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Transverse Momentum Dependent splitting kernels from k_T-factorization

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Transverse Momentum Dependent parton distribution functions allow to take into account apart from the proton momentum fraction also transverse momenta of initial partons in the description of hadronic cross-section. They are therefore a promising tool to obtain a more precise description of kinematics of hadronic observables. In this talk we present our most recent results in the determination of transverse-momentum-dependent splitting kernels. Our approach is based on a combination of high energy and collinear factorization and aims at the formulation of a generalized TMD framework.

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1. Introduction

Parton distributions functions (PDFs) provide indispensable input to the description of hard scattering processes at hadron colliders. Given the hierarchy $M \gg \Lambda_{\rm QCD}$ where M denotes the hard scale and $\Lambda_{\rm QCD}$ the QCD characteristic scale of the order of a few hundred MeV, hadronic cross-sections can be factorized into convolutions of parton distribution functions and corresponding partonic matrix elements. The latter can be calculated within perturbative QCD. Current collider phenomenology is predominantly based on the framework of collinear factorization [1–3], where the incoming parton momenta are treated to be collinear to the momentum of the respective mother hadron. Through the determination of hard matrix elements to higher perturbative order in the strong coupling constant α_s, it is possible to improve systematically on the precision of the theoretical prediction. Among other aspects, such as reducing the scale dependence due to the ambiguity in factorization- and renormalization scales, these higher order corrections further serve to improve the kinematic approximation inherent to the leading order description, where initial parton momenta are taken to be collinear with the respect to the mother hadron.

As an alternative to improving the kinematic description through the calculation of higher order corrections, one may attempt to treat kinematics exactly from the very beginning. This implies to include the bulk of kinematic effects already in the leading order description and to achieve in this way possibly improved convergence of the perturbative series. An example of such kinematic effects on which we will focus on in the following, is the transverse momentum k_T of the initial state partons. Within collinear factorization, this transverse momentum is set to zero. Schemes which provide an improved kinematic description already at the leading order involve in general Transverse-Momentum-Dependent (TMD) or 'unintegrated' PDFs. We also note that a possible implementation of such a transverse momentum dependent factorization with various phenomenological applications is currently being realized within the so-called parton branching method, see [4, 5], where evolution is based on collinear splitting kernels.

2. Kinematic considerations for TMD factorization

In the following we take a closer look at the kinematics required for a transverse momentum dependent factorization. TMD PDFs arise naturally in regions of phase space which are characterized by a hierarchy of scales. A region of phase space where a TMD factorization is essentially obtained as a by-product, is provided by the so called low x region. Here x is the ratio of the hard scale M^2 of the process over the center-of-mass energy squared s. One therefore considers the hierarchy $s \gg M^2 \gg \Lambda^2_{\rm QCD}$. For this kinematical setup, large logarithms $\ln 1/x$ can compensate for the smallness of the perturbative strong coupling α_s and it is necessary to resum terms $(\alpha_s \ln 1/x)^n$ to all orders to maintain the predictive power of the perturbative expansion. Such a resummation is achieved by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [6–9] evolution equation. Its formulation is based on factorization of QCD amplitudes in the high energy limit, $s \gg M^2$. In this kinematic limit, QCD cross-sections are expressed as convolutions in transverse momentum, similarly to convolutions in momentum fraction of conventional collinear factorization. As a consequence, cross-sections are automatically factorized into k_T dependent impact factors and the so-called BFKL

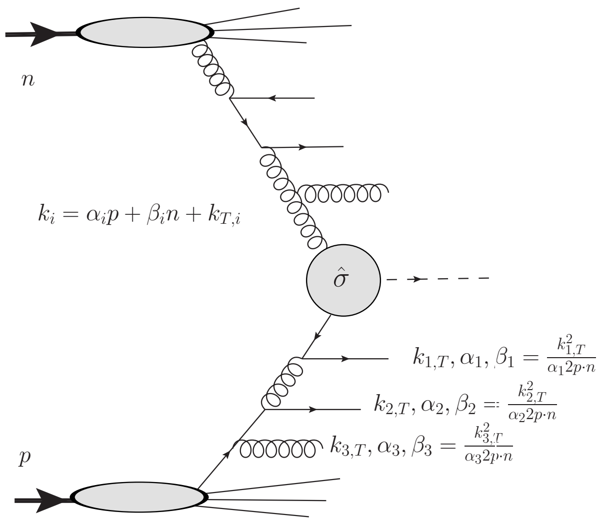

Figure 1: Parton cascade

Green's function. Matching of high energy factorization to collinear factorization which identifies properly normalized impact factors and Green's function with unintegrated gluon density and k_T-dependent perturbative coefficients is then achieved by so-called k_T-factorization [11].

Even though high energy factorization provides a well defined framework for calculations of evolution kernels and coefficient functions and can be extended to higher perturbative order, see [12,13] and [14–17], the applicability of the results is naturally limited to the low x limit of hard scattering events. Within the so-called Multi-Regge-Kinematics, which underlies the formulation of the BFKL evolution equation, the proton momentum fraction α_i are strongly ordered,

$$x \ll \alpha_1 \ll \alpha_2 \ll \ldots, \tag{2.1}$$

see Fig. 1 for the precise definition. As a consequence one resums only the terms enhanced by logarithms in $\ln 1/x$. Isolating the logarithmically enhanced contribution of the proton momentum fraction, the latter are not conserved and the information about proton fractions are lost along the chain. On the other hand there is no ordering in transverse momenta; within the Multi-Regge-Kinematics, transverse momenta are generally taken to be of the same order of magnitude $k_{T,i} \sim k_{T,j}$. Collinear factorization covers on the other hand the complimentary case: transverse momenta are strongly ordered along the parton cascade

$$M \gg k_{1,T} \gg k_{2,T} \gg \ldots, \tag{2.2}$$

as needed for a resummation of logarithms in the hard scale $\ln M^2$. As a consequence, information about transverse momenta is lost. Proton momentum fraction are on the other hand taken to be of the same order of magnitude $\alpha_i \sim \alpha_j$ and are kept exactly.

To arrive at a more precise treatment of kinematics along the parton cascade, it is clearly needed to arrive at a unification of both evolution schemes. A first suitable variable, which might serve as

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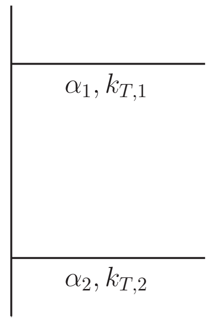

Figure 2: subsequent emissions

an evolution parameter along the chain might be given by the rapidity of the emitted parton,

$$\eta = \frac{1}{2}\ln\frac{\alpha}{\beta} = \ln\frac{\alpha(2p\cdot n)}{|k_T|}. \tag{2.3}$$

For two subsequent emissions along a parton chain, strong ordering in rapidity implies,

$$\eta_{21} = \eta_2 - \eta_1 = \ln\frac{\alpha_2|k_{T,1}|}{\alpha_1|k_{T,2}|} \gg 1, \tag{2.4}$$

see also Fig. 2. It is now straight forward to realize that this condition covers both the collinear limit (strong ordering in transverse momenta, t proton momentum fraction of the same order of magnitude) and the high energy limit (strong ordering in proton momentum fractions, transverse momenta of the same order of magnitude). As an alternative to ordering in rapidity, one might instead consider ordering in the momentum fraction of the collision partner, *i.e.* making use of the notation of Fig. 1 in the β_is. Since

$$\beta_1 \gg \beta_2 \gg \dots \tag{2.5}$$

translates into

$$\frac{k_{T,1}^2}{\alpha_1 2n\cdot p} \gg \frac{k_{T,2}^2}{\alpha_2 2n\cdot p} \gg \dots \tag{2.6}$$

for real produced partons, ordering in the Sudakov parameters β allows for a combination of both DGLAP and high energy kinematics.

3. Transverse momentum dependent splitting kernels

Our starting point for the formulation of transverse momentum dependent parton evolution is given by the 2PI-expansion of [10].We include so-called kinematic higher twist effects to all orders, but restrict ourselves for the moment to the dilute region where exchange of multiple chains is subleading. β-ordering corresponds then for a certain splitting kernel K_{ij}, $i, j = q, g$ to treat the outgoing momentum q (in the t-channel) exactly, while the β-component of the incoming momentum k is ignored; the momentum is taken within the typical kinematics of high energy factorization

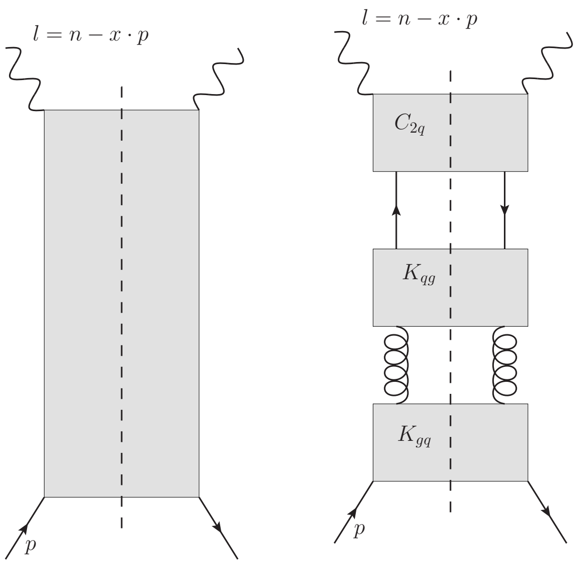

Figure 3: Factorization of a 4-point correlator

$k = xp + k_T$. Requiring agreement in the well-established limiting cases, *i.e.* the high energy and collinear limit, as well as ensuring current conservation in presence of off-shell t-channel partons, the following angular averaged real splitting kernels have been determined:

$$\bar{P}_{qg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = T_R \left(\frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2}\right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2\frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right], \tag{3.1}$$

$$\bar{P}_{gq}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = C_F \left[\frac{2\tilde{\boldsymbol{q}}^2}{z|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} - \frac{(2-z)\tilde{\boldsymbol{q}}^4 + z(1-z^2)\boldsymbol{k}^2\tilde{\boldsymbol{q}}^2}{\left(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2\right)^2}\right], \tag{3.2}$$

$$\bar{P}_{qq}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = C_F \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \times \left[\frac{\tilde{\boldsymbol{q}}^2 + (1-z^2)\boldsymbol{k}^2}{(1-z)|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} + \frac{z^2\tilde{\boldsymbol{q}}^2 - z(1-z)(1-3z+z^2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)}\right]. \tag{3.3}$$

$$\bar{P}_{gg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = C_A \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \left[\frac{(2-z)\tilde{\boldsymbol{q}}^2 + (z^3 - 4z^2 + 3z)\boldsymbol{k}^2}{z(1-z)\left|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2\right|} + \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\boldsymbol{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)}\right] \tag{3.4}$$

The obtained splitting functions do not only incorporate the correct high energy and collinear limits, but furthermore re-produce in the limit of vanishing transverse momentum of the emitted gluon (for the quark-to-quark and gluon-to-gluon splitting) the corresponding terms of the CCFM kernel [21, 22] and will allow in the future for a first exploration of transverse momentum dependent splitting kernels in the parton chain.

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References

[1] R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer and G. G. Ross, "Perturbation Theory and the Parton Model in QCD," Nucl. Phys. B **152**, 285 (1979). doi:10.1016/0550-3213(79)90105-6

[2] J. C. Collins, D. E. Soper and G. F. Sterman, "Factorization for Short Distance Hadron - Hadron Scattering," Nucl. Phys. B **261** (1985) 104. doi:10.1016/0550-3213(85)90565-6

[3] G. T. Bodwin, "Factorization of the Drell-Yan Cross-Section in Perturbation Theory," Phys. Rev. D **31** (1985) 2616 Erratum: [Phys. Rev. D **34** (1986) 3932]. doi:10.1103/PhysRevD.34.3932, 10.1103/PhysRevD.31.2616

[4] F. Hautmann, H. Jung, A. Lelek, V. Radescu and R. Zlebcik, JHEP **1801**, 070 (2018) doi:10.1007/JHEP01(2018)070 [arXiv:1708.03279 [hep-ph]].

[5] A. Bermudez Martinez, P. Connor, H. Jung, A. Lelek, R. Zlebcik, F. Hautmann and V. Radescu, Phys. Rev. D **99** (2019) no.7, 074008 doi:10.1103/PhysRevD.99.074008 1804.11152.

[6] V. S. Fadin, E. A. Kuraev and L. N. Lipatov, "On the Pomeranchuk Singularity in Asymptotically Free Theories," Phys. Lett. **60B**, 50 (1975). doi:10.1016/0370-2693(75)90524-9

[7] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, "Multi - Reggeon Processes in the Yang-Mills Theory," Sov. Phys. JETP **44**, 443 (1976) [Zh. Eksp. Teor. Fiz. **71**, 840 (1976)].

[8] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, "The Pomeranchuk Singularity in Nonabelian Gauge Theories," Sov. Phys. JETP **45**, 199 (1977) [Zh. Eksp. Teor. Fiz. **72**, 377 (1977)].

[9] I. I. Balitsky and L. N. Lipatov, "The Pomeranchuk Singularity in Quantum Chromodynamics," *Sov.J.Nucl.Phys.* **28** (1978) 822–829.

[10] G. Curci, W. Furmanski, and R. Petronzio, "Evolution of Parton Densities Beyond Leading Order: The Nonsinglet Case," *Nucl. Phys.* **B175** (1980) 27–92.

[11] S. Catani, M. Ciafaloni, and F. Hautmann, "High-energy factorization and small x heavy flavor production," *Nucl. Phys.* **B366** (1991) 135–188.

[12] V. S. Fadin, L. N. Lipatov, "BFKL pomeron in the next-to-leading approximation," Phys. Lett. B **429** (1998) 127, doi:10.1016/S0370-2693(98)00473-0 hep-ph/9802290;

[13] M. Ciafaloni, G. Camici, "Energy scale(s) and next-to-leading BFKL equation," Phys. Lett. B **430** (1998) 349, doi:10.1016/S0370-2693(98)00551-6, hep-ph/9803389.

[14] M. Hentschinski and A. Sabio Vera, "NLO jet vertex from Lipatov's QCD effective action," *Phys. Rev.* **D85** (2012) 056006, 1110.6741;

[15] G. Chachamis, M. Hentschinski, J. D. Madrigal Martinez, and A. Sabio Vera, "Next-to-leading order corrections to the gluon-induced forward jet vertex from the high energy effective action," *Phys. Rev.* **D87** (2013), no. 7, 076009, 1212.4992; "Forward jet production and quantum corrections to the gluon Regge trajectory from Lipatov's high energy effective action," *Phys. Part. Nucl.* **45** (jul, 2014) 788–799, 1211.2050; "Quark contribution to the gluon Regge trajectory at NLO from the high

energy effective action," *Nucl. Phys.* **B861** (2012) 133–144, 1202.0649; "Gluon Regge trajectory at two loops from Lipatov's high energy effective action," *Nucl. Phys.* **B876** (2013) 453–472, 1307.2591.

[16] M. Hentschinski, J. D. M. Martínez, B. Murdaca and A. Sabio Vera, "The gluon-induced Mueller-Tang jet impact factor at next-to-leading order," Nucl. Phys. B **889** (2014) 549 doi:10.1016/j.nuclphysb.2014.10.026 1409.6704; "The quark induced Mueller-Tang jet impact factor at next-to-leading order," Nucl. Phys. B **887** (2014) 309 doi:10.1016/j.nuclphysb.2014.08.010 1406.5625. "The next-to-leading order vertex for a forward jet plus a rapidity gap at high energies," Phys. Lett. B **735** (2014) 168 doi:10.1016/j.physletb.2014.06.022 1404.2937.

[17] A. van Hameren, "Calculating off-shell one-loop amplitudes for k_T-dependent factorization: a proof of concept," 1710.07609.

[18] F. Hautmann, M. Hentschinski, and H. Jung, "Forward Z-boson production and the unintegrated sea quark density," *Nucl. Phys.* **B865** (2012) 54–66, 1205.1759.

[19] O. Gituliar, M. Hentschinski, and K. Kutak, "Transverse-momentum-dependent quark splitting functions in k_T-factorization: real contributions," *JHEP* **01** (2016) 181, 1511.08439.

[20] M. Hentschinski, A. Kusina, and K. Kutak, "Transverse momentum dependent splitting functions at work: quark-to-gluon splitting," *Phys. Rev.* **D94** (2016), no. 11, 114013, 1607.01507.

[21] M. Ciafaloni, "Coherence Effects in Initial Jets at Small q**2 / s," *Nucl. Phys.* **B296** (1988) 49–74.

[22] S. Catani, F. Fiorani, and G. Marchesini, "QCD Coherence in Initial State Radiation," *Phys. Lett.* **B234** (1990) 339–345.