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Flavour-Exotic Tetraquarks in Large-N_c QCD: Do They Exist?

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Flavour-exotic tetraquark mesons, by definition bound states of two quarks and two antiquarks of four mutually different quark flavours, are, for given Lorentz features, subject to two incompatible constraints: On the one hand, within quantum chromodynamics a formation of compact tetraquark states is most easily envisaged by merging two colour-antisymmetric two-quark clusters, a diquark and an antidiquark. This path, however, leads to merely a single tetraquark state of chosen Lorentz characteristics. On the other hand, in the limit of the number of colour degrees of freedom growing beyond bounds, internal consistency at leading order calls for the presence of (at least) two of such tetraquark states of *identical* quark-flavour composition. The failure of attempts to reconcile these two contradictory insights suggests the nonexistence of compact flavour-exotic tetraquark mesons.

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1. Simplified Look at Tetraquarks by Quantum Chromodynamics in Large-N_c Limit

Quantum chromodynamics (QCD), the quantum field theory governing the strong interactions, supports as admissible bound states of its quark and gluon degrees of freedom not only conventional hadrons (i.e., the quark–antiquark mesons and three-quark baryons) but also multiquark states (such as tetraquarks, pentaquarks, hexaquarks) and hadrons hybridized by gluon excitations. The first and foremost boundary condition to each variant of exotic construction is: it has to form a colour singlet. We are interested in tetraquark mesons exhibiting flavour quantum numbers $a,b,c,d \in \{u,d,s,c,b\}$,

$$T = [\overline{q}_a\, q_b\, \overline{q}_c\, q_d] \ , \tag{1.1}$$

compact states tightly binding two quarks q_b, q_d and two antiquarks $\overline{q}_a, \overline{q}_c$, of masses m_a, m_b, m_c, m_d.

Our goal is to derive information on the main features of tetraquark mesons [1–6] by inspection of their contributions, in form of poles, to the amplitudes of the scattering of two ordinary mesons of momenta p_1 and p_2 into two ordinary mesons of momenta p_1' and p_2'. Supposing these four ordinary mesons to be created from the vacuum by adequate interpolating operators or currents, we do this by investigating Green functions of four quark-bilinear currents, here generically symbolized by j. For the meson $M_{\overline{a}b}$ composed of an antiquark $\overline{q}_a$ and a quark q_b with associated field operators $\overline{q}_a(x)$ and $q_b(x)$, its coupling strength $f_{M_{\overline{a}b}}$ to any appropriate quark-bilinear operator $j_{\overline{a}b}(x)$, upon notationally suppressing immaterial reference to Lorentz nature of generic form $j_{\overline{a}b}(x) \equiv \overline{q}_a(x)\, q_b(x)$, is given by

$$f_{M_{\overline{a}b}} \equiv \langle 0|\, j_{\overline{a}b}(0) |M_{\overline{a}b}\rangle \ .$$

The primary task in that undertaking is to formulate a criterion that allows for the unambiguous identification of those contributions to the above scattering amplitudes that encapsulate information about tetraquark mesons [1]. Feynman diagrams passing this criterion are labelled *tetraquark-phile*:

> A Feynman diagram is tetraquark-phile [2,5] if, as function of the Mandelstam variable
>
> $$s \equiv (p_1 + p_2)^2 = (p_1' + p_2')^2 \ ,$$
>
> it depends *non-polynomially* on s and exhibits a branch cut starting at the *branch point* $\hat{s}$ defined by the square of the sum of the masses of the involved bound-state constituents,
>
> $$\hat{s} = (m_a + m_b + m_c + m_d)^2 \ ,$$
>
> possibly contributing to a *tetraquark pole* by support of a four-quark intermediate state.

The (non-) existence of such a branch cut can be decided by application of the Landau equations [7].

In the present context, considerable profit will be drawn from allowing the number N_c of colour degrees of freedom of QCD to become arbitrarily large and harvesting any finding of the formulated theory, large-N_c QCD [8,9], in both its limit $N_c \to \infty$ and its expansion in powers of $1/N_c$ thereabout. Consistency calls for constancy of the product of N_c and the square of the strong coupling g_s [8], i.e.,

$$\alpha_s \equiv \frac{g_s^2}{4\pi} = O(N_c^{-1}) \xrightarrow[N_c \to \infty]{} 0 \ .$$

An immediate implication of this is the large-N_c behaviour of all ordinary-meson couplings $f_{M_{\overline{a}b}}$ [9],

$$f_{M_{\overline{a}b}} = O(\sqrt{N_c}) \qquad \text{for} \quad N_c \to \infty \ .$$

Compared with its point of origin, large-N_c QCD is of a significantly lesser complexity; this enabled (at least, qualitative) analyses of different features of hadrons, notably, of multiquark states [10–14].

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2. *Flavour-Exotic* Tetraquark Poles in Correlators: Large-N_c Leading Contributions

Evidently, our next move must be to define the tetraquarks' flavour content we are interested in. Table 1 of Ref. [4] compiles the conceivable quark-flavour combinations. From this list, particularly tantalizing to us seems to be the case of flavour-exotic tetraquarks, where the flavours a, b, c, d of the two quarks and two antiquarks forming any such meson differ from each other. This case belongs to those configurations that betray, beyond all doubt, the *nonconventional* nature of some hadron state.

We study mere two-ordinary-meson scattering but (for the sake of completeness) with either an identical or a differing distribution of the four available quark flavours to the two ordinary mesons in initial and final state. As a consequence, we have to take into account two categories[1] of correlators:

$$\text{flavour-preserving Green functions} = \left\{ \langle T(j_{\bar{a}b}\, j_{\bar{c}d}\, j^\dagger_{\bar{a}b}\, j^\dagger_{\bar{c}d}) \rangle,\ \langle T(j_{\bar{a}d}\, j_{\bar{c}b}\, j^\dagger_{\bar{a}d}\, j^\dagger_{\bar{c}b}) \rangle \right\}, \tag{2.1}$$

$$\text{flavour-reordering Green functions} = \left\{ \langle T(j_{\bar{a}d}\, j_{\bar{c}b}\, j^\dagger_{\bar{a}b}\, j^\dagger_{\bar{c}d}) \rangle \right\}. \tag{2.2}$$

Any *potential* intermediate-state tetraquark satisfies: if it contributes to *both* of the flavour-retaining correlators (2.1), then it will contribute to the flavour-reshuffling correlator (2.2) too, and *vice versa*.

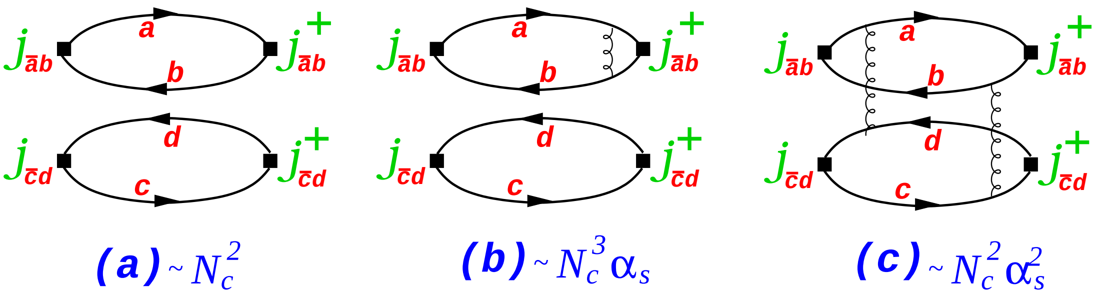

Figure 1: Flavour-preserving Green functions $\langle T(j_{\bar{a}b}\, j_{\bar{c}d}\, j^\dagger_{\bar{a}b}\, j^\dagger_{\bar{c}d}) \rangle$ of four quark-bilinear currents j: examples of contributions by, at order $O(N_c^2) = O(N_c^3\, \alpha_s)$ necessarily, *non-tetraquark-phile* Feynman diagrams (a,b) as well as *tetraquark-phile* Feynman diagrams (c), of the N_c-*leading tetraquark-phile* order $O(N_c^0) = O(N_c^2\, \alpha_s^2)$.

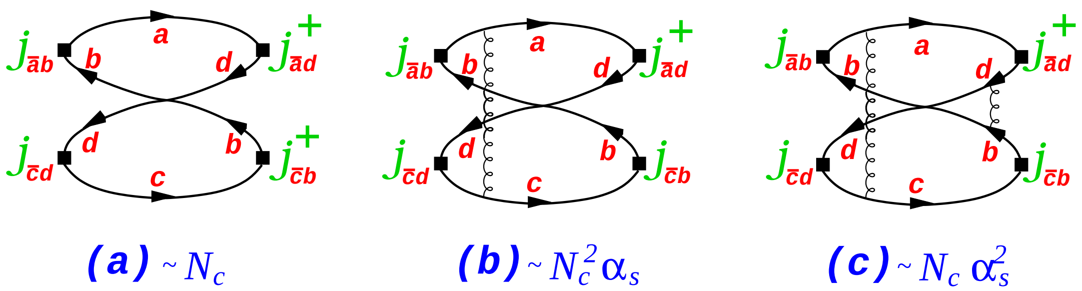

Figure 2: Flavour-reordering Green functions $\langle T(j_{\bar{a}d}\, j_{\bar{c}b}\, j^\dagger_{\bar{a}b}\, j^\dagger_{\bar{c}d}) \rangle$ of four quark-bilinear currents j: examples of contributions by [at order $O(N_c) = O(N_c^2\, \alpha_s)$ exclusively] *non-tetraquark-phile* Feynman diagrams (a,b) as well as *tetraquark-phile* Feynman diagrams (c) of the N_c-leading tetraquark-phile order $O(N_c^{-1}) = O(N_c\, \alpha_s^2)$.

[1] In order to prevent confusion, recall that in Ref. [1] these sets got named "direct" and "recombination", respectively.

For the elements of the two disjoint sets (2.1) and (2.2) of Green functions, the Laurent series of the $1/N_c$ expansions of their contributions start at order $O(N_c^2)$ for the flavour-preserving (Fig. 1) set (2.1) but at order $O(N_c)$ for the flavour-reordering (Fig. 2) set (2.2), respectively. The corresponding decisive subsets of *tetraquark-phile* contributions prove to be [1–4] of maximum order $O(N_c^0)$ in the flavour-retaining case (Fig. 3) but of maximum order $O(N_c^{-1})$ in the flavour-reordering case (Fig. 4). Imagining a tetraquark-phile Feynman diagram in the form of a cylinder (Fig. 3, bottom row; Fig. 4, left) facilitates its inspection, in particular, the crucial distinction of its planar and nonplanar gluons. Disregarding (hardly justifiable) adjustments by hand of N_c dependencies leads us to the finding [1]:

> The N_c-*leading* members of the two classes of *tetraquark-phile* Feynman diagrams, the flavour-preserving and the flavour-rearranging ones, exhibit a different N_c dependence.

A consequence of this outcome is that the implied constraints on the large-N_c limit of the Green functions cannot be solved under the assumption of the presence of only a single tetraquark. Rather, for a selected quark flavour combination one needs *at least* two tetraquarks, discriminable, however, by the large-N_c dependence of their transitions to the two conceivable pairs of conventional mesons.

For any flavour-exotic quark content, let us focus to the minimal option, i.e., two tetraquarks T_A and T_B of masses m_{T_A} and m_{T_B}, respectively. Their impact on the correlators (2.1) and (2.2) yields, in terms of all tetraquark–two-ordinary-meson transition amplitudes $A(T_\square \longleftrightarrow M_{\overline{\square}\square} M_{\overline{\square}\square})$ of interest,

- for the leading large-N_c behaviour of the *flavour-preserving* tetraquark-phile Green functions,

$$\frac{\langle T(j_{\bar{a}b}\, j_{\bar{c}d}\, j^\dagger_{\bar{a}b}\, j^\dagger_{\bar{c}d})\rangle}{f_M^4} = \frac{|A(T_A \longleftrightarrow M_{\bar{a}b} M_{\bar{c}d})|^2}{p^2 - m_{T_A}^2} + \frac{|A(T_B \longleftrightarrow M_{\bar{a}b} M_{\bar{c}d})|^2}{p^2 - m_{T_B}^2} + \cdots = O(N_c^{-2}) \,,$$

$$\frac{\langle T(j_{\bar{a}d}\, j_{\bar{c}b}\, j^\dagger_{\bar{a}d}\, j^\dagger_{\bar{c}b})\rangle}{f_M^4} = \frac{|A(T_A \longleftrightarrow M_{\bar{a}d} M_{\bar{c}b})|^2}{p^2 - m_{T_A}^2} + \frac{|A(T_B \longleftrightarrow M_{\bar{a}d} M_{\bar{c}b})|^2}{p^2 - m_{T_B}^2} + \cdots = O(N_c^{-2}) \,,$$

- for the leading large-N_c behaviour of the *flavour-reordering* tetraquark-phile Green functions,

$$\frac{\langle T(j_{\bar{a}d}\, j_{\bar{c}b}\, j^\dagger_{\bar{a}b}\, j^\dagger_{\bar{c}d})\rangle}{f_M^4} = \frac{A(T_A \longleftrightarrow M_{\bar{a}b} M_{\bar{c}d})\, A(T_A \longleftrightarrow M_{\bar{a}d} M_{\bar{c}b})}{p^2 - m_{T_A}^2} + \frac{A(T_B \longleftrightarrow M_{\bar{a}b} M_{\bar{c}d})\, A(T_B \longleftrightarrow M_{\bar{a}d} M_{\bar{c}b})}{p^2 - m_{T_B}^2} + \cdots = O(N_c^{-3}) \,.$$

For a single tetraquark, the conflict is evident. In contrast, for two tetraquarks coupling unequally to two ordinary mesons, one solution for the large-N_c behaviour of the four transition amplitudes reads

$$\underbrace{A(T_A \longleftrightarrow M_{\bar{a}b} M_{\bar{c}d}) = O(N_c^{-1})}_{\Longrightarrow \quad \Gamma(T_A) = O(N_c^{-2})\,,} \quad \overset{\boxed{N_c \text{ order}}}{>} \quad A(T_A \longleftrightarrow M_{\bar{a}d} M_{\bar{c}b}) = O(N_c^{-2})$$

$$A(T_B \longleftrightarrow M_{\bar{a}b} M_{\bar{c}d}) = O(N_c^{-2}) \quad \overset{\boxed{N_c \text{ order}}}{<} \quad \underbrace{A(T_B \longleftrightarrow M_{\bar{a}d} M_{\bar{c}b}) = O(N_c^{-1})}_{\Longrightarrow \quad \Gamma(T_B) = O(N_c^{-2})\,,}$$

if assuming both tetraquark masses m_{T_A} and m_{T_B} to retain their finite values in the limit $N_c \to \infty$. The N_c dependence of the total decay widths $\Gamma(T_A)$ and $\Gamma(T_B)$ of the two tetraquarks T_A and T_B, governed by their N_c-leading two-meson channels, would be parametrically equal: $\Gamma(T_A) = \Gamma(T_B) = O(N_c^{-2})$.

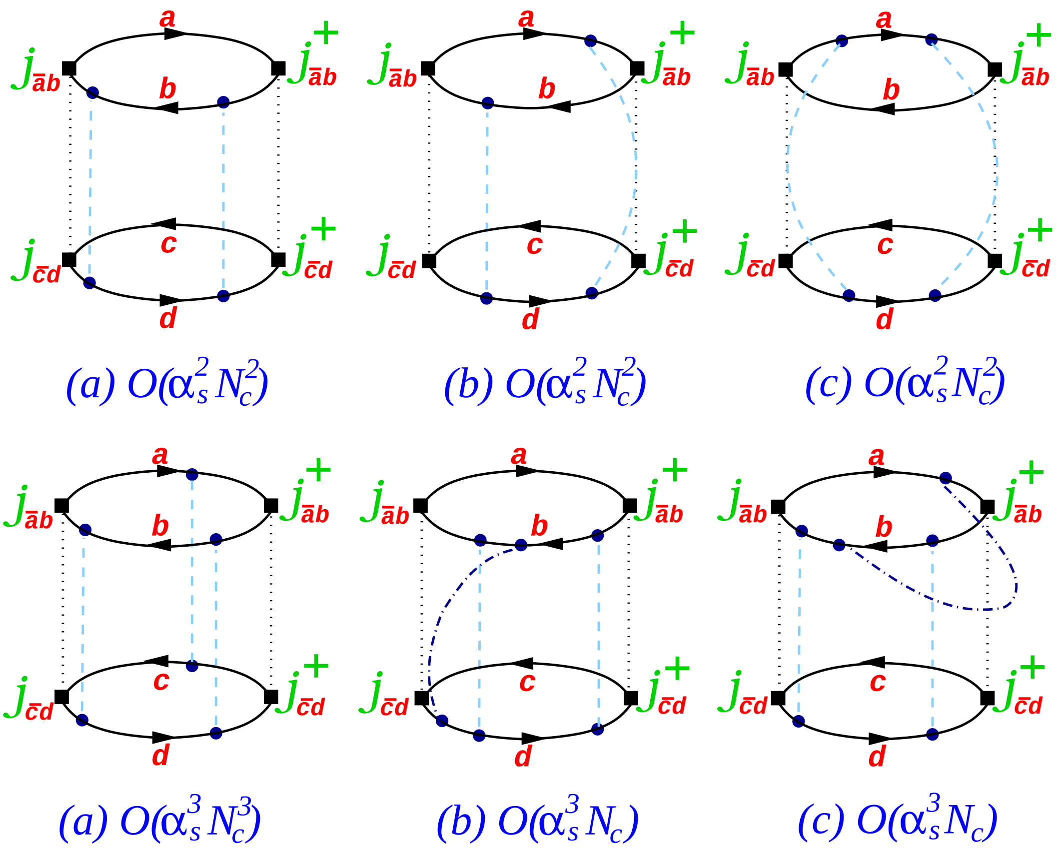

Figure 3: Cylinder interpretation (with cylinder surfaces indicated by dotted black lines) of typical examples of the flavour-retaining tetraquark-phile Feynman diagrams [5] of (top) N_c-leading order $O(\alpha_s^2 N_c^2) = O(N_c^0)$, involving two planar gluons (pale blue dashed lines), and an amendment of Feynman diagram (a) by (bottom) one additional (a) planar gluon (pale blue dashed lines) or (b,c) nonplanar gluon (dark-blue dot-dashed lines).

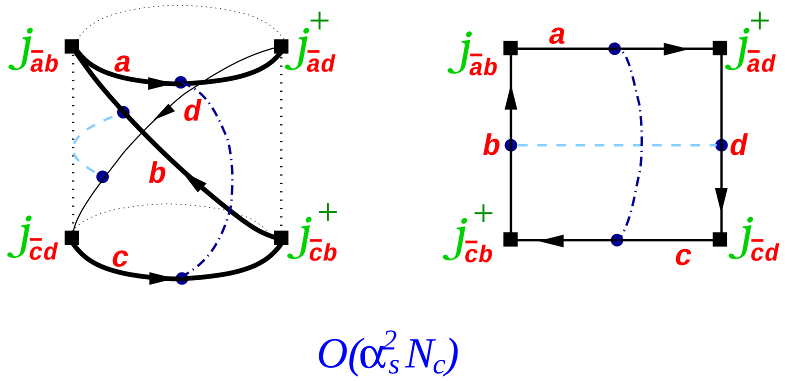

Figure 4: Generic example of flavour-reshuffling tetraquark-phile Feynman diagrams [5] of N_c-leading order $O(\alpha_s^2 N_c) = O(N_c^{-1})$, involving one planar gluon (pale blue dashed lines) and one nonplanar gluon (dark-blue dot-dashed lines), in cylinder (left, with the cylinder contours indicated by dotted black lines) and in unfolded (right) — therefore the inevitable nonplanarity of one of the exchanged gluons emphasizing — interpretation.

3. Bitter Pill to Swallow: Mismatch of Tetraquark Formation and Large-N_c Insights

Finally, we turn to the formation path of a *compact* tetraquark meson. From the group-theoretic point of view, a state combining two quarks and two antiquarks transforming according to either the 3-dimensional fundamental representation **3** of the QCD gauge group SU(3) or that representation's complex conjugate $\overline{\mathbf{3}}$ forms a reducible 81-dimensional representation of SU(3) whereas, according to confinement, any hadron is an SU(3) singlet **1**: we must identify the singlets in the tensor product

$$q_b\, q_d\, \overline{q}_a\, \overline{q}_c \sim \mathbf{3}\otimes\mathbf{3}\otimes\overline{\mathbf{3}}\otimes\overline{\mathbf{3}} = \mathbf{81} = \mathbf{1}\oplus\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}\ .$$

Tracking the tetraquark formation through the possible intermediate stages, SU(3) singlets can arise along only two routes. Either the two quarks merge to a *diquark* in the antisymmetric 3-dimensional representation $\overline{\mathbf{3}}$ and the two antiquarks to an *antidiquark* in the complex conjugate representation **3**, which yields a tightly bound, compact tetraquark state, or the supposed tetraquark constituents form two colour-singlet *quark–antiquark* states that may end up as a loosely bound, molecular-type state. As a matter of fact, this reasoning applies identically for all choices $N_c \geq 2$ of the number of colours.

4. Conclusion: Bad Hand for Tetraquark Mesons of Genuinely Flavour-Exotic Type

So, large-N_c QCD causes, in N_c-leading evaluation, *flavour-exotic* tetraquarks to exist pairwise [1–4] but provides just one *compact* tetraquark structure. The possible resolution of that clash might explain the lack of any reliable experimental observation of flavour-exotic tetraquark candidates [5]:

Large-N_c QCD does not support the existence of any narrow *flavour-exotic* tetraquarks.

Maybe, an evident *side remark* is in order. For a pretty simple reason, our above considerations do not apply to any case of *identical* flavours of either the two quarks or the two antiquarks, and thus also not to any case of identical flavours of *both* the two quarks and the two antiquarks, in particular, not to any case of four identical flavours, that is, to all tetraquarks exhibiting the flavour composition

$$T = [\overline{q}_a\, q_b\, \overline{q}_c\, q_b] \quad (a, c \text{ arbitrary}) \qquad \text{or} \qquad T = [\overline{q}_a\, q_b\, \overline{q}_a\, q_d] \quad (b, d \text{ arbitrary})\ : \qquad (4.1)$$

upon allowing for flavour identity $a = c$ or $b = d$ in Eq. (1.1), flavour-retaining Green functions (2.1) and flavour-rearranging Green functions (2.2) become indistinguishable, whence a single tetraquark suffices to satisfy the N_c-leading constraints. Tetraquarks of the kind (4.1) with diquark–antidiquark binding have been explored since long, and may have been spotted by lattice QCD recently [15–17].

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