectrum: $q^2 = [0, 7, 14] \text{ GeV}^2$ with the given BCL coefficients from the LCSR calculation. The resulting vector is

named \mathbf{f}_{LCSR} . Their correlation \mathbf{C}_{LCSR} is determined by generating random samples from a multivariate normal distribution using the covariance matrix from the LCSR calculation. The associated χ_{LCSR}^2 term is constructed as

$$\chi_{\text{LCSR}}^2 = \Delta \mathbf{f}_t^T \mathbf{C}_{\text{LCSR}}^{-1} \Delta \mathbf{f}_t, \quad (2.1)$$

with $\Delta \mathbf{f}_t = \mathbf{f}(\mathbf{a}) - \mathbf{f}_{\text{LCSR}}$ denoting the vectorial difference involving a prediction vector $\mathbf{f}(\mathbf{a})$ depending on the expansion coefficients \mathbf{a} . The χ^2 terms for the individual experimental measurements of References [4, 5, 6] are constructed as

$$\chi_{\text{exp}}^2(\mathbf{a}) = \Delta \Gamma_e^T C_{\text{exp}}^{-1} \Delta \Gamma_e, \quad (2.2)$$

with $\Delta \Gamma_e = \Gamma(\mathbf{a}) - \Gamma_{\text{exp}}$ denoting the vectorial difference of the predicted and measured rates. The global χ^2 function takes the form

$$\chi^2(\mathbf{a}) = \chi_{\text{LCSR}}^2(\mathbf{a}) + \sum_{\text{exp}} \chi_{\text{exp}}^2(\mathbf{a}). \quad (2.3)$$

The form factors pre- and post-fit are given in Figure 2. It is clearly visible that corrections on the form factors can be extracted from the experimental measurements.

3. Predicting R_ρ and R_ω

3.1 SM Prediction

We predict R_ρ and R_ω in two different q^2 intervals:

- $q^2 \in [0, q_{\text{max}}^2]$ GeV. The ratio is independent of the shape of the differential distributions.
- $q^2 \in [m_\tau^2, q_{\text{max}}^2]$ GeV. The evaluation of the differential rate in the same q^2 region for tauonic and semileptonic final states increases the correlation between nominator and denominator in the ratio and thus larger cancellation of uncertainties is possible. However, the prediction then depends on the actual shape of the light-lepton differential rate, due to the introduced cut.

We define R and \hat{R} as:

$$R_V = \frac{\int_{m_\tau^2}^{q_{\text{max}}^2} \frac{d\Gamma(B \rightarrow V \tau \nu)}{dq^2} dq^2}{\int_0^{q_{\text{max}}^2} \frac{d\Gamma(B \rightarrow V \ell \nu)}{dq^2} dq^2},$$

$$\hat{R}_V = \frac{\int_{m_\tau^2}^{q_{\text{max}}^2} \frac{d\Gamma(B \rightarrow V \tau \nu)}{dq^2} dq^2}{\int_{m_\tau^2}^{q_{\text{max}}^2} \frac{d\Gamma(B \rightarrow V \ell \nu)}{dq^2} dq^2}.$$

The predictions for R_V and \hat{R}_V are tabulated in Table 1.

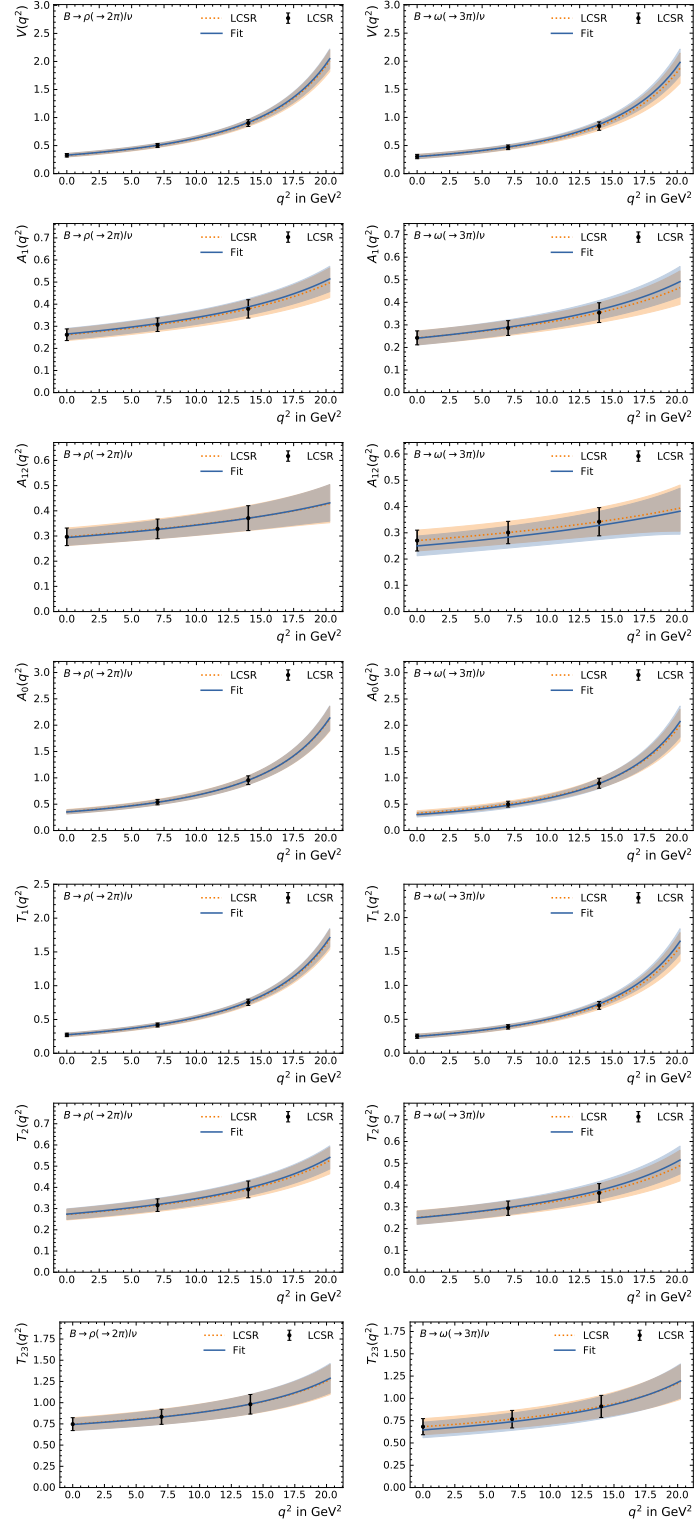


Figure 2: Pre- and post-fit form factors. It is clearly visible that corrections on the form factors can be extracted from a combined theory and data fit.

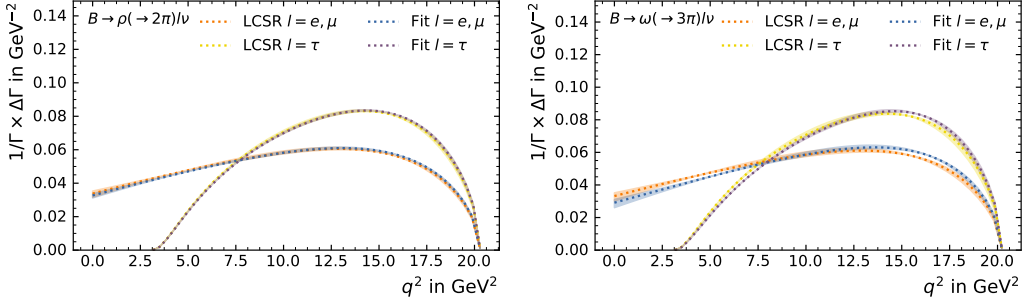


Figure 3: The normalized q^2 spectra for $B \rightarrow \rho \ell \nu$ and $B \rightarrow \rho \tau \nu$ (left) and $B \rightarrow \omega \ell \nu$ and $B \rightarrow \omega \tau \nu$ (right) with the form factor expansion coefficients from light cone sum rules (dotted orange and yellow) and with the expansion coefficients extracted from the theory plus data fit (dotted blue and purple).

R_V	LCSR	Fit	Improvement
R_ρ	0.532 ± 0.011	0.535 ± 0.008	25%
R_ω	0.534 ± 0.018	0.546 ± 0.015	16%
\hat{R}_ρ	0.605 ± 0.007	0.606 ± 0.006	6%
\hat{R}_ω	0.606 ± 0.012	0.612 ± 0.011	7%

Table 1: The predictions for the ratio of tauonic to leptonic final states. The column ‘LCSR’ uses only the theory prediction for the form factors from [3]. The column ‘Fit’ uses the coefficients extracted from the fit described in Section 2.

3.2 New Physics Contribution to R_ρ and R_ω

The following predictions assume that new physics only contributes to the heavy leptons, i.e. the light leptons are free from any new physics contribution. The complete basis of the four-Fermi operators mediating the $b \rightarrow q \tau \nu_\tau$ is given by

$$2i\sqrt{2}V_{ub}G_F[\bar{q}\chi_j^i\gamma^\mu P_j b][\bar{\ell}\lambda_l^k\gamma_\mu P_l \nu], \quad (3.1)$$

where χ_j^i and λ_l^k are the new physics coupling constants to the quark and lepton current, respectively. These new physics couplings are normalized to the SM coupling strengths. Furthermore, the indices $j, l = \{L, R\}$ denote the helicity of the b quark and the neutrino. The indices $i, k = \{S, V, T\}$ indicate that the current of the interaction is either a scalar, vector, or tensor current. The influence of the new physics contribution on R_ρ and R_ω is the same, except for mass differences, since both are vector-like particles. The influence of new physics on the ratio R for each individual coupling is shown in Figure 4.

4. Summary and Outlook

We have demonstrated that we were able to improve the precision of BCL expansion coefficients by combining theory predictions with experimental measurements. This leads to more precise predictions of R_ρ and R_ω and \hat{R}_ρ and \hat{R}_ω , reducing the uncertainties by 20% and 7%, re-

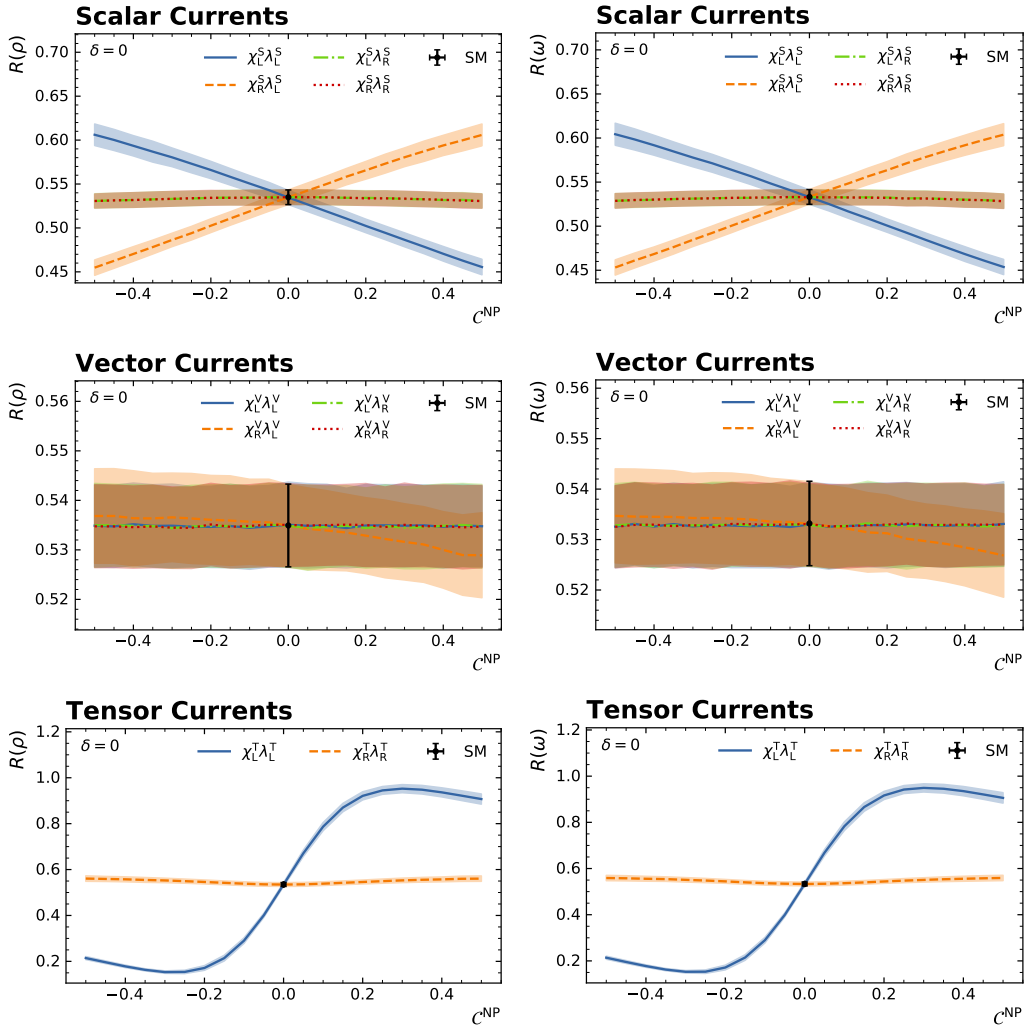


Figure 4: The impact of a new physics current to the ratio of taonic to leptonic final states in different new physics scenarios. The slight differences between the ρ and ω final states originates solely from the kinematic differences in the decay. The influence of new physics itself is the same, due to their identical vector-particle nature.

spectively, for R and \hat{R} . In addition, we investigated the impact of new physics contributions on R for all four-Fermi operators.

References

- [1] C. Bourrely, I. Caprini and L. Lellouch, *Model-independent description of $B \rightarrow \pi \ell \nu$ decays and a determination of $|V(ub)|$* , *Phys. Rev.* **D79** (2009) 013008 [0807.2722].
- [2] The helicity amplitudes are simplified by omitting the complex i .
- [3] A. Bharucha, D. M. Straub and R. Zwicky, *$B \rightarrow V \ell^+ \ell^-$ in the Standard Model from light-cone sum rules*, *JHEP* **08** (2016) 098 [1503.05534].

- [4] BELLE collaboration, *Study of Exclusive $B \rightarrow X_u \ell \nu$ Decays and Extraction of $\|V_{ub}\|$ using Full Reconstruction Tagging at the Belle Experiment*, *Phys. Rev.* **D88** (2013) 032005 [1306.2781].
- [5] BABAR collaboration, *Study of $B \rightarrow \pi \ell \nu$ and $B \rightarrow \rho \ell \nu$ Decays and Determination of $|V_{ub}|$* , *Phys. Rev.* **D83** (2011) 032007 [1005.3288].
- [6] BABAR collaboration, *Branching fraction measurement of $B^+ \rightarrow \omega \ell^+ \nu$ decays*, *Phys. Rev.* **D87** (2013) 032004 [1205.6245].