Measuring the muon precession frequency in the E989 Fermilab g-2 experiment

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Since more than 50 years the electron and muon anomalies, $a_e$ and $a_\mu$, defined in terms of the gyromagnetic factor $g_i$ for particle $i$ as $a_i = (g_i - 2)/2$, have provided a deep insight into the quantum structure of elementary particles. They have been, and continue to be, a milestone for the development of the Standard Model of Particle Physics against which all new theories have to be compared. For almost 20 years, the experimental value of $a_\mu$ has shown a tantalizing discrepancy of more than 3$\sigma$ from the theoretical prediction making it mandatory for experimentalists to improve the current result, dominated by the E821 experiment at BNL[1]. The Muon $g - 2$ E989 experiment at Fermilab will use the same storage ring technique used at BNL, and previously in the CERN-III experiment, with the goal of decreasing by a factor of 4 the current error on $a_\mu$, which will allow for a finer comparison with the theoretical prediction. E989 started collecting data in winter 2018 accumulating, in the period April-July 2018 (Run1) almost twice the statistics of the previous experiment (before application of data quality cuts).

In this document, the experiment will be briefly described, underlying the improvements which will allow to reduce the systematic error, and some preliminary result will be shown.

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†on behalf of the Muon $g - 2$ collaboration

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1. Introduction

A particle with electric charge $Q$ and spin $\vec{s}$ is characterized by a magnetic moment

$$\vec{\mu} = g \frac{Q}{2m} \vec{s}$$

(1.1)

where $g$ is the gyromagnetic factor.

For an elementary spin 1/2 particle, Dirac theory predicts that the gyromagnetic ratio is exactly $g = 2$. However, the development of the Quantum ElectroDynamic theory (QED) led to the prediction, and then to the observation, of virtual diagrams in which photons, as well as other particles, are emitted and reabsorbed. These diagrams modify the effective magnetic moment and therefore the coupling of the particle to an external magnetic field.

This was first predicted by Schwinger\cite{2} and measured by Kusch and Foley\cite{3} in 1948. At first level in perturbation theory, the anomaly $a$ was predicted by Schwinger to be:

$$a = \frac{g - 2}{2} = \frac{\alpha}{2\pi} = 0.00116 \pm 0.00004$$

(1.2)

The measured value was:

$$a = 0.00118 \pm 0.00003$$

(1.3)

It was the first great success of QED.

With time, the measurement has been refined over and over reaching the astonishing value of

$$a_e = (115965218073 \pm 28) \times 10^{-14}$$

for the electron \cite{4} and

$$a_\mu = (116592080 \pm 63) \times 10^{-11}$$

for the muon \cite{1}.

Although the muon anomaly can be measured less precisely than the electron one, mostly because of the particle finite lifetime, it was soon realized that a new particle (boson) contributing to the anomaly in a virtual correction would have an effect which, in general, can be proportional to the square of the mass ratio:

$$\alpha_{NP} \simeq \left( \frac{m_\mu}{M} \right)^2$$

due to the chirality flip in the boson emission. Therefore the muon anomaly, although less precisely measured, is more sensitive to New Physics contributions than the electron one.

The current precision with which the anomaly is known is summarized in table 1. The QED contribution has been evaluated at 5 loops (more than 12000 diagrams!), the electroweak contribution is well under control while the hadronic vacuum polarizion and the light-by-light scattering are the largest sources of uncertainty in the $a_\mu^{\text{ew}}$ determination.

The theoretical prediction shows a tantalizing discrepancy of 3.7$\sigma$ from the experimental result quoted above, which calls for a new experiment to possibly confirm, with a larger significance, the current difference.

The Muon $g - 2$ experiment at Fermilab is designed to measure the muon anomaly with an error 4 times smaller than the current one by using the same experimental technique used in BNL.
Table 1: Theoretical determination of muon anomaly $a_\mu$.

<table>
<thead>
<tr>
<th>contribution</th>
<th>value ($\times 10^{-11}$)</th>
<th>error ($\times 10^{-11}$)</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>11658471.90</td>
<td>0.01</td>
<td>[5]</td>
</tr>
<tr>
<td>EW</td>
<td>15.36</td>
<td>0.10</td>
<td>[6]</td>
</tr>
<tr>
<td>LO HLbL</td>
<td>9.80</td>
<td>2.60</td>
<td>[7]</td>
</tr>
<tr>
<td>NLO HLbL</td>
<td>0.30</td>
<td>0.20</td>
<td>[8]</td>
</tr>
<tr>
<td>LO HVP</td>
<td>693.27</td>
<td>2.46</td>
<td>[9]</td>
</tr>
<tr>
<td>NLO HVP</td>
<td>-9.82</td>
<td>0.04</td>
<td>[9]</td>
</tr>
<tr>
<td>NNLO HVP</td>
<td>1.24</td>
<td>0.01</td>
<td>[10]</td>
</tr>
<tr>
<td>Total</td>
<td>11659182.05</td>
<td>3.56</td>
<td>[9]</td>
</tr>
</tbody>
</table>

as well as in the CERNIII experiment, briefly described in the next section, but improving both on the statistical and on the systematical error. In particular, the E821 total error was dominated by the statistical component, therefore the first goal of the Fermilab experiment is to increase the collected statistics by a factor of 21, while the systematical error will have to be improved “just” by a factor of 3 to reach the final sensitivity.

2. The experiment

The experiment is based on the principle that the spin of a muon moving in a constant magnetic field $\vec{B}$, in the presence of a static electric field $\vec{E}$, precesses around $\vec{B}$ with an angular velocity $\omega_s$ which is slightly faster than the momentum precession (cyclotron frequency) $\omega_p$ around the same vector. More precisely, the spin vector projection on the momentum axis changes with time according to (from [11] eq.11.171):

$$
\frac{d}{dt}(\hat{\beta} \cdot \vec{s}) = -\frac{e}{mc} \vec{s}_{\perp} \cdot \left[ \left( \frac{g}{2} - 1 \right) \hat{\beta} \times \vec{B} + \left( \frac{g \beta}{2} - 1 \right) \vec{E} \right] 
$$

(2.1)

For a muon beam of momentum $p_\mu = 3.095$ GeV/c, called magic momentum, corresponding to a value of $\beta$ which cancels out the second term of equation 2.1, and assuming that all muons follow the ideal circular path in a plane perpendicular to $\vec{B}$, then the above expression greatly simplifies into:

$$
\omega_a = \frac{ea_\mu B}{m}
$$

(2.2)

where $\omega_a = \omega_s - \omega_p$ is the difference between the spin precession and the cyclotron frequency and where quantities are taken as absolute values (no sign). By inverting the simplified equation 2.2, the the muon anomaly $a_\mu$ is given by:

$$
a_\mu = \frac{m\omega_a}{eB}
$$

(2.3)

In reality, the beam will have dimensions both in the radial and in the vertical directions, as well as a momentum spread, therefore the simple expression given above is only a first order approximation which will need to be carefully corrected. The most evident correction to the motion is the so-called Coherent Betatron Oscillation (CBO), which is due to the radial and vertical movement of particles within the beam. This will be briefly discussed in section 4.
3. The E989 experiment at Fermilab

The E989 experiment at Fermilab is largely built on the legacy of E821. During the summer of 2013, the 14-m diameter superconducting coils from the E821 storage magnet were moved from Brookhaven National Laboratory in New York to Fermilab, near Chicago. Performing the experiment at Fermilab provides a number of advantages, including the ability to produce more muons and to eliminate the pion contamination of the muon beam injected into the storage ring, which was a major limiting factor for E821.

The upgraded linear accelerator and booster ring structure of FNAL will deliver proton pulses (8 GeV, $4 \times 10^{12}$ protons per pulse, 1.3 s pulse separation) impinging on the production target. The secondary $\pi^+$ beam will be focused with a pulsed lithium lens into the transport beam line which accepts $\pi^+$ with a momentum spread of $\pm 0.5\%$ around 3.11 GeV/c. In the transport beam line and in the delivery ring section the in-flight-decay of $\pi^+$ generates the $\mu^+$ beam, polarized due to the V-A structure of the weak current. The $\approx 10$ times longer flight distance at FNAL compared to BNL allows the residual hadronic contamination in the muon beam to decay away before it reaches the muon storage ring. This will essentially eliminate the so-called hadronic flash in the positron calorimeters after muon beam injection which was a major source of background for the BNL experiment. The muons are injected into the storage ring through an inflector magnet which locally cancels out the main dipole field, thus allowing the muons to enter the storage ring perpendicularly to its radius at a value which is 77 mm larger than the nominal one. A set of kickers then kicks the muons into the right orbit. Muons then circulate in the storage ring decaying with a lifetime $\tau = \gamma \tau_0 \approx 64 \mu$s. The high-energy positrons from the muon decay are emitted preferentially along the spin direction, again because of the V-A structure of the weak current, with an asymmetry $A$ which depends on the positron fractional energy.

Twenty-four individual calorimeter stations[12], each consisting of an array of $6 \times 9$ PbF2 crystals (25.4 mm $\times$ 25.4 mm $\times$ 152.4 mm), will be spaced equidistantly around the inner radius of the storage ring in order to capture the emitted positrons. Each crystal is individually instrumented with a silicon photomultiplier (SiPM) to detect the Cerenkov light generated by the high energy positrons. The high segmentation allows hit position discrimination while the fast SiPM response can separate events as close as 3 ns (800 MHz digitization rate) which will allow to address pile-up related systematic effects.

A sophisticated laser system will be used to calibrate in energy and to align in time the response of the 1296 crystals. This is of paramount importance as the single largest systematic error in the BNL experiment was the calorimeter “gain stability”, corresponding to 120 ppb error contribution out of a total of $\sigma_{\text{tot}}^{\text{syst}} = 180$ ppb [1]. Thanks to the laser system and to the new calorimeter, the budget for this error is 20 ppb: a reduction of a factor 6!

Straw tracker stations will be operated in front of two positron calorimeters which will allow for the precise reconstruction of the positron flight path and of the muon beam distribution. Retractable fiber harp detectors will be installed in the muon storage region to measure the muon distribution in the storage region.
4. The analysis strategy

Starting from the simplified equation 2.3, the determination of the muon anomaly $a_\mu$ requires the measurement of the magnetic field $B$ and of the spin precession frequency, relative to momentum, $\omega_a$. The magnetic field has to be precisely mapped over the full phase space defined by the muon beam; this is accomplished by measuring the precession frequency of free protons, so-called $\omega_p$, using Nuclear Magnetic Resonance techniques. In this way, $a_\mu$ is given by the ratio of two frequencies measured in the same magnetic field. The determination of $\omega_p$ will not be discussed here; see, for example, reference [13].

To measure $\omega_a$ it is necessary to know the spin direction of decaying muons. This can be obtained by exploiting the V-A structure of weak interactions in the muon decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$. The produced neutrino is left-handed, while the anti-neutrino and the positron are right-handed\(^1\), as a consequence, for momentum conservation, the positron direction is correlated to the muon spin. These positrons spiralize inward and hit the calorimeters, which register their energy and the arrival time. The number of observed positrons decreases exponentially with a decay time $\tau = 64.4 \, \mu s$, due to the boosted muon lifetime, and it is modulated by the spin precession with an amplitude which depends on the positron energy and of its correlation with the muon spin direction. The general expression is:

$$N(t) = N_0(y) e^{-t/\tau} \cdot (1 + A(y) \cos(\omega_a t + \phi)) \quad (4.1)$$

The amplitude (also called “asymmetry”) $A$ and the normalization $N_0$ depend on the positron energy which is normally parametrized as $y = E/E_{\text{MAX}}$, with $E_{\text{MAX}} = 3.1$ GeV representing the positron maximum energy, i.e. the full muon momentum.

The value of $A$ depends on the kinematics and on the detector acceptance, as only a fraction of positrons is actually detected, therefore the standard analysis consists in counting positrons above a threshold which has been set to $E = 1.7$ GeV, after an optimization process.

The analysis method just described - counting positrons above threshold - has been used also in past experiments and it is known as $T$-method. An example of this distribution is shown in fig.1, together with a fit with function 4.1. The plot refers to a subset of Run1 corresponding to 60 hours of data taking. The $\chi^2$/ndf, equals to 8791/3814, shows that the simple 5-parameters expression is not sufficient to describe the details of the time evolution; corrections related to a more realistic beam description are required and will be briefly described in the next section. Note also that the $\omega_p$ parameter does not appear explicitly in the fit result. In order not to be biased by the result, the real parameter is blinded with a specific procedure and only a new parameter $R$, which represents the shift in ppm from an unknown (i.e. blinded) value, is provided as outcome of the fit. The error on $R$, however, is meaningful and it tells us that with 60h of data taking we can reach a statistical precision of 1.27 ppm on the determination of $\omega_a$. \(^2\)

In the next section, the full fit will be discussed.

5. Fitting $\omega_a$

The Fourier Trasform of the residuals of the 5 parameter fit shown in fig.1, is plotted in fig:2.

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\(^1\)This is strictly true only for massless particles; neutrino masses are negligible, while the electron mass provides a correction which is absorbed into the amplitude $A$ of eq.4.1.

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Figure 1: Arrival time spectrum of high energy positrons from a subset of data. The data are fit with an exponential decay modulated by a sinusoidal function describing the muon spin precession. The fit parameter $\tau$ in the insert corresponds to the boosted muon lifetime $\gamma \tau \mu$, while the parameter $R$ is the blinded version of the precession frequency $\omega_a$.

By plotting the residuals in the frequency domain, the beam oscillation frequencies become evident. The largest peak is the so-called Coherent Betatron Oscillation (CBO), corresponding to the beam radial oscillations around the nominal radius which has a typical frequency $f_{CBO} = 300$ kHz. The two side peaks represent the beating between this frequency and the spin rotation $f_a$, appearing at frequencies $f_{CBO} - f_a$ and $f_{CBO} + f_a$. The other peak at $f_V = 2.3$ MHz is caused by the vertical oscillations, which have higher frequencies but lower amplitude.

These additional peaks are taken into account in the fitting by additional terms which have a structure similar to the 5-parameter one:

$$f_i(t) = N_i \cdot e^{-t/\zeta_i} \cdot (1 - A_i \cos(\omega_i t + \phi_i))$$

(5.1)

where $i = (CBO, V)$, to represent the radial and vertical oscillations, respectively.

The normalization parameters $N_i$ are absorbed into the global $N_0$, so these terms add 8 new parameters to the fit.

An additional effect which distorts the simple fit is due to the “lost muons”. In fact the number of muons in the beam decreases not only because of their decay, but also because they hit the ring collimators, or other obstacles, loosing momentum and curling inward. This happens with higher probability in the first part of the fill, thus the exponential decay has to be corrected as follows:

$$N(t) = N_0(y) \cdot f_{CBO}(t) \cdot c_{dot f_V(t)} \cdot e^{-t/\zeta} \cdot A(t) \cdot (1 + A(y) \cos(\omega_d t + \phi))$$

(5.2)
Figure 2: Residuals between the fit function and data. The red histogram, with the peaks due to beam motion, corresponds to the simple 5 parameter fit, while the black one corresponds to the full fit.

The function $\Lambda(t)$ is evaluated with data by selecting particles which cross two or more calorimeters with an energy $E \approx 170$ GeV, typical of a Minimum Ionizing Particle (MIP) and with a time difference $\Delta t \approx 6.2$ ns, compatible with the travel time of a relativistic particle.

By including all these effects, the fit has a $\chi^2/ndf = 3451/3467$ and residuals that don’t show any structure (see fig. 2).

6. Conclusions

E989 started collecting data in February 2018. After few months of commissioning, the first real data started to accumulate in April of the same year which allowed to reach by the end of Run1 in July 2018, a raw integral number of positrons which, after quality cuts, is similar to the total sample of the previous BNL experiment.

A new run started in April 2019 (Run2) and collected almost twice the statistics of Run1.

The analysis is currently going on, but a preliminary analysis of the first collected data shows that all E989 subsystems (segmented calorimeter, laser calibration system, straw tracker, ...) are working as expected and they seem to be able to keep the systematic error at or below their budget.

If the E989 will confirm the previously measured value, then the discrepancy from the Standard Model will start to become relevant. The experiment will collect data in Run3 starting Autumn 2019, and Run4, the year after, with the final goal of integrating 15-20 times the statistics collected at BNL, thus reducing the error by a factor $\approx 4$ and possibly by providing a strong indication for new, as yet undiscovered, particles in loops which contribute to the muon anomaly.
7. Acknowledgments

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References