PROCEEDINGS OF SCIENCE

Mathematical aspects of the scattering amplitude for $H \to gg$ within the loop-tree duality

PoS(EPS-HEP2019)545

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In this talk, we study the decay width of $H \to gg$ at next-to-leading order in the large top quark mass effective limit. We follow the ideas of the four-dimensional-unsubtraction method, in which a carefully study of infrared and ultraviolet singularities is carried out. In fact, the cancellation of these singularities is performed at integrand level.

European Physical Society Conference on High Energy Physics - EPS-HEP2019 -
10-17 July, 2019
Ghent, Belgium

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[†]Talk based on a collaboration with R. Hernandez, A. Renteria and G. Rodrigo.

1. Introduction

The calculation of observables has became very important in the physics that the Large hadron collider (LHC) delivered with run 2. Hence, in view of the new run and the prospectives for the design of the new experiments, the scientific community needs to motive their construction. To this end, a possible strategy one could take is increasing the precision in theoretical predictions. In other words, considering processes with the highest possible accuracy, which start combining EW and QCD corrections. Nevertheless, when increasing the accuracy more calculations, from the perturbation theory, must be performed. Then, the search for efficient methods is desirable. Therefore, as an alternative, we use the loop-tree duality (LTD) theorem [1, 2], a relation that combines loop with phase space integrals. It is worth to remark that recent progress in physical process at two-loop level have been done in this direction [3 – 5]. Also, alternative procedures of applying LTD have also discussed [6, 7].

The phenomenological application of the Higgs boson production via gluon fusion has been extensively studied in the full theory of the standard model and in the effective field theory approach where the Higgs boson couples directly to gluons. The latter is straightforwardly obtained by considering the heavy top mass limit. In this talk, following the ideas of universal dual amplitudes in the loop-tree duality formalism [8], we analyse the one-loop amplitude $H \to gg$ in the large top mass limit. In fact, we show that independently of the particles running in the loop (scalar, quarks, vector bosons and gluons), we recover the same functional structure. We present the decay width of $H \to gg$, in which local ultraviolet (UV) renormalisation and local infrared (IR) cancellation are done by means of the four-dimensional-unsubtraction (FDU) method [9, 10]. We compare our results with [11].

Although in this talk we show preliminary results, we mainly focus on the local UV renormalisation and the IR subtraction one achieves from straightforward change of variables. The latter is inspired by the FDU method. An extensive study is currently carried out in [12].

2. Decay width at LO

Let us first recall the d-dimensional phase-space associated to $1 \to 2$ and $1 \to 3$,

$$\int d\Phi_{1\to 2} = \frac{\Gamma(1-\varepsilon)}{2(4\pi)^{1-\varepsilon}\Gamma(2-2\varepsilon)} \left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon}, \tag{2.1}$$

$$\int d\Phi_{1\to 3} = \frac{s_{12}}{2(4\pi)^{3-2\varepsilon}\Gamma(2-2\varepsilon)} \left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon} \left(y'_{12}y'_{1r}y'_{2r}\right)^{-\varepsilon} dy'_{1r}dy'_{2r}, \tag{2.2}$$

with the recursive relation

$$\int d\Phi_{1\to 3} = \frac{(4\pi)^{\varepsilon-2} s_{12}}{\Gamma(1-\varepsilon)} \left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon} \left(\int d\Phi_{1\to 2}\right) \left(y'_{12}y'_{1r}y'_{2r}\right)^{-\varepsilon} dy'_{1r}dy'_{2r}, \tag{2.3}$$

Therefore, the decay widh of $H \to gg$ at LO turns out to be

$$\Gamma^{(0)}_{gg} = \int d\Phi_{1\to 2} \left|\mathscr{A}^{(0)}_{gg}\right|^2 = \frac{G_F m_H^2}{36\sqrt{2}\pi} \left(\frac{\alpha_S}{2\pi}\right)^2 C_A C_F\,. \tag{2.4}$$

with $C_A = 3$, $C_F = 4/3$ and

$$\left|\mathscr{A}_{gg}^{(0)}\right|^2 = \frac{1}{2}(d-2)\, g_{\rm EFT}^2\, m_H^4\, C_A C_F\,, \tag{2.5}$$

we recover the known result [13, 14].

In the following, we use

$$c_\Gamma = \frac{\Gamma(1-\varepsilon)^2\Gamma(\varepsilon+1)}{(4\pi)^{-\varepsilon}\Gamma(1-2\varepsilon)}\,. \tag{2.6}$$

3. Decay width at NLO

3.1 Virtual correction of Γ_{gg}

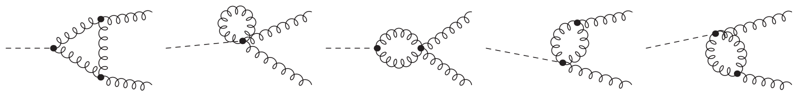

Figure 1: One-loop correction to $H \to gg$.

The QCD one-loop correction to the amplitude $H \to gg$ after summing over internal states and accounting for the finite renormalisation term is,

$$\Gamma_{gg}^{(1)} = C_A\,\Gamma_{gg}^{(0)}\, c_\Gamma \left(\frac{\alpha_S}{2\pi}\right)\left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon}\left(-\frac{2}{\varepsilon_{\rm IR}^2} + \frac{2}{\varepsilon_{\rm UV}} - \frac{2}{\varepsilon_{\rm IR}} + \frac{11}{3} + \pi^2\right)\,. \tag{3.1}$$

Then, to remove the UV singularity, we locally construct the UV counter-term

$$\Gamma_{gg}^{(1,{\rm ct})} = iC_A\, g_s^2 \int_{\ell_{\rm UV}} G_F^2\left(q_{3;{\rm UV}}\right)\left(c_3\, G_F\left(q_{3;{\rm UV}}\right) + c_2\right)\,, \tag{3.2}$$

with

$$c_3 = \frac{8(3d-10)(p_1\cdot q_1)(p_2\cdot q_1)}{(d-2)s_{12}}\,, \tag{3.3}$$

$$c_2 = \frac{(d-10)(d-3)}{d-2}\,, \tag{3.4}$$

where $c_3^{(\rm sub)}$ is a subleading term that fixes the finite part of the counter-term.

Furthermore, the c-t (3.2) with the use of IBPs can be cast into

$$\Gamma_{gg}^{(1,{\rm ct})} = iC_A\, g_s^2 \int_{\ell_{\rm UV}} G_F\left(q_{3;{\rm UV}}\right) c_1\,, \tag{3.5}$$

with

$$c_1 = \frac{1}{\mu_{\rm UV}^2}\left((d-10)d+20\right)\,. \tag{3.6}$$

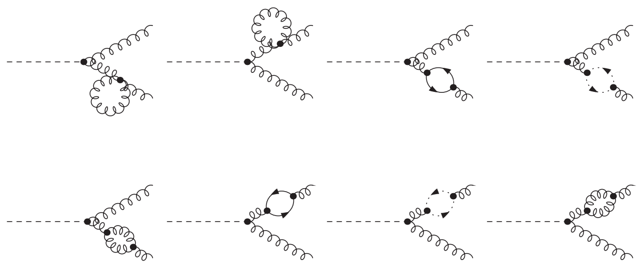

Figure 2: One-loop wave function correction to the decay of $H \to gg$.

3.2 Massless bubbles

Since we are independently studying UV and IR divergencies, we have to keep diagrams that usually vanish in dimensional regularisation, i.e. scaleless integrals. To this end, we consider the massless bubbles of Fig. 2. However, these diagrams are not well defined but can be collapsed into

$$\Gamma_{gg}^{(1,0)} = -i\,\Pi_2^{(1)}\left(p_i;r_i\right)\Gamma_{gg}^{(0)}\,, \tag{3.7}$$

where $\Pi_2^{(1)}\left(p_i;q_i\right)$ is the unrenormalised one-loop vacuum polarisation with an external massless momentum, $p_i^2 = 0$, and an auxiliary massless momentum r_i.

Due to the way how Π_2 is constructed, we can decomposed as

$$\Pi_2^{(1)}\left(p_i;q_i\right) = C_A\,\Pi_2^{(1,g)}\left(p_i;q_i\right) + n_f\,\Pi_2^{(1,f)}\left(p_i;q_i\right)\,, \tag{3.8}$$

with

$$\begin{aligned}
\Pi_2^{(1,g)}\left(p_1;p_2\right) &= \frac{g_s^2}{2s_{12}^2}\int_\ell G_F\left(q_1\right)G_F\left(q_3\right)\left(16(d-2)p_2\cdot q_1\, p_2\cdot q_3 + (d-6)s_{12}^2\right) \\
&= i\,c_\Gamma\left(\frac{\alpha_S}{4\pi}\right)\left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon}\left(-\frac{5}{3\varepsilon_{\rm IR}} + \frac{5}{3\varepsilon_{\rm UV}}\right)\,,
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\Pi_2^{(1,f)}\left(p_1;p_2\right) &= -\frac{16g_s^2}{s_{12}^2}\int_\ell G_F\left(q_1\right)G_F\left(q_3\right)p_2\cdot q_1\, p_2\cdot q_3\,. \\
&= i\,c_\Gamma\left(\frac{\alpha_S}{4\pi}\right)\left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon}\left(\frac{2}{3\varepsilon_{\rm IR}} - \frac{2}{3\varepsilon_{\rm UV}}\right)\,,
\end{aligned} \tag{3.10}$$

Then, by applying a local UV renormalisation we find

$$\Pi_2^{(x,{\rm ct})} = iC_A\,g_s^2\int_{\ell_{\rm UV}} G_F^2\left(q_{3;{\rm UV}}\right)\Big(c_4\,G_F^2\left(q_{3;{\rm UV}}\right) + c_3\,G_F\left(q_{3;{\rm UV}}\right) + c_2 + c_3^{({\rm sub})}\,G_F\left(q_{3;{\rm UV}}\right)\Big)\,, \tag{3.11}$$

with

$$c_4^{(f)} = -\frac{64(p_1\cdot q_1)^2(p_2\cdot q_1)^2}{s_{12}^2}\,, \qquad c_4^{(g)} = \frac{32(d-2)(p_1\cdot q_1)^2(p_2\cdot q_1)^2}{s_{12}^2}\,,$$

$$
\begin{aligned}
c_3^{(f)} &= \frac{16(p_1\cdot q_1)(p_2\cdot q_1)}{s_{12}}\,, & c_3^{(g)} &= -\frac{8(d-2)(p_1\cdot q_1)(p_2\cdot q_1)}{s_{12}}\,, \\
c_2^{(f)} &= 0\,, & c_2^{(g)} &= \frac{(d-6)}{2}\,, \\
c_3^{(f,\mathrm{sub})} &= 0\,, & c_3^{(g,\mathrm{sub})} &= -\frac{(d-2)\mu_{\mathrm{UV}}^2}{3}\,,
\end{aligned}
\tag{3.12}
$$

3.3 Real corrections

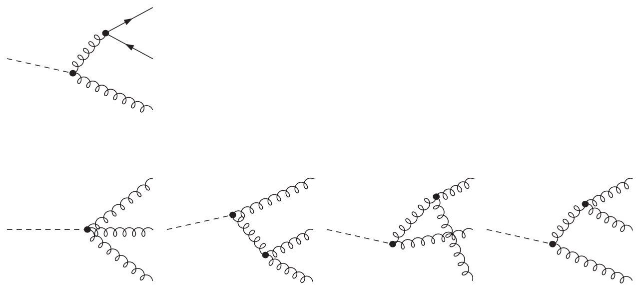

Figure 3: n_f and C_A contribution to the real radiation

For the real corrections we evaluate the diagrams of Fig 3, obtaining

$$
\left|\mathscr{A}_{q\bar{q}g}^{(0)}\right|^2 = \frac{n_f\, g_s^2}{m_H^4 s_{12}}\left(s_{13}^2 + s_{23}^2 + 2\frac{(d-4)}{(d-2)}s_{13}s_{23}\right)\left|\mathscr{A}^{(0)}\right|^2\,, \tag{3.13}
$$

$$
\left|\mathscr{A}_{ggg}^{(0)}\right|^2 = \frac{4C_A\, g_s^2}{m_H^4}\left[\left(\frac{s_{12}^3}{s_{13}s_{23}} + \frac{3s_{13}s_{23} + 2\left(s_{13}^2+s_{23}^2\right)}{s_{12}}\right) + 4\frac{(2d-5)}{(d-2)}s_{12}\right]\left|\mathscr{A}^{(0)}\right|^2 \tag{3.14}
$$

$$
+\text{cycl. perm.}\,, \tag{3.15}
$$

which generate

$$
\Gamma_{q\bar{q}g}^{(0)} = \int d\Phi_{1\to 3}\left|\mathscr{A}_{q\bar{q}g}^{(0)}\right|^2 = n_f\,\Gamma_{gg}^{(0)}\, c_\Gamma \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)\left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon}\left(-\frac{2}{3\varepsilon_{\mathrm{IR}}} - \frac{7}{3}\right)\,, \tag{3.16}
$$

$$
\Gamma_{ggg}^{(0)} = \int d\Phi_{1\to 3}\left|\mathscr{A}_{q\bar{q}g}^{(0)}\right|^2 = C_A\,\Gamma_{gg}^{(0)}\, c_\Gamma \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)\left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon}\left(\frac{2}{\varepsilon_{\mathrm{IR}}^2} + \frac{11}{3\varepsilon_{\mathrm{IR}}} - \pi^2 + \frac{73}{6}\right)\,, \tag{3.17}
$$

4. Full decay width

Adding up all contributions, the decay width becomes

$$
\begin{aligned}
\Gamma_{gg}^{(\mathrm{NLO})} &= \Gamma_{gg}^{(1)} + 2\Gamma_{gg}^{(1,0)} + \Gamma_{q\bar{q}g}^{(0)} + \Gamma_{ggg}^{(0)}\,, \\
&= \Gamma_{gg}^{(0)}\, c_\Gamma \left(\frac{\alpha_{\mathrm{S}}}{4\pi}\right)\left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon}\left(C_A\left(\frac{95}{3}\right) - n_f\left(\frac{7}{3}\right)\right)\,.
\end{aligned}
\tag{4.1}
$$

5. UV behaviour

Let us add up all counter-terms computed in the previous sections,

$$\begin{aligned}\Gamma_{gg}^{(\mathrm{ct})} =& \Gamma_{gg}^{(1,\mathrm{ct})} + \Gamma_{gg}^{(1,0,\mathrm{ct})} \\ =& \Gamma_{gg}^{(0)} c_\Gamma \left(\frac{\alpha_S}{2\pi}\right) \left(\frac{s_{12}}{\mu^2}\right)^{-\varepsilon} \left[C_A \left(\frac{11}{3}\right) - n_f \left(\frac{2}{3}\right)\right] \frac{1}{\varepsilon_{\mathrm{UV}}}, \end{aligned} \tag{5.1}$$

6. Conclusions

In this talk we showed a procedure to locally renormalise ultraviolet (UV) and locally subtract infrared (IR) singularities. We discussed the decay of $H \to gg$ at next-to-leading order in the large top quark mass effective limit. We highlighted the relevant results that are going to be summarised and extensively discussed in [12] by means of loop-tree duality and the four-dimensional-unsubtraction method.

On top of it, we observed that the local UV renormalisation can straightforwardly be applied to effective field theories as well. Likewise, the use of alternative approaches to compute renormalised multi-loop amplitudes can take advantage of the procedure described in [3] and summarised here.

Acknowledgments

This work is supported by the Spanish Government (Agencia Estatal de Investigación) and ERDF funds from European Commission (Grants No. FPA2017-84445-P and SEV-2014-0398), Generalitat Valenciana (Grant No. PROMETEO/2017/053), Consejo Superior de Investigaciones Científicas (Grant No. PIE-201750E021), the COST Action CA16201 PARTICLEFACE and the "Juan de la Cierva Formación" program (FJCI-2017-32128).

The Feynman diagrams depicted in this paper were generated using FeynArts [15].

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