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On the spin correlations of final leptons produced in the annihilation processes $e^+e^- \to \mu^+\mu^-, e^+e^- \to \tau^+\tau^-$

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The electromagnetic processes of annihilation of (e^+e^-) pairs, generated in various high-energy reactions and decays, into heavy flavor lepton pairs are theoretically studied in the one-photon approximation, applying the technique of helicity amplitudes . For the process $e^+e^- \to \mu^+\mu^-$, it is shown that – in the case of the unpolarized electron and positron – the final muons are also unpolarized but their spins are strongly correlated. For the final $(\mu^+\mu^-)$ system, the structure of triplet states is analyzed and explicit expressions for the components of the spin density matrix and correlation tensor are derived; besides, the formula for angular correlation at the decays of final muons μ^+ and μ^- is obtained.

It is demonstrated that here the spin correlations of muons have the purely quantum character, since one of the Bell-type incoherence inequalities for the correlation tensor components is always violated (i.e. there is always one case when the modulus of sum of two diagonal components exceeds unity). Besides, the additional contribution of the weak interaction of lepton neutral currents through the virtual Z^0 boson is considered in detail, and it is established that, when involving the weak interaction contribution, the qualitative character of the muon spin correlations does not change.

Analogous analysis can be wholly applied as well to the annihilation process with the formation of a tau-lepton pair ($e^+e^- \to \tau^+\tau^-$), which becomes possible at considerably higher energies.

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1. Helicity amplitudes for the annihilation process $e^+e^- \to \mu^+\mu^-$ and structure of the triplet states of the final $(\mu^+\mu^-)$ system

In the first non-vanishing approximation over the electromagnetic constant $e^2/\hbar c$, the process of conversion of the (e^+e^-) pair into the muon pair is described by the well-known one-photon Feynman diagram [1]. Due to the electromagnetic current conservation, the virtual photon with a time-like momentum transfers the total angular momentum $J = 1$ and negative parity. Since the internal parities of muons μ^+ and μ^- are opposite, the $(\mu^+\mu^-)$ pair is generated in the triplet states (total spin $S = 1$) with $J = 1$ and the orbital angular momenta $L = 0$ and $L = 2$.

The respective helicity amplitudes have the following structure:

$$f_{\Lambda'\Lambda}(\theta,\phi) = R_{\Lambda'\Lambda}(E)\, d^{(1)}_{\Lambda'\Lambda}(\theta)\exp(i\Lambda\phi), \tag{1.1}$$

where θ, ϕ are the polar and azimuthal angles of the flight direction of the positive muon (μ^+) in the c.m. frame of the reaction with respect to the initial positron momentum; $d^{(1)}_{\Lambda'\Lambda}(\theta)$ are the Wigner functions for $J = 1$; Λ is the difference of helicities of the positron and electron, coinciding with the projections of total spin and total angular momentum of the (e^+e^-) pair onto the direction of positron momentum in the c.m. frame; Λ' is the difference of helicities of the muons μ^+ and μ^-, coinciding with the projection of total angular momentum of the $(\mu^+\mu^-)$ pair onto the direction of momentum of μ^+ in the c.m. frame (see, e.g., [1,2]).

In Eq. (1.1), $R_{\Lambda'\Lambda}(E) = r^{(\mu)}_{\Lambda'}(E)\, r^{(e)}_{\Lambda}(E)$ ($\Lambda', \Lambda = +1, 0, -1$) – due to the factorizability of the Born amplitude, and here $r^{(\mu)}_{+1} = r^{(\mu)}_{-1} = r^{(\mu)}_{1}, r^{(e)}_{+1} = r^{(e)}_{-1} = r^{(e)}_{1}$ – owing to the space parity conservation in the electromagnetic interactions . Further, in accordance with the structure of electromagnetic current for the pairs (e^+e^-) and $(\mu^+\mu^-)$ in the c.m. frame [1], the following relations are valid:

$$r^{(\mu)}_0 = \frac{m_\mu}{E} r^{(\mu)}_1 = \sqrt{1-\beta_\mu^2}\, r^{(\mu)}_1, \quad r^{(e)}_0 = \frac{m_e}{E} r^{(e)}_1, \tag{1.2}$$

where m_μ, m_e are the muon and electron masses and β_μ is the muon velocity in the c.m. frame. Thus, since we always have $E \geq m_\mu \gg m_e$ for the given process, the contribution of (e^+e^-)-states with equal helicities can be neglected, i.e. $R_{\Lambda 0}(E) \approx 0$.

The one-photon diagram calculation gives (here e is the electron charge) :

$$r^{(\mu)}_1(E) = r^{(e)}_1(E) = \frac{|e|}{\sqrt{2}E}\,. \tag{1.3}$$

Using the expressions (1.1) – (1.3) and the formulas for d–functions at $J = 1$ [1,2], we may find the effective cross section of the reaction $e^+e^- \to \mu^+\mu^-$ and the angular distribution of muon emission, normalized by unity, in the c.m. frame (see [3]) .

Taking into account Eqs. (1.1) – (1.3), it is clear that, in the cases of total polarization of both the positron and electron along the positron momentum in the c.m. frame and in the direction being antiparallel to the positron momentum, the $(\mu^+\mu^-)$ system is produced, respectively, in the triplet states of the following form [3] :

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2\sin^2\theta}}\left(\frac{1+\cos\theta}{2}|+1\rangle - \sqrt{1-\beta_\mu^2}\,\frac{\sin\theta}{\sqrt{2}}|\,0\,\rangle + \frac{1-\cos\theta}{2}|-1\rangle\right). \qquad (1.4)$$

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2\sin^2\theta}}\left(\frac{1-\cos\theta}{2}|+1\rangle + \sqrt{1-\beta_\mu^2}\,\frac{\sin\theta}{\sqrt{2}}|\,0\,\rangle + \frac{1+\cos\theta}{2}|-1\rangle\right). \qquad (1.5)$$

Here β_μ is the velocity of each of the muons – just as in Eq. (1.2), and $|+1\rangle, |-1\rangle, |\,0\,\rangle$ are the states with the projection of total spin of the $(\mu^+\mu^-)$ pair onto the direction of momentum of μ^+ in the c.m. frame, equaling $+1$, -1 and 0, respectively.

2. Correlation tensor of the $(\mu^+\mu^-)$ pair and violation of "classical" incoherence inequalities

If the positron and electron are not polarized, then, since $r_0^{(e)} \approx 0$, the final state of the $(\mu^+\mu^-)$ pair represents an incoherent mixture of spin states $|\Psi\rangle^{(+1)}$ (1.4) and $|\Psi\rangle^{(-1)}$ (1.5), each of them being realized with the relative probability of $1/2$.

The components of the correlation tensor for two particles with spin $1/2$ are defined as: $T_{ik} = \langle \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)} \rangle$ ($i,k \to \{1,2,3\} \to \{x,y,z\}$; the symbol $\langle ... \rangle$ denotes the averaging over the quantum ensemble). For the final $(\mu^+\mu^-)$ pair under consideration, the axis z is directed along the momentum of μ^+ in the c.m. frame of the reaction $e^+e^- \to \mu^+\mu^-$, and the axis y – along the normal to the reaction plane.

It is easy to see that, in the case of unpolarized primary positron and electron, the produced muons μ^+ and μ^- are also unpolarized but their spins are correlated, and the correlation tensor components have the following form [3] :

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2-\beta_\mu^2)\sin^2\theta}{2-\beta_\mu^2\sin^2\theta}, \quad T_{zz}^{(\mu^+\mu^-)} = \frac{2\cos^2\theta + \beta_\mu^2\sin^2\theta}{2-\beta_\mu^2\sin^2\theta},$$

$$T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2\sin^2\theta}{2-\beta_\mu^2\sin^2\theta}, \quad T_{xz}^{(\mu^+\mu^-)} = -\frac{(1-\beta_\mu^2)^{1/2}\sin 2\theta}{2-\beta_\mu^2\sin^2\theta}, \;\; T_{xy} = T_{yz} = 0\,. \qquad (2.1)$$

Just as it should hold for triplet states, the "trace" of the correlation tensor is equal to unity: $T^{(\mu^+\mu^-)} = T_{xx}^{(\mu^+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1$. Let us note that the "trace" of the correlation tensor T determines the angular correlation between flight directions for the products of decay of two unstable particles with spin $1/2$ under the space parity nonconservation (see [4-8]). In particular, for the decays of the muons μ^- and μ^+ ($\mu^- \to e^-\nu_\mu\bar{\nu}_e$, $\mu^+ \to e^+\bar{\nu}_\mu\nu_e$), produced in the process $e^+e^- \to \mu^+\mu^-$, we obtain [3] (in accordance with the general formula for angular correlation [4,5,7,8]) :

$$dW^{(\mu^+\mu^-)} = \frac{1}{2}\left(1 + \frac{\alpha_1\alpha_2 T}{3}\cos\delta\right) d(-\cos\delta) = \frac{1}{2}\left(1 - \frac{1}{27}\cos\delta\right) d(-\cos\delta). \qquad (2.2)$$

Here α_1 and α_2 are the angular asymmetry coefficients for the decays of the first (μ^-) and second (μ^+) particle (here we have: $\alpha_1 = -1/3$, $\alpha_2 = +1/3$ [9]), $\cos\delta = \mathbf{n}_1\mathbf{n}_2$, $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors along the momenta of particles formed in the first and second decay (in our case – e^- and e^+), which are defined, respectively, in the rest frames of the first and second unstable particle and specified with respect to a unified system of spatial coordinate axes [7,8] (for further details, see also [3]).

Regarding the components (2.1) of the correlation tensor, we observe here the violation of the "classical" incoherence inequalities (for incoherent mixtures of factorizable states of two particles with spin $1/2$, the modulus of sum of any two (and three) diagonal components of the correlation tensor cannot exceed unity [4,5]) . Indeed, according to Eqs. (2.1), one of the incoherence inequalities is always violated at $\theta \neq 0$ [3, 10–13] :

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1 - T_{yy}^{(\mu^+\mu^-)} = \frac{2}{2-\beta_\mu^2 \sin^2\theta} > 1. \tag{2.3}$$

Thus, we see that the spin correlations of muons in the process $e^+e^- \to \mu^+\mu^-$ have the strongly pronounced quantum character.

Certainly, the above consideration can be wholly applied also to the process $e^+e^- \to \tau^+\tau^-$ – with the replacements $m_\mu \to m_\tau$, $\beta_\mu \to \beta_\tau$.

At very high energies $E \gg m_\mu$ (m_τ), when $\beta_\mu, \beta_\tau \to 1$, the nonzero components of the correlation tensor for the final lepton pair take – in accordance with Eqs. (2.1) – the following values:

$$T_{xx} = \frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{yy} = -\frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{zz} = 1, \tag{2.4}$$

and we see that one of the incoherence inequalities is still violated : $T_{xx} + T_{zz} \geq 1$.

3. Incorporation of the weak interaction of lepton neutral currents through the virtual Z^0 boson

At very high energies the annihilation processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ are conditioned not only by the electromagnetic interaction through the virtual photon, but also by the weak interaction of neutral currents through the Z^0 boson [9].

The interference of amplitudes of the purely electromagnetic and weak interaction leads to the charge asymmetry in lepton emission and to the effects of space parity violation. In the framework of the standard model of electroweak interaction, at the electron-positron pair annihilation the pairs $\mu^+\mu^-$, $\tau^+\tau^-$ are produced in the states ${}^3S_1$, ${}^3D_1$ with the negative space parity and, due to the weak interaction, also in the state ${}^3P_1$ with the positive space parity. In doing so, the total angular momentum is $J = 1$ and CP parity of the pairs is positive.

If the weak interaction contribution is neglected, then the lepton pairs, generated at the annihilation of the unpolarized positron and electron, are correlated but unpolarized. Analysis shows that, due to the weak interaction through the exchange by the virtual Z^0 boson with the nonconservation of space parity, the final leptons acquire the longitudinal polarization. Since the lepton pairs are produced in the triplet states, the polarization vectors of the positively and negatively charged leptons are the same, and their average helicities $\lambda_+ = -\lambda_-$ have different signs in consequence of the opposite directions of momenta in the c.m. frame [3].

The structure of the correlation tensor of the final leptons is, on the whole, similar to that for the case of purely electromagnetic annihilation at very high energies ($\beta_\mu \to 1, \beta_\tau \to 1$). In doing so, the nonzero components of the correlation tensor are: $T_{zz} = 1$, $T_{xx} = -T_{yy}$, as before. Again one of the incoherence inequalities for the correlation tensor components is violated: $T_{xx} + T_{zz} > 1$ (see [3]).

Thus, in the spin correlations of lepton pairs generated in the annihilation processes $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$, the consequences of quantum-mechanical coherence for two-particle quantum systems with nonfactorizable internal states manifest themselves very distinctly, and they can be verified experimentally (see also our papers [3] and [10 – 13]) .

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