Production of $\chi_c i \chi_c j$ pairs in proton-proton collisions in $k_T$-factorization and collinear approaches

Wolfgang Schäfer*  
Institute of Nuclear Physics Polish Academy of Sciences,  
ul. Radzikowskiego 152, PL-31-342 Kraków, Poland  
E-mail: wolfgang.schafer@ifj.edu.pl

Anna Cisek  
College of Natural Sciences, Institute of Physics, University of Rzeszów, ul. Pigmia 1,  
PL-35-310 Rzeszów, Poland  
E-mail: acisek@ur.edu.pl

Izabela Babiarz  
Institute of Nuclear Physics Polish Academy of Sciences,  
ul. Radzikowskiego 152, PL-31-342 Kraków, Poland  
E-mail: izabela.babiarz@ifj.edu.pl

Antoni Szczurek  
Institute of Nuclear Physics Polish Academy of Sciences,  
ul. Radzikowskiego 152, PL-31-342 Kraków, Poland  
E-mail: antoni.szczurek@ifj.edu.pl

Here we present recent results of our calculations of $\chi_c$ pair production, mainly in the single parton scattering (SPS) mode. An important feature is that the single-gluon exchange mechanism can to some extent mimic the behaviour of double parton scattering (DPS) production. Off-shell matrix elements for $g^* g^* \rightarrow \chi_c i \chi_c j$ were derived and then used in the $k_T$-factorization approach for the $pp \rightarrow \chi_c i \chi_c j$ reaction. Different combination of the $\chi_c$ mesons are considered. A similar analysis is repeated for the collinear factorization approach, but now including the associated production with a gluon (jet). The leading order contributions ($2 \rightarrow 2$ processes) are rather small, compared to the $k_T$-factorization result. But the addition of $2 \rightarrow 3$ processes helps to recover the latter results.

European Physical Society Conference on High Energy Physics - EPS-HEP2019 -  
10-17 July, 2019  
Ghent, Belgium

*Speaker.

© Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License (CC BY-NC-ND 4.0). https://pos.sissa.it/
1. Introduction

In this contribution we review our recent studies of $\chi_c$-pair production in $k_T$-factorization [1] and of the of $\chi_c$-pairs associated with a gluon (jet) in collinear factorization [2].

One of the motivations of these works lies in the recent interest in double parton scattering processes (DPS). Indeed, the charmonium and open charm production is expected to be an important probe of DPS mechanisms. For example, the production of $J/\psi$-pairs has been suggested as a probe of double-parton scattering (DPS) processes [3]. The importance of the DPS production mode in the open charm production has been stressed in [4]. As far as charmonia are concerned, the cross sections for production of $J/\psi$-pairs were measured at the Tevatron [5] and the LHC [6, 7, 8, 9].

There are a number of puzzles in the description of these data, though. For example the single parton scattering (SPS) leading order of $O(\alpha_s^2)$ (the so-called “box-mechanism”, see e.g. [10, 11]) does not describe all the kinematical distributions well in the case of the ATLAS and CMS data. In particular, at large rapidity distance $\Delta y$ between two $J/\psi$ mesons the “box mechanism” falls short of experimental data.

If one ascribes all the discrepancy between data and the box-mechanism SPS mode to DPS processes, the normalization of DPS comes out a factor $\sim 2.5$ larger than in other hard processes.

It is still an open issue at the moment whether this points to a nonuniversality of DPS effects or whether there are additional single parton scattering mechanisms not taken into account up to now. This is part of the motivation why we looked into SPS processes which contribute at large rapidity distance between charmonia.

Beyond that, the $\chi_c J^+$-production is a convenient testbed for calculations of SPS and DPS mechanisms, as the leading order production mechanism (in the color singlet model) is just a $\rightarrow 1$ gluon fusion ($gg \rightarrow \chi$) process.

2. Production of $\chi_c$-pairs

In the standard hard scattering approach, the cross section of production of a pair of quarkonia $a, b$ is calculated from a convolution of parton densities with a parton-level cross section (see the left diagram in Fig. 1). However at high energies, favored by the rise of the gluon distribution at small $x$ there is a sizable contribution from processes in which two or more hard processes proceed...
in the same proton-proton collision (see the right diagram in Fig. 1). We can think of it as an inelastic interaction involving the simultaneous participation of two parton chains.

One often assumes the factorized ansatz for the production cross section in the DPS mode:

$$\frac{d\sigma_{DPS}(pp \rightarrow abX)}{dy_abd^2\vec{p}_{at}d^2\vec{p}_{bt}} = \frac{1}{1 + \delta_{ab}} \frac{1}{\alpha_{eff}} \frac{d\sigma(pp \rightarrow aX) d\sigma(pp \rightarrow bX)}{dy_abd^2\vec{p}_{at}d^2\vec{p}_{bt}}.$$ (2.1)

The DPS cross section is written as a product of the inclusive single-particle spectra, and the cross section is normalized by the “effective cross section” $\alpha_{eff}$. In the simplest model, the inverse of the effective cross section is related to the overlap of the parton densities of the participating hadrons in the transverse plane, $t_N(\vec{b})$:

$$\frac{1}{\alpha_{eff}} = \int d^2\vec{b} T_{NN}^2(\vec{b}), \quad T_{NN}(\vec{b}) = \int d^2\vec{s} t_N(\vec{s}) t_N(\vec{b} - \vec{s}).$$ (2.2)

The most important features of DPS are immediately obvious from Eq. 2.1. Each of the single particle spectra is a broad function of $y_{a,b}$, and thus the DPS distribution in rapidity distance $\Delta y = y_b - y_a$ will have a long range as well. As far as the size of the effective cross section is concerned, it is usually taken in the ballpark of $\alpha_{eff} = 15 \text{ mb}$. This is consistent with a fair amount of hard processes, see e.g. a table in [7].

In the case of $J/\psi$-pair production the lowest-order “box-diagram” mechanism suggests a very clean separation of SPS versus DPS modes. Indeed, explicit calculations performed in $k_T$-factorization [11], show, that the $J/\psi$-pair distribution is sharply peaked around $\Delta y = 0$.

A main point of this presentation is the fact that the situation looks completely different for the case of production of a pair of $\chi_c$ mesons. Indeed, the $\chi_{cJ}$ states, which come in three different spins $J = 0, 1, 2$ have positive $C$-parity and thus couple to two gluons in a color singlet state. Consequently the mechanism of Fig. 2 with a $t$-channel exchange of a single gluon is possible. This $t$-channel exchange mechanism yields a $gg \rightarrow \chi \chi$ cross section which is independent of cm-energy in the high-energy limit. The matrix element for $\chi$-pair production thus puts no penalty on large rapidity distance $\Delta y$ between the $\chi_c$-mesons.

The relevant amplitudes can be obtained from effective $g^+ g^+ \rightarrow \chi_{cJ}$ vertices for the fusion of two spacelike off-shell gluons. We adopt the color singlet model, where in the NRQCD limit the relevant vertices take the form

$$V_{\mu\nu}(J,J_c; q_1, q_2) = -i4\pi\alpha_S \sqrt{3} \cdot T_{\mu\nu}(J,J_c; q_1, q_2).$$ (2.3)

Here $R'(0)$ is the derivative of the $p$-wave radial wave function at the origin. It is constrained e.g. by the $\chi_c, 0, 2 \rightarrow \gamma\gamma$ decay widths. The explicit form of tensors $T_{\mu\nu}(J,J_c; q_1, q_2)$ has been obtained in Ref. [1] for all possible spin-states of the $\chi_c$ family. Notice that for the spin-1 meson $\chi_{c1}$ the vertex vanishes when both gluons go on-shell (as required by the Landau-Yang theorem), but it is generally non-zero off-shell.

These off-shell vertices are a necessary input for our $k_T$-factorization calculations including transverse momenta of incoming (off-shell) gluons.

We now come to the discussion of some selected results. In the left panel of Fig. 3 we show the distribution in rapidity distance $\Delta y$ between mesons. Notice that here we only show as an
Production of $\chi_c$ pairs

Wolfgang Schäfer

**Figure 2:** The gluon $t$-channel exchange mechanism for the production of $\chi_c$ pairs.

**Figure 3:** Distribution of $\chi_c$-pairs in rapidity difference between mesons. Top panel: SPS mode, lower panel: DPS mode.

The large rapidity distance between mesons corresponds to a large phase space for emission of additional gluons. We therefore studied in Ref.[2] the associated production of $\chi_c$ pairs with a gluon in the standard collinear factorization. There are two main contributions, shown in the diagrams of Fig. 4, firstly, the emission of a “leading gluon”, where the gluon jet carries a large fraction of the momentum carried by one of the incoming gluons. The amplitude, say of diagram A in Fig.4 takes the form

$$\mathcal{A}_A = i g_s f a b c e^\mu (\lambda_a, q_a) \Gamma_{\mu\nu\rho} (q_a, p_R) n^{-\rho} e^{\nu^*} (\lambda_{q_b}, p_R) \frac{1}{t_1} n^{+\mu'} .\mathcal{M}_{\mu\nu\rho}^{b b} (p_g - q_{a}, q_b; p_1, p_2) e^{\nu^*} (\lambda_b, q_b)$$

$$= i g_s f a b c 2 q_a^+ \delta_{\lambda_a \lambda_{q_b}} \frac{1}{t_1} n^{+\mu'} e^{\nu^*} (\lambda_{b}, q_b) .\mathcal{M}_{\mu\nu\rho}^{b b} (p_g - q_{a}, q_b; p_1, p_2),$$

(2.4)

where $\Gamma_{\mu\nu\rho}$ is the standard three-gluon vertex, and the $2 \to 2$ amplitude $\mathcal{M}_{\mu\nu\rho}^{b b}$ is constructed from

example the production of pairs of identical mesons, the full array of all possible combinations can be found in Ref. [1]. In the right panel of Fig. 3 we show distributions in $\Delta y$ for the DPS mode, using $\sigma_{\text{eff}} = 15 \text{mb}$. We see, that these distributions are very broad and the overall magnitude is in the same ballpark as the SPS contribution. Of course there is no minimum at $\Delta y = 0$ for the DPS distributions. Thus we observe rather similar distributions in $\Delta y$ for single and double parton scattering production of different $\chi_c$-quarkonia states. This shows that both contributions must be included in analysis of future data for $\chi_{cJ} \chi_{cJ}$ production.
Production of $\chi_{c1}\chi_{c1}$ pairs

Wolfgang Schäfer

Figure 4: Feynman diagrams for the production of a $\chi_{c}$-pair associated with a gluon.

the above quoted $g^* g^* \chi_{cJ}$ vertices.

Secondly there is a contribution from the production of “central” gluons, which are emitted in rapidity space between the two mesons with a large difference in rapidity from either one. The diagram C of Fig. 4 leads to the amplitude

$$M_C = i g S f^{abc}_c V_1^{ab}(q_a, p_1) C^p(q_a - p_1, q_b - p_2) \epsilon_\nu^*(\lambda_g, p_g^1) \frac{1}{t_2} V_2^{bb}(q_b, p_2) , \quad (2.5)$$

where

$$V_1^{ab}(q_a, p_1) = \epsilon^\mu(\lambda_a, q_a) V_2^{ad}(J_1, q_1; q_a, p_1 - q_a) n^{-\mu}$$

$$V_2^{bb}(q_b, p_2) = \epsilon^\nu(\lambda_b, q_b) V_2^{bb}(J_2, q_2; q_b, p_2 - q_b) n^{+\nu} , \quad (2.6)$$

and $C^p$ is the $g^* g^* g$-Lipatov vertex (see e.g. [12]).

Some distributions, again in rapidity distance $\Delta y$ between mesons are shown in Fig. 5. The production of leading gluons adds to the Born-result to recover the $k_T$-factorization result, while the production of central gluons gives rise to an about 20% enhancement of the cross section. Here one may think of $\alpha_S \cdot \Delta y$ as a large parameter which could be resummed in the future using a BFKL formalism.

3. Conclusions

Pair production of quarkonia is a topic that still poses puzzles to theorists. A quantitative understanding of DPS contributions requires not only a reliable formalism for its calculation but also a good understanding of SPS processes that can show similar behavior as DPS in many kinematic variables.

For the theoretically simplest case, the production of $\chi_c$-pairs, we showed that the cross sections for different combinations of $\chi_c$ quarkonia the SPS and DPS cross sections are of the similar size, and both involve very broad distributions in rapidity distance $\Delta y$.

We have also shown, that an enhancement of the pair production cross section for $\chi_c$-pairs can be expected from the higher order corrections, due to the large phase-space of gluon emission.

However, it turns out, that feed-down from $\chi$-pairs into the $J/\psi$-pair channel does not resolve the discrepancy between different determinations of $\sigma_{\text{eff}}$ [13].
Production of $\chi_{c0}\chi_{c1}$ pairs

Wolfgang Schäfer

Figure 5: Distribution in rapidity between $\chi_0$ mesons (top panel) and $\chi_{c2}$ mesons for the following different processes: Born-level production of $\chi_{c}$-pairs, production of $\chi_{c}$ pairs with a leading gluon, and production of $\chi_{c}$-pairs with a central gluon.

It might be necessary to look deeper into the fundamentals of DPS theory (see e.g. [14]) to understand the peculiar behaviour of charmonium pair production.

References