PROCEEDINGS OF SCIENCE

The $\gamma^*\gamma^* \to \eta_c(1S,2S)$ transition form factors for two spacelike photons

Izabela Babiarz*
The Henryk Niewodniczański Institute of Nuclear Physics Polish Academy of Sciences
E-mail: izabela.babiarz@ifj.edu.pl

Victor P. Goncalves
Instituto de Fisica e Matematica – Universidade Federal de Pelotas (UFPel)
E-mail: barros@ufpel.edu.br

Roman Pasechnik
Department of Astronomy and Theoretical Physics,Lund University, SE-223 62 Lund, Sweden
E-mail: roman.pasechnik@thep.lu.se

Wolfgang Schäfer
The Henryk Niewodniczański Institute of Nuclear Physics Polish Academy of Sciences E-mail: Wolfgang.Schafer@ifj.edu.pl

Antoni Szczurek
The Henryk Niewodniczański Institute of Nuclear Physics Polish Academy of Sciences
E-mail: antoni.szczurek@ifj.edu.pl

We discuss $\gamma^*\gamma^* \to \eta_c(1S)$, $\eta_c(2S)$ transition form factors for both virtual photons. The general formula is given. We use different models for the $c\bar{c}$ wave function obtained from the solution of the Schrödinger equation for different $c\bar{c}$ potentials: harmonic oscillator, Cornell, logarithmic, power-law, Coulomb and Buchmüller-Tye. We showed some examples of wave functions in the Light Front representation as well as in the rest frame of $c\bar{c}$ pair. We compare our results to the BaBar experimental data for $\eta_c(1S)$, for one real and one virtual photon, and to the values collected by the Particle Data Group for $F(0,0)$, decay width $\Gamma_{\gamma\gamma}$ and decay constant f_{η_c}. We also considered the non-relativistic limit for $F(0,0)$ form factor with the wave function at the origin $R(0)$.

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*Speaker.

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1. Introduction

During the last few years, the pseudoscalar charmonium states $\eta_c(1S)$ and its radial excitation $\eta_c(2S)$ have attracted a lot of attention from both theoretical [1, 2] and experimental [3, 4, 5] communities. So far, CLEO, BABAR, Belle, L3 collaborations have extracted the transition form factor for light mesons (π^0, η, η') from events, where only one of the leptons in the final state could be measured in the electromagnetic process depicted in Fig. 1. A similar analysis was done for $\eta_c(1S)$ by the BABAR collaboration. The study of transition form factor for both off-shell photons is motivated by a possibility for an accurate measurement of the double-tag mode, considering high luminosity at the Belle2 experiment.

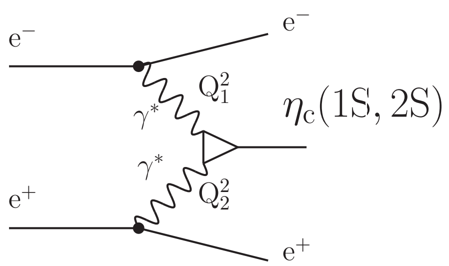

Figure 1: *Feynman diagram for the process $e^+e^- \to e^+\,\eta_c(1S,2S)\,e^-$*

The matrix element for the $\gamma^*\gamma^* \to \eta_c$ fusion can be written in terms of the $F(Q_1^2, Q_2^2)$ form factor as follows

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\, F(Q_1^2,Q_2^2)\,. \tag{1.1}$$

Here, $Q_i^2 = -q_i^2 > 0\,, i = 1,2$ are space like virtualities of the initial photons. In order to construct the $\gamma^*\gamma^* \to \eta_c(1S)\,,\,\eta_c(2S)$ amplitude, one can use the Light-Front (LF) wave function $\psi(z,k_\perp)$ such that the corresponding form factor $F(Q_1^2,Q_2^2)$ reads [7]:

$$\begin{aligned} F(Q_1^2,Q_2^2) \;&= e_c^2\sqrt{N_c}4m_c \cdot \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3}\,\psi(z,\mathbf{k})\Big\{\frac{1-z}{(\mathbf{k}-(1-z)\mathbf{q}_2)^2+z(1-z)\mathbf{q}_1^2+m_c^2} \\ &+\frac{z}{(\mathbf{k}+z\mathbf{q}_2)^2+z(1-z)\mathbf{q}_1^2+m_c^2}\Big\}\,. \end{aligned} \tag{1.2}$$

Here, $(z,\mathbf{k})$ are the LF variables, z and $1-z$ are the longitudinal momentum fractions of c and $\bar{c}$, respectively, and $\mathbf{k}$ is the relative momentum between c and $\bar{c}$ in the center-of-mass of the $c\bar{c}$ pair. In Fig. 2 we present our main new result on the dependence of the transition form factor on both photon virtualities Q_1^2 and Q_2^2 in the case of the Buchmüller-Tye interquark interaction potential [6] (for more details and results, see Ref. [7]). Below, we discuss some basic details of our analysis.

2. Radial momentum-space wave function and boosting

The radial wave function in the rest frame of the quark-antiquark pair is obtained by solving the Schrödinger equation

$$\frac{\partial^2 u(r)}{\partial r^2} = (V_{\rm eff}(r)-\varepsilon)u(r)\,, \qquad u(r) = \sqrt{4\pi}\,r\psi(r)\,, \tag{2.1}$$

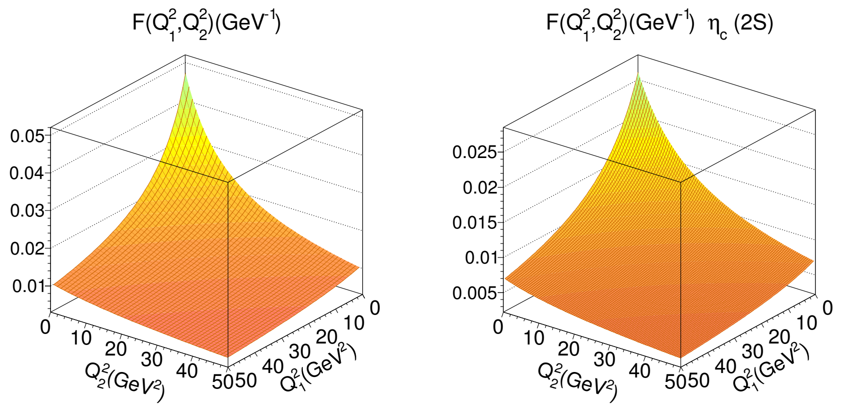

Figure 2: *Transition form factor for $\eta_c(1S)$ and $\eta_c(2S)$ computed by using the Buchmüller-Tye potential [6].*

in terms of the effective interquark interaction potential $V_{\text{eff}}(r)$ (for more details, see e.g. Ref. [8]). Then one turns to the momentum space representation preserving the normalisation of the wave function

$$\int_0^\infty |u(r)|^2 dr = 1 \quad \Rightarrow \quad \int_0^\infty |u(p)|^2 dp = 1\,. \tag{2.2}$$

One notices in Fig. 3 that the wave function $u(p)$ found numerically for each potential has a somewhat different behaviour which, in particular, is sensitive to the value of the constituent c-quark mass.

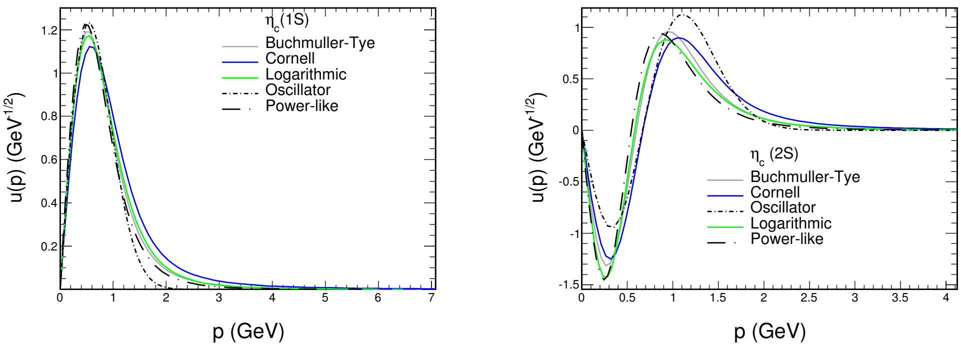

Figure 3: *The radial momentum-space wave function for $\eta_c(1S)$ (left panel) and for $\eta_c(2S)$ (right panel) states for different potentials.*

In further calculations we have used the popular Terent'ev prescription [9] giving rise to the LF quarkonium wave function in the following form

$$\psi(z,k_\perp) = \frac{\pi}{\sqrt{2M_{c\bar{c}}}} \frac{u(p)}{p}\,, \tag{2.3}$$

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using the kinematical quantities

$$p_\perp = k_\perp, \quad p_z = (z - \frac{1}{2})M_{c\bar{c}}, \quad M_{c\bar{c}}^2 = \frac{k_\perp + m_c^2}{z(1-z)}, \tag{2.4}$$

and properly accounting for the Jacobian of transformation of the integration variables. An example of the LF wave function is shown in Fig. 4, for the Buchmüller-Tye potential model [6]. One can observe that, quite naturally, the wave function is strongly peaked around $z \sim 1/2$.

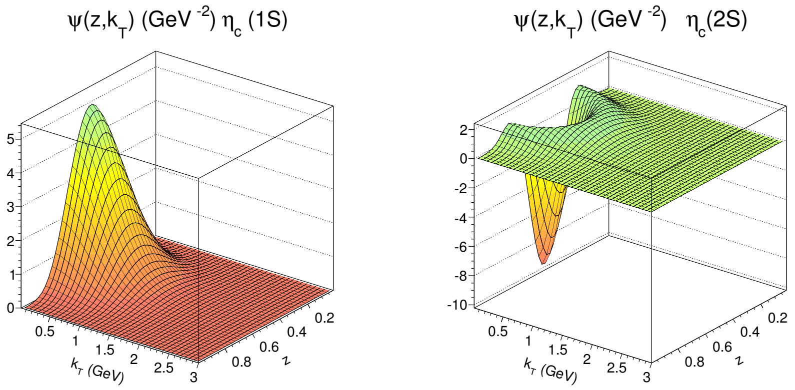

Figure 4: The radial LF wave function computed by using the Buchmüller-Tye potential [6].

3. Transition form factor for both on-shell photons

In order to write down the formula for both on-shell photons, we can directly simplify Eq. (1.2) as follows

$$F(0,0) = e_c^2 \sqrt{N_c}\, 4m_c \cdot \int \frac{dz d^2k_\perp}{z(1-z)16\pi^3} \frac{\psi(z,k_\perp)}{k_\perp^2 + m_c^2}, \tag{3.1}$$

and then the relation between the two-photon decay width and $F(0,0)$ can be found in the form

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{\pi}{4} \alpha_{\rm em}^2 M_{\eta_c}^3 |F(0,0)|^2. \tag{3.2}$$

The so-called meson decay constant f_{η_c} can be extracted numerically as follows

$$f_{\eta_c} \varphi(z,\mu_0^2) = \frac{1}{z(1-z)} \frac{\sqrt{N_c}\, 4m_c}{16\pi^3} \int d^2k_\perp\, \theta(\mu_0^2 - k_\perp^2)\, \psi(z,k_\perp), \qquad \int_0^1 dz\, \varphi(z,\mu_0^2) = 1. \tag{3.3}$$

The on-shell form factor $F(0,0)$ can further be rewritten in terms of the radial momentum-space wave function $u(p)$:

$$F(0,0) = e_c^2 \sqrt{2N_c} \frac{2m_c}{\pi} \int_0^\infty \frac{dp\, p\, u(p)}{\sqrt{M_{c\bar{c}}^3}(p^2 + m_c^2)} \frac{1}{2\beta} \log\left(\frac{1+\beta}{1-\beta}\right), \quad \beta = \frac{p}{\sqrt{p^2 + m_c^2}}, \tag{3.4}$$

in terms of β being the velocity v/c of the quark in the $c\bar{c}$ center of mass frame. In the non-relativistic limit, corresponding to $\beta \ll 1$, $p^2/m_c^2 \ll 1$, and $2m_c = M_{c\bar{c}}$ (or $2m_c = M_{\eta_c}$), we obtain

$$F(0,0) = e_c^2\sqrt{N_c}\sqrt{2}\frac{4}{\pi\sqrt{M_{\eta_c}^5}}\int_0^\infty dp\, p\, u(p) = e_c^2\sqrt{N_c}\frac{4R(0)}{\sqrt{\pi M_{\eta_c}^5}}\,, \tag{3.5}$$

where $R(0)$ is the radial wave function at the origin. The values of the transition form factor for both on-shell photons, the decay constant as well as the decay width $\Gamma_{\gamma\gamma}$ are collected in Table 1 for $\eta_c(1S)$ and in Table 2 for $\eta_c(2S)$ states.

Table 1: *Transition form factor $|F(0,0)|$ for $\eta_c(1S)$ at $Q_1^2 = Q_2^2 = 0$.*

potential type	m_c [GeV]	$\lvert F(0,0)\rvert$ [GeV^{-1}]	$\Gamma_{\gamma\gamma}$ [keV]	f_{η_c} [GeV]
harmonic oscillator	1.4	0.051	2.89	0.2757
logarithmic	1.5	0.052	2.95	0.3373
power-like	1.334	0.059	3.87	0.3074
Cornell	1.84	0.039	1.69	0.3726
Buchmüller-Tye	1.48	0.052	2.95	0.3276
experiment	-	0.067 ± 0.003 [10]	5.1 ± 0.4 [10]	0.335 ± 0.075 [11]

Table 2: *Transition form factor $|F(0,0)|$ for $\eta_c(2S)$ at $Q_1^2 = Q_2^2 = 0$.*

potential type	m_c [GeV]	$\lvert F(0,0)\rvert$ [GeV^{-1}]	$\Gamma_{\gamma\gamma}$ [keV]	f_{η_c} [GeV]
harmonic oscillator	1.4	0.03492	2.454	0.2530
logarithmic	1.5	0.02403	1.162	0.1970
power-like	1.334	0.02775	1.549	0.1851
Cornell	1.84	0.02159	0.938	0.2490
Buchmüller-Tye	1.48	0.02687	1.453	0.2149
experiment [10]	-	0.03266 ± 0.01209	2.147 ± 1.589	-

We have also calculated the normalized transition form factor $F(Q^2,0)/F(0,0)$ with the aim of comparison of our results to the experimental data obtained by the BABAR collaboration [12], see Fig. 5. The right panel in Fig. 5 presents the prediction for the normalized transition form factor for $\eta_c(2S)$ meson. The results are rather different between the predictions obtained with each model of the interquark interaction potential. We noticed that the best description of the data in provided by the model with $m_c = 1.334$ GeV. We observe a strong dependence on the charm quark mass.

4. Conclusion

In this report, we present our recent results on the transition form factor for different wave functions obtained as a solution of the Schrödinger equation for the $c\bar{c}$ system for different phenomenological $c\bar{c}$ potentials from the literature. More details and results can be found in Ref. [7].

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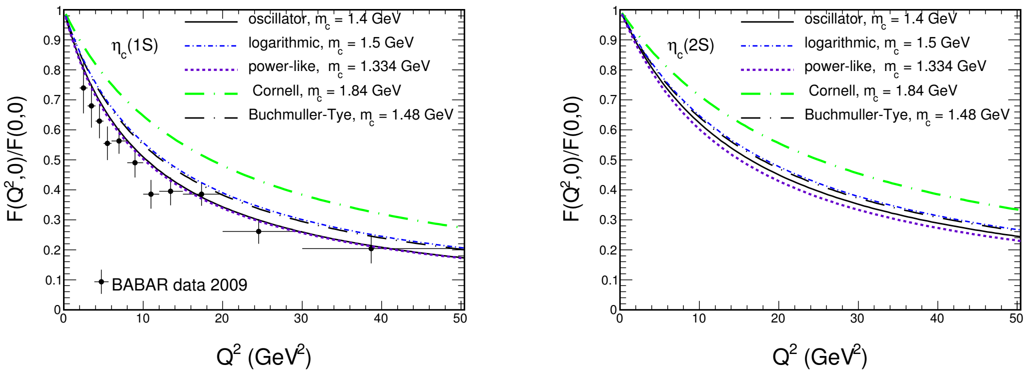

Figure 5: *Normalized transition form factor $F(Q^2,0)/F(0,0)$ as a function of photon virtuality Q^2. The BABAR data [12] are shown for comparison.*

We have studied the transition form factors for $\gamma^*\gamma^* \to \eta_c(1S,2S)$ for two space-like virtual photons, which can be accessed experimentally in future measurements of the cross section for the $e^+e^- \to e^+e^-\eta_c$ process in the double-tag mode. The form factor for only one off-shell photon as a function of its virtuality has been compared to the BaBar data for the $\eta_c(1S)$ case.

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References

[1] V. L. Chernyak and S. I. Eidelman, Prog. Part. Nucl. Phys. **80**, 1 (2014).

[2] H. Y. Ryu, H. M. Choi and C. R. Ji, Phys. Rev. D **98**, no. 3, 034018 (2018).

[3] R. Aaij *et al.* [LHCb Collaboration], Phys. Lett. B **769**, 305 (2017).

[4] Y. Hu [BESIII Collaboration], arXiv:1906.08998 [hep-ex].

[5] Z. Metreveli, eConf C **070805**, 16 (2007).

[6] W. Buchmuller, S.H.H. Tye, Phys. Rev. D **24**, 132 (1981).

[7] I. Babiarz, V. P. Goncalves, R. Pasechnik, W. SchÃd’fer and A. Szczurek, Phys. Rev. D **100**, no. 5, 054018 (2019).

[8] J. Cepila, J. Nemchik, M. Krelina and R. Pasechnik, Eur. Phys. J. C **79**, no. 6, 495 (2019).

[9] M.V. Terentev, Sov. J. Nucl. Phys. **24**, 106 (1976). [Yad. Fiz. **24**,207 (1976)].

[10] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no.3, 030001 (2018).

[11] K. W. Edwards *et al.* [CLEO Collaboration], Phys. Rev. Lett. **86**, 30 (2001).

[12] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **81**, 052010 (2010).