Recent results on hadronic cross sections measurements at BABAR for the $g - 2$ calculation

V. P. Druzhinin$^{ab}$, for BABAR collaboration

$^a$ Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia
$^b$ Novosibirsk State University, Novosibirsk, 630090, Russia
E-mail: druzhinin@inp.nsk.su

A program of measuring the light hadrons production in exclusive $e^+e^- \rightarrow \text{hadrons}$ processes is in place at BABAR with the aim to improve the calculation of the hadronic contribution to the muon $g - 2$. We present the most recent results obtained by using the full data set of about 470 fb$^{-1}$ collected by the BABAR experiment at the PEP-II $e^+e^-$ collider at a center-of-mass energy of about 10.6 GeV. In particular, we report the results on the channels $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$, $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$, $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0\eta$, and $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\eta$. The first reaction, in particular, presently gives the main uncertainty on the total hadronic cross section in the energy region between 1 and 2 GeV.
1. Introduction

The precise measurement of $e^+e^-$ annihilation into hadrons is needed, in particular, for Standard Model (SM) calculation of the anomalous magnetic moment of muon $a_\mu = (g_\mu - 2)/2$. Currently, 3.3–4.1σ difference is observed between experiment [1] and the SM calculations [2, 3, 4], and the experimental and theoretical accuracies are close to each other. A new measurement is currently carried out at Fermilab [5], which is expected to improve the $a_\mu$ accuracy by a factor of at least 4. Another measurement is planned at J-PARC [6]. More than 50% of the SM $a_\mu$ error comes from the leading-order hadronic vacuum polarization contribution $d_{\mu}^{\text{had,LO}}$, which cannot be obtained accurately from theory alone. It is calculated using dispersion relation from experimental measurements of the total cross section $e^+e^-$ annihilation into hadrons. Low energies, below 2 GeV, give the dominant contribution to $d_{\mu}^{\text{had,LO}}$. In this energy region the total hadronic cross section is determined as a sum of exclusive hadronic cross sections.

The BABAR detector [7] collected data at the PEP-II asymmetric $e^+e^-$ collider at SLAC (9 GeV $e^-$ and 3.1 GeV $e^+$) in 1999-2008. It has an extensive program of measurement of exclusive hadronic cross sections at low energy based on the initial state radiation (ISR) technique. A data sample of 469 fb$^{-1}$ recorded near or at a center-of-mass energy of 10.58 GeV is used in these measurements. In the ISR process $e^+e^- \rightarrow f\gamma$, the mass spectrum of the hadronic system $f$ is related to the cross section of the reaction $e^+e^- \rightarrow f$. This allow to perform the measurement of the low-energy hadronic cross section in a wide energy range at a high-luminosity collider operating at a fixed energy. BABAR has studied more than 30 final hadronic states: all two- and three-body, almost all four-body, partly five- and six-body. The main goal is to measure (together with other experiments) all final states contributing to $d_{\mu}^{\text{had,LO}}$ below 2 GeV.

2. Study of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ and $e^+e^- \rightarrow \pi^+\pi^-\eta$ reactions

The process $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ has a largest hadronic cross section in the energy region 1.2–2.2 GeV, and is very important for the $d_{\mu}^{\text{had,LO}}$ calculation. Its cross section measured by BABAR [8] is shown in Fig. 1 (left) in comparison with previous measurements. The BABAR results are the

Figure 1: Left panel: The $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ cross section measured by BABAR in comparison with previous measurements. Right panel: The $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ cross section measured by BABAR in comparison with previous measurements.
most precise and cover a wider energy range. The systematic uncertainty is 3.1% in the 1.2–2.7 GeV energy range. The contribution to $d_{\mu}^{had,LO}$ for $1.02 < \sqrt{s} < 1.8$ GeV is measured to be $(179 \pm 6) \times 10^{-11}$ (3.4% precision). This significantly improves the previous result $(180 \pm 12) \times 10^{-11}$.

The reaction $e^+e^- \rightarrow \pi^+\pi^-\eta$ is studied [9] in the $\eta \rightarrow \gamma\gamma$ mode. It is expected to proceed via the $\rho(770)\eta$ intermediate state and is important for spectroscopy of excited $\rho$-like states.

The BABAR results on the $e^+e^- \rightarrow \pi^+\pi^-\eta$ cross section shown in Fig. 1 (right) agrees well with the previous measurements, but is more precise and covers a wider energy range. The systematic uncertainty near the cross-section maximum, 1.35–1.80 GeV, is 4.5%.

The $\pi^+\pi^-$ mass spectrum for data from the energy region 1.4–2.0 GeV is shown in Fig. 2 (left) in comparison with the simulated signal spectrum. The simulation uses the model of the $\rho(770)\eta$ intermediate state. The observed difference between data and simulated spectra may be explained by the contribution of other intermediate states, for example $\rho(1450)\eta$, and their interference with the dominant $\rho(770)\eta$ amplitude. This effect was observed previously in the SND experiment [10].

![Figure 2](image) Left panel: The two-pion invariant mass distribution for data (points with error bars) and simulated (histogram) events from the mass range $1.4 < m_{\pi^+\pi^-} < 2.0$ GeV. Right panel: The comparison of the $B(\tau^+ \rightarrow \pi^+\pi^-\eta\nu_\tau)$ values calculated using CVC hypothesis from the $e^+e^- \rightarrow \pi^+\pi^-\eta$ cross section with direct measurements.

The conserved vector current (CVC) hypothesis and isospin symmetry allow to use data on the $e^+e^- \rightarrow \pi^+\pi^-\eta$ cross section to predict the branching fraction for the decay $\tau^- \rightarrow \pi^-\pi^0\eta\nu_\tau$. The BABAR results on this branching fraction based on the $e^+e^- \rightarrow \pi^+\pi^-\eta$ measurement in the $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^+\pi^-\pi^0$ [11] modes is $B(\tau^- \rightarrow \pi^-\pi^0\eta\nu_\tau) = 0.163 \pm 0.008\%$. Its comparison with direct measurements and previous CVC based calculations is presented in Fig. 2 (right). The difference between the PDG value [12] and our calculation is $1.8\sigma$.

### 3. Study of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\eta^0$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\eta^0$ reactions

The $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\eta^0$ cross section was measured previously in the M3N, MEA experiments with very limited precision. The BABAR result [13] is shown in Fig. 3 (left). The systematic uncertainty of the BABAR measurement below 2 GeV is 10%. The four intermediate states contribute into the cross section: $\pi^+\pi^-\eta$, $\omega\pi^0\pi^0$, $\rho^+\pi^+\pi^0\pi^0$ and $\rho^+\rho^-\pi^0$. The states with $\rho$ meson dominate above 2 GeV. The fraction of $\rho^+\rho^-\pi^0$ events relative to the number of $\rho^+\pi^+\pi^0\pi^0$ events is 50% below 2.5 GeV. This fraction decreases with increase of energy and is 20% near 3 GeV. Below 1.8 GeV the $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0\eta^0$ cross section is saturated by the $e^+e^- \rightarrow \pi^+\pi^-\eta$ and

---

**Recent results on hadronic cross sections measurements at BABAR**

V. P. Druzhinin

---

POSS(EPS-HEP2019)535
$e^+ e^- \rightarrow \omega \pi^0 \pi^0$ reactions. The $e^+ e^- \rightarrow \pi^+ \pi^- \eta$ cross section measured in the $\eta \rightarrow 3\pi^0$ mode agrees well with the measurements discussed in the previous section. The $e^+ e^- \rightarrow \omega \pi^0 \pi^0$ cross section measured for the first time is presented in Fig. 3 (right).

**Figure 3:** The $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0 (\eta)$ (left) and $e^+ e^- \rightarrow \omega \pi^0 \pi^0 (\eta)$ (right) cross sections measured by BABAR.

The cross section for the process $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \eta$ shown in Fig. 4 (left) is measured for the first time [13]. The systematic uncertainty of this measurement below 2.5 GeV is 13%. This process contributes significantly to the total hadronic cross section near 2 GeV. The $\pi^+ \pi^- \pi^0 \pi^0 \eta$ final state, like that for $\pi^+ \pi^- \omega \pi^0 \pi^0$, has a rich substructure. The dominant intermediate state below 2 GeV is $\omega \pi^0 \pi^0$. The $e^+ e^- \rightarrow \omega \pi^0 \pi^0$ cross section measured by BABAR is shown in Fig. 4 (right) in comparison with previous results from SND [14]. The SND data are seen to lie systematically above our data. A significant fraction of the $e^+ e^- \rightarrow \omega \pi^0 \eta$ events contain $a_0(980)$ decaying to $\pi^0 \eta$. Above 2.5 GeV the dominant mechanism is $\rho^+ \pi^- \pi^0$. Some fraction of these events contain two $\rho$ meson, i.e. proceed via $\rho^+ \rho^- \eta$ intermediate state. Between 2 and 3 GeV there is also signal of the OZI-suppressed process $e^+ e^- \rightarrow \phi \pi^0 \eta$, which contribution to the $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \eta$ cross section reaches 15%.

**Figure 4:** The $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \eta (\text{left})$ and $e^+ e^- \rightarrow \omega \pi^0 \pi^0 (\text{right})$ cross sections measured by BABAR. The latter cross section are compared with previous SND measurement [14].

### 4. Summary

Precise low-energy $e^+ e^-$ hadronic cross section data are needed to obtain an accurate SM prediction for $\alpha_{\mu}^{\text{rad,LO}}$. Recent BABAR results on $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0$ and other processes reduce the respective uncertainty in $\alpha_{\mu}^{\text{rad,LO}}$. Two previously unmeasured processes $e^+ e^- \rightarrow \pi^+ \pi^- \omega \pi^0 \pi^0$ and $e^+ e^- \rightarrow \pi^+ \pi^- \omega \pi^0 \pi^0 \eta$ contributing to the total hadronic cross section below 2 GeV have been studied. Several ISR processes are under analysis or planned to be studied at BABAR: $e^+ e^- \rightarrow \pi^+ \pi^-,$
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ with the full BABAR dataset, $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-3\pi^0$, $e^+e^- \rightarrow KSK^+\pi^-\pi^0\pi^0$, etc. Currently, the sum of exclusive cross sections near 2.0 GeV shows a systematic deviation from the QCD predictions. BABAR measurements of previously unmeasured processes may reduce this deviation.

References