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Intermediate dynamics of the four pions production in e^+e^- annihilation and tau decay processes

E.A.Kozyrev[1,2], **E.P.Solodov**[1,2], **R.R.Akhmetshin**[1], **A.N.Amirkhanov**[1,2], **A.V.Anisenkov**[1,2], **V.M.Aulchenko**[1,2], **V.S.Banzarov**[1], **N.S.Bashtovoy**[1], **D.E.Berkaev**[1,2], **A.E.Bondar**[1,2], **A.V.Bragin**[1], **S.I.Eidelman**[1,2], **D.A.Epifanov**[1,2], **L.B.Epshteyn**[1,2,3], **A.L.Erofeev**[1,2], **G.V.Fedotovich**[1,2], **S.E.Gayazov**[1,2], **A.A.Grebenuk**[1,2], **S.S.Gribanov**[1,2], **D.N.Grigoriev**[1,2,3], **F.V.Ignatov**[1], **V.L.Ivanov**[1,2], **S.V.Karpov**[1], **A.S.Kasaev**[1], **V.F.Kazanin**[1,2], **A.A.Korobov**[1,2], **I.A.Koop** [1], **A.N.Kozyrev**[1,2], **P.P.Krokovny**[1,2], **A.E.Kuzmenko**[1,2], **A.S.Kuzmin**[1,2], **I.B.Logashenko**[1,2], **P.A.Lukin**[1,2], **A.P.Lysenko**[1], **K.Yu.Mikhailov**[1,2], **V.S.Okhapkin**[1], **E.A.Perevedentsev**[1,2], **Yu.N.Pestov**[1], **A.S.Popov**[1,2], **G.P.Razuvaev**[1,2], **Yu.A.Rogovsky**[1,2], **A.A.Ruban**[1], **N.M.Ryskulov**[1], **A.E.Ryzhenenkov**[1,2], **V.E.Shebalin**[1,2], **D.N.Shemyakin**[1,2], **B.A.Shwartz**[1,2], **D.B.Shwartz**[1,2], **A.L.Sibidanov**[4], **Yu.M.Shatunov**[1], **A.A.Talyshev**[1,2], **A.I.Vorobiov**[1], **Yu.V.Yudin**[1,2]

Author affiliation

[1]*Budker Institute of Nuclear Physics, SB RAS, Novosibirsk, 630090, Russia*
[2]*Novosibirsk State University, Novosibirsk, 630090, Russia*
[3]*Novosibirsk State Technical University, Novosibirsk, 630092, Russia*
[4]*University of Victoria, Victoria, British Columbia, Canada V8W 3P6*
E-mail: e.a.kozyrev@inp.nsk.su

In this article we print out preliminary results of an amplitude analysis of the $e^+e^- \to 4\pi$ process. The amplitude analysis is performed using experimental data from the CMD-3 detector at the VEPP-2000 collider in the energy range 0.9–2.007 GeV. In the study the dominance of the $e^+e^- \to \omega\pi^0 \to 4\pi$ and $e^+e^- \to a_1(1260)\pi \to 4\pi$ amplitudes is proved, as well as sizable contributions of the $\rho^+\rho^-, \rho f_0(980), \rho\sigma$ and etc. states are observed. The obtained amplitude is used for the test of the isospin relations and hypothesis of conserved vector current (CVC).

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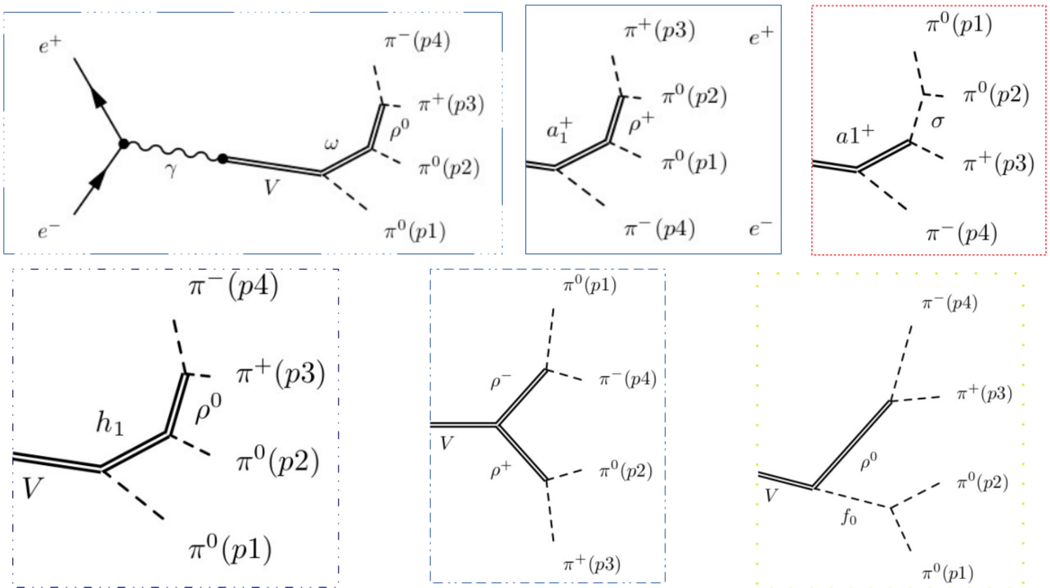
1. Introduction

The processes $\mathrm{e^+e^-} \to 2\pi^0\pi^+\pi^-$ and $\mathrm{e^+e^-} \to 2\pi^+2\pi^-$ dominate the hadronic cross section at low center-of-mass energies $E_{\mathrm{c.m.}} < 2$ GeV. Previously, amplitude analysis of $e^+e^- \to 4\pi$ was performed by the CMD-2, BaBar and CLEO collaborations. BaBar [1, 2] carried out the most precise cross section measurement in the region $\mathrm{E_{c.m.}} = 0.85 \div 4.5$ GeV, but analysis of the intermediate amplitudes was not performed, though simple calculations (by selection cuts) of the $\omega\pi^0$, $\rho f_0(980)$, $\rho^+\rho^-$ states were made. Some evidence for the presence of the ρf_2 state was also obtained at $\mathrm{E_{c.m.}} > 1.8$ GeV. The CMD-2 collaboration also published the results of the amplitude analysis in the energy range $1.05 < \mathrm{E_{c.m.}} < 1.38$ GeV based on 22k events [3]. The dominance of the $\omega\pi^0$ and $\mathrm{a}_1\pi$ was observed and other intermediate states were not found. Another analysis performed by CLEO [4] using about 24.5k events of the decay $\tau \to \pi^-3\pi^0\nu_\tau$ considered additional amplitudes. This analysis found that the models, providing the best description of the data, are dominated by $\omega\pi^0$ and $\mathrm{a}_1\pi$ with small additional contributions of the $\sigma\rho$, $f_0(980)\rho$ or non-resonant $\rho\pi\pi$ channels.

A large data sample of their events collected by the CMD-3 experiment at the VEPP-2000 e^+e^- collider allows for an amplitude analysis (AA) at $0.95 < E_{\mathrm{c.m.}} < 2.007$ GeV. This study uses about 43 pb^{-1} of an integrated luminosity accumulated in four scans at 85 energy points. This work is aimed to probe the structure of the hadronic current of the $e^+e^- \to 4\pi$. Different parts of the current are partially schematically demonstrated in the Fig. 1. In the assumption of the dominance



of the two-body decays we consider the following list of the intermediate states:

- $\omega[J^{PC}=1^{--}]$ $\pi^0[0^{-+}]$ (only $\to 2\pi^\pm 2\pi^0$)
- $a_1(1200)[1^+]$ $\pi[0^-]$
- $\rho[1^{--}]$ $f_0(980)[0^{++}]$
- $\rho[1^{--}]$ $\sigma(500)[0^{++}]$
- $\pi(1300)^\pm[0^-]\pi^\mp[0^-]$
- $\rho[1^{--}]$ $f_2(1270)[2^{++}]$
- $\rho^+[1^-]$ $\rho^-[1^-]$ (only $\to 2\pi^\pm 2\pi^0$)
- $a_2(1320)[2^{++}]$ $\pi[0^-]$
- $h_1(1170)[1^{+-}]$ $\pi^0[0^{-+}]$ (only $\to 2\pi^\pm 2\pi^0$)

2. Amplitude analysis

The description of events selection for the AA can be found in the paper [5]. The AA is performed by minimization of the unbined Likelihood function for a particular model

$$L = -\ln \prod_{\rm i}^{\rm e^+e^-\to 2\pi^0\pi^+\pi^-} \frac{|\sum_\alpha \mathbf{V}_\alpha \mathrm{A}^0_\alpha(\Omega_{\rm i})|^2}{\frac{1}{\mathrm{N_{MC}^{gen}}}\sum_{\rm k}^{\text{rec. ph. space MC}} |\sum_\alpha \mathbf{V}_\alpha \mathrm{A}^0_\alpha(\Omega_{\rm k})|^2} - \ln \prod_{\rm j}^{\rm e^+e^-\to 2\pi^+2\pi^-} \frac{|\sum_\alpha \mathbf{V}_\alpha \mathrm{A}^\pm_\alpha(\Omega_{\rm j})|^2}{\frac{1}{\mathrm{N_{MC}^{gen}}}\sum_{\rm k}^{\text{rec. ph. space MC}} |\sum_\alpha \mathbf{V}_\alpha \mathrm{A}^\pm_\alpha(\Omega_{\rm k})|^2}, \tag{2.1}$$

where i and j run over all selected events. The sum α runs over all intermediate states, $\mathbf{V}_\alpha$ - a complex model parameter. The quantities $\mathbf{V}_\alpha A^\pm_\alpha\Big(p_1(\pi^+), p_2(\pi^-), p_3(\pi^+), p_4(\pi^-)\Big)$ and $\mathbf{V}_\alpha A^0_\alpha\Big(p_1(\pi^0), p_2(\pi^0), p_3(\pi^+), p_4(\pi^-)\Big)$ are the specific components α of the total matrix elements at particular points in phase space for the $\mathrm{e^+e^-} \to 2\pi^+2\pi^-$ and $\mathrm{e^+e^-} \to 2\pi^0\pi^+\pi^-$ channels, respectively. The sum in the denominator runs over all MC events, flatly generated in phase space and passed all selection criteria above. The effects of detection resolution and initial state radiation are under control by the AA of the toy MC samples. An example of an amplitude for decay $\rho' \to \rho f_2(1270)$ in isobar model: $A(\rho'\rho f_2) = \mathbf{V}_{\rho'\rho\mathbf{f_2}} \cdot \rho'_\alpha \rho^\star_\beta \cdot \omega^\star_{\alpha\beta}, \cdot \delta^{ab}\phi^a_{\rho'}\phi^{\star b}_\rho$, where $\omega_{\alpha\beta}$ - the polarization tensor of f_2-meson; a,b - isospin indexes.

The parametrization of the different components of the matrix element is the same as used in Ref. [3]. Used amplitudes satisfy requirements from C invariance: $A^0_\alpha(p_1,p_2,p_3,p_4) = -A^0_\alpha(p_1,p_2,p_4,p_3)$, Bose symmetry: $A^0_\alpha(p_1,p_2,p_3,p_4) = A^0_\alpha(p_2,p_1,p_3,p_4)$, Gauge invariance: $q^\mu A_\mu(p_1,p_2,p_3,p_4) = 0$, where $q = p_{e^-} + p_{e^+}$. In the isospin limit: $A^\pm_\alpha\Big(p_1(\pi^+), p_2(\pi^-), p_3(\pi^+), p_4(\pi^-)\Big) = A^0_\alpha\Big(p_1,p_2,p_3,p_4\Big) + A^0_\alpha\Big(p_3,p_2,p_1,p_4\Big) + A^0_\alpha\Big(p_1,p_4,p_3,p_2\Big) + A^0_\alpha\Big(p_3,p_4,p_1,p_2\Big)$. Masses and central values of resonance widths are fixed according to PDG. A fraction f_X of an individual component of the matrix element is calculated as

$$f_X^{\pm/0} = \frac{\int |\mathbf{V_X} A_X^{\pm/0}(\Omega)|^2 d\Omega}{\int |\sum_\alpha \mathbf{V}_\alpha A_\alpha^{\pm/0}(\Omega)|^2 d\Omega}. \tag{2.2}$$

and presented in the Fig. 1. Values of the obtained phases of the amplitudes are not shown in the article due to the page limit.

3. data-MC comparison

The data-MC comparison of the invariant mass spectra for the process $\mathrm{e^+e^-} \to 2\pi^0\pi^+\pi^-$ and $\mathrm{e^+e^-} \to 2\pi^+2\pi^-$ in the energy ranges $\sqrt{s} = 1.7 \div 1.8$ GeV is shown in Fig. 2. In general build model is well agree with data, though two pion mass spectra demonstrates remaining data-MC discrepancy. The analysis is still ongoing and updated results is expected soon.

We generate 10-30 toy Monte Carlo samples at each energy point. Each sample is generated with parameters obtained in the fit to the data and the number of events in each sample is equal to that observed in the data. The goodness-of-fit is estimated as a fraction of samples where $G = \sum_i^{N_{exp}}(log|M_i|^2)$ is less than in the data, where $M = |\sum_\alpha \mathbf{V}_\alpha A_\alpha(\Omega_k)|^2$ - is the total matrix element of the $e^+e^- \to 4\pi$. The G takes values around 0.5±0.3 for different energy points.

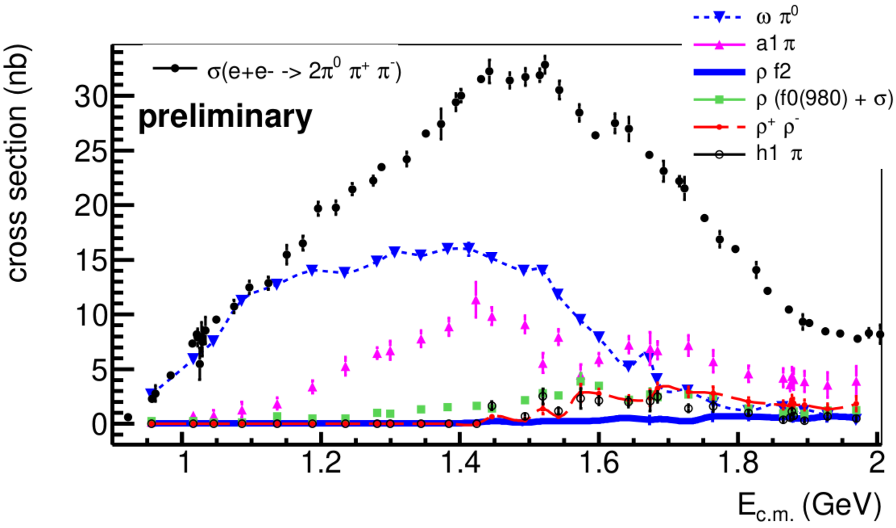


Figure 1: $f_X^0 \cdot \sigma(e^+e^- \to 2\pi^0\pi^+\pi^-)$ - cross sections calculated for different components of the matrix element. The total cross section of the process $e^+e^- \to 2\pi^0\pi^+\pi^-$, measured in this analysis (filled circles).

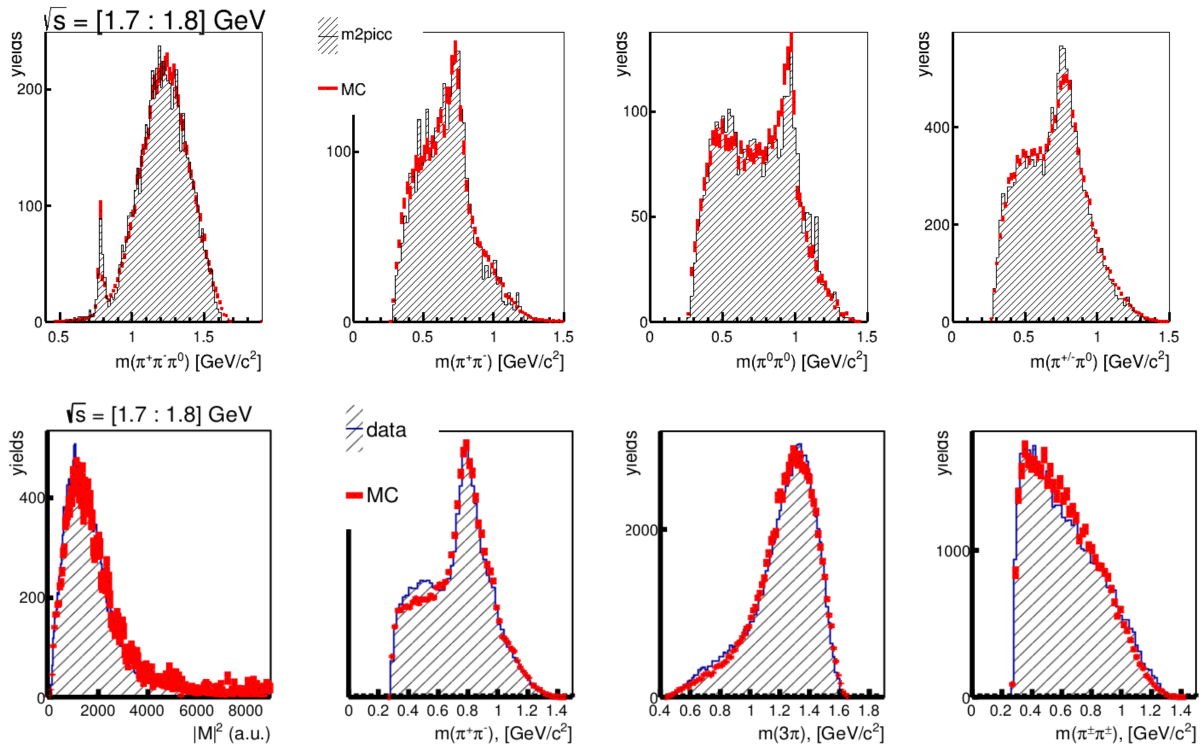


Figure 2: The data-MC comparison of the invariant mass spectra for the process $e^+e^- \to 2\pi^0\pi^+\pi^-$ (top) and $e^+e^- \to 2\pi^+2\pi^-$ (bottom) in the energy ranges $\sqrt{s} = 1.7 \div 1.8$ GeV.

4. The prediction of the τ decays $R_X = \frac{\Gamma(\tau^- \to \nu_\tau X) \cdot B(X \to 4\pi)}{\Gamma(\tau^- \to \nu_\tau 4\pi)}$

The obtained amplitudes $A^{\pm/0}(\Omega)$ and CVC hypothesis allow to predict the matrix elements of the charged vector current relevant for τ decay. Fractions $f_X^{\pm/0}$ can be used for the calculation of τ decays to intermediate states:

$$R_X = \frac{\Gamma(\tau^- \to \nu_\tau X) \cdot B(X \to 4\pi)}{\Gamma(\tau^- \to \nu_\tau 4\pi)} == \frac{\int_0^{m_\tau^2} dQ^2 V(Q^2)\Big(f_X^{\pm}(Q^2) \cdot \sigma(e^+e^- \to 2\pi^+ 2\pi^-)(Q^2) + f_X^0(Q^2) \cdot \sigma(e^+e^- \to 2\pi^0 \pi^+ \pi^-)(Q^2)\Big)}{\int_0^{m_\tau^2} dQ^2 V(Q^2)\Big(\sigma(e^+e^- \to 2\pi^+ 2\pi^-)(Q^2) + \sigma(e^+e^- \to 2\pi^0 \pi^+ \pi^-)(Q^2)\Big)}$$

$$V(Q^2) = Q^2 \cdot (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2)$$

The predictions can be checked in future detailed studies of tau decays. The predictions is listed in the following table, where only statistical uncertainty of f_X us taken into account:

X	R_X
$\omega\pi^-$	0.41 ± 0.01
$a_1(1260)\pi$	0.197 ± 0.008
$\rho^-[\sigma(500) + f_0(980)]$	0.044 ± 0.001
$\rho^- f_2(1270)$	0.00076 ± 0.00008
$\rho^-\rho^0$	0.0045 ± 0.0005
$h_1(1170)\pi^-$	0.006 ± 0.001

5. Conclusion

- The preliminary study of internal dynamics is performed for the process $e^+e^- \to 4\pi$ at $E_{\text{c.m.}} < 2$ GeV;
- The dominance of states $\omega\pi^0$ and $a_1\pi$ is observed;
- The estimation of the contributions of $\rho(\sigma(500) + f_0(980)), \rho^+\rho^-, \rho f_2(1270)$ and $h_1(1170)\pi^0$ is shown.

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References

[1] J. P. Lees *et al.* (BABAR Collaboration), Phys. Rev. D **376**, 092009 (2017).

[2] J. P. Lees *et al.* (BABAR Collaboration), Phys. Rev. D **85**, 112009 (2012).

[3] R.R. Akhmetshin *et al.* (CMD-2 Collaboration), Physics Letters B **466** 392âĂŞ402 (1999).

[4] K. W. Edwards *et al.* (CLEO Collaboration), Phys. Rev. D **61**, 072003 (2000).

[5] E. A. Kozyrev, *et al.* EPJ Web Conf. **212** 03008 (2019).