Production of $W^+W^-$ and $t\bar{t}$ pairs via photon-photon processes in proton-proton scattering and corresponding gap survival factor

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We review our recent results for production of $W^+W^-$ and $t\bar{t}$ pairs via photon-photon fusion mechanism. A sketch of theoretical approach is presented. We include transverse momenta of photons in calculation of fluxes of photons. Then we present our results for $W^+W^-$ production. Results for different parametrizations of proton structure functions are used to calculate inelastic fluxes of photons. A discussion on rapidity gap survival probability due to remnant fragmentation is presented. A similar discussion is presented for $t\bar{t}$ production.
1. Introduction

It was realized rather recently that the electroweak corrections are important for precise calculations of cross sections in different processes. The $pp \rightarrow W^+W^-$ process is a good example (see e.g. [1]). Then $\gamma\gamma \rightarrow W^+W^-$ is the relevant subprocess. This subprocess is important also in the context of searches beyond Standard Model [2, 3]. By imposing special conditions on the final state this contribution can be observed experimentally [4, 5].

In [6, 7] we developed a formalism for calculating $pp \rightarrow l^+l^-$ processes proceeding via photon-photon fusion. In [8] we used the same technique to calculate cross section for $pp \rightarrow W^+W^-$ reaction proceeding via photon-photon fusion. In order to make reference to real “measurements” of the photon-photon contribution one has to include in addition the gap survival probability caused by extra emissions. In [9] we concentrated on the effect related to remnant fragmentation and its destroying of the rapidity gap.

In [10] we calculated cross section for the photon-photon contribution for the $pp \rightarrow t\bar{t}$ reaction including also effects of gap survival probability.

Here we briefly review our results obtained in [8, 9, 10].

2. A sketch of the formalism

In our analyses we included different types of processes shown in Fig.1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{diagram1.png}
\caption{Diagrams representing different types of photon-photon induced mechanisms for production of $W^+W^-$ pairs.}
\end{figure}

In our approach we include transverse momenta of (virtual) photons. Then the differential cross section for $W^+W^-$ production can be written as:

\[
\frac{d\sigma^{(i,j)}}{dy_1dy_2d^2p_T1d^2p_T2} = \int \frac{d^2\tilde{q}_{T1}}{\pi \tilde{q}_{T1}^2} \frac{d^2\tilde{q}_{T2}}{\pi \tilde{q}_{T2}^2} \mathcal{F}_r^{(i)}(x_1, \tilde{q}_{T1}) \mathcal{F}_r^{(j)}(x_2, \tilde{q}_{T2}) \frac{d\sigma^*(p_1, p_2; \tilde{q}_{T1}, \tilde{q}_{T2})}{dy_1dy_2d^2p_T1d^2p_T2} d^2(\tilde{q}_{T2} - \tilde{q}_{T1}) \delta(2)
\]

where $i, j = \text{elastic, inelastic}$ and the longitudinal momentum fractions are expressed in terms of rapidities and transverse momenta of $W$ bosons. The elementary off-shell cross section in (2.1) is written as:

\[
\frac{d\sigma^*(p_1, p_2; \tilde{q}_{T1}, \tilde{q}_{T2})}{dy_1dy_2d^2p_T1d^2p_T2} = \frac{1}{16\pi^2(x_1x_2s)^2} \sum_{\lambda_W^+, \lambda_W^-} |M(\lambda_W^+, \lambda_W^-)|^2 \delta(2)(\tilde{p}_{T1} + \tilde{p}_{T2} - \tilde{q}_{T1} - \tilde{q}_{T2})
\]
Above the helicity-dependent off-shell matrix elements were calculated as:

\[
M(\lambda_{W^+}, \lambda_{W^-}) = \frac{1}{|q_{\perp 1}||q_{\perp 2}|} \sum_{A_1, A_2} (\bar{e}_{\perp}^{\dagger} (\lambda_1) \cdot \vec{q}_{\perp 1}) (\bar{e}_{\perp}^{\dagger} (\lambda_2) \cdot \vec{q}_{\perp 2}) M(\lambda_1, \lambda_2; \lambda_{W^+}, \lambda_{W^-})
\]

\[
= \frac{1}{|q_{\perp 1}||q_{\perp 2}|} \sum_{A_1, A_2} q_{\perp 1} q_{\perp 2} e_i(\lambda_1) e_j(\lambda_2) M(\lambda_1, \lambda_2; \lambda_{W^+}, \lambda_{W^-}). \tag{2.2}
\]

Initial and final state helicity-dependent matrix elements were discussed e.g. in [11]. The \(k_T\)-factorization W-boson helicity dependent matrix elements were calculated with the help of the above [8].

The unintegrated inelastic flux of photons is expressed as:

\[
\mathcal{F}_{\gamma - A}^{\text{in}}(z, \vec{q}_T) = \frac{\alpha_{em}}{\pi} \left\{ (1 - z) \left( \frac{\vec{q}_T^2}{\vec{q}_T^2 + z(M_W^2 - m^2) + z^2 m^2_p} \right) \frac{2 F_2(x_{Bj}, Q^2)}{Q^2 + M_W^2 - m^2_p} + \frac{z^2}{4 x_{Bj}^2 \vec{q}_T^2 + z(M_W^2 - m^2) + z^2 m^2_p} \frac{2 x_{Bj} F_1(x_{Bj}, Q^2)}{Q^2 + M_W^2 - m^2_p} \right\}.
\tag{2.3}
\]

The main ingredients of the formula are \(F_1\) and \(F_2\) proton structure functions.

The unintegrated elastic flux of photons is expressed as:

\[
\mathcal{F}_{\gamma - A}^{\text{el}}(z, \vec{q}_T) = \frac{\alpha_{em}}{\pi} \left\{ (1 - z) \left( \frac{\vec{q}_T^2}{\vec{q}_T^2 + z(M_W^2 - m^2) + z^2 m^2_p} \right) \frac{2 4 m_p^2 G_E^2(Q^2) + Q^2 G_M^2(Q^2)}{4 m_p^2 + Q^2} \right\}.
\tag{2.4}
\]

In this case the main ingredients are \(G_E\) and \(G_M\) electromagnetic form factors of proton.

To calculate inelastic fluxes of photons one needs numerical representation of structure functions of protons. Different parametrizations of \(F_2\) structure functions are available in the literature, see e.g. [12, 13, 14].

3. Selected results

The integrated cross sections obtained in our approach are collected in Table 1.

Without any gap survival effects:

\[
\sigma(\text{inel. - inel.}) > \sigma(\text{inel. - el.}) + \sigma(\text{el. - inel.}) > \sigma(\text{el. - el.}). \tag{3.1}
\]

Many differential distributions were calculated in [8]. Here, in Fig.3, we show only invariant mass distribution for double dissociation processes (inelastic-inelastic) for different parametrizations of the structure functions from the literature.

The \(k_T\)-factorization result is similar to the collinear one for the same structure function (LUX-like). The rather old MRST04-QED collinear approach [15] predicted larger cross section. The reasons were discussed in [8].
W$^+W^-$ and $t\bar{t}$ production

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Table 1: Cross sections (in $pb$) for different contributions and different $F_2$ structure functions: LUX, ALLM97 and SU, compared to the relevant collinear distributions with MRST04 QED and LUXqed distributions.

<table>
<thead>
<tr>
<th>contribution</th>
<th>8 TeV</th>
<th>13 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>LUX-like</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{\ell}$</td>
<td>0.214</td>
<td>0.409</td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{t}$</td>
<td>0.214</td>
<td>0.409</td>
</tr>
<tr>
<td>$\gamma_{t}\gamma_{t}$</td>
<td>0.478</td>
<td>1.090</td>
</tr>
<tr>
<td>ALLM97 F2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{\ell}$</td>
<td>0.197</td>
<td>0.318</td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{t}$</td>
<td>0.197</td>
<td>0.318</td>
</tr>
<tr>
<td>$\gamma_{t}\gamma_{t}$</td>
<td>0.289</td>
<td>0.701</td>
</tr>
<tr>
<td>SU F2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{\ell}$</td>
<td>0.192</td>
<td>0.420</td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{t}$</td>
<td>0.192</td>
<td>0.420</td>
</tr>
<tr>
<td>$\gamma_{t}\gamma_{t}$</td>
<td>0.396</td>
<td>0.927</td>
</tr>
<tr>
<td>LUXqed collinear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\ell}+\gamma_{\ell}+\gamma_{t}$</td>
<td>0.366</td>
<td>0.778</td>
</tr>
<tr>
<td>MRST04 QED collinear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{\ell}$</td>
<td>0.171</td>
<td>0.341</td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{t}$</td>
<td>0.171</td>
<td>0.341</td>
</tr>
<tr>
<td>$\gamma_{t}\gamma_{t}$</td>
<td>0.548</td>
<td>0.980</td>
</tr>
<tr>
<td>Elastic- Elastic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{\ell}$ (Budnev)</td>
<td>0.130</td>
<td>0.273</td>
</tr>
<tr>
<td>$\gamma_{\ell}\gamma_{\ell}$ (DZ)</td>
<td>0.124</td>
<td>0.267</td>
</tr>
</tbody>
</table>

Figure 2: $M_{WW}$ invariant mass distribution for double dissociative contribution obtained with different parametrizations of structure functions.
As an example in Fig.3 we show distribution in virtualities of photons. Rather large virtualities of photons come into game. The large virtualities of photons seem to contradict collinear approach.

The remnant fragmentation [9] was done with the help of PYTHIA 8 program. Including only parton (jet) emission is already a quite good approximation.

The gap survival probability for single dissociative process is calculated as:

$$S_R(\eta_{\text{cut}}) = 1 - \frac{1}{\sigma} \int_{-\eta_{\text{cut}}}^{\eta_{\text{cut}}} \frac{d\sigma}{d\eta_{\text{jet}}} d\eta_{\text{jet}}.$$  (3.2)

Jet emissions were considered also in [17].

The gap survival factor associated with jet emission is shown in Fig.4.

We find (see also Table 1)

$$S_{R,DD} \approx (S_{R,SD})^2.$$  (3.3)

Such an effect is expected when the two fragmentations are independent, which is the case by the model construction. So far we have not included the soft gap survival factors. They are relatively
easy to calculate only for double elastic (DE) contribution [16]. For the “soft” gap survival factors we expect:

\[ S_{\text{soft}}(DD) < S_{\text{soft}}(SD) < S_{\text{soft}}(DE) . \]

\[ (3.4) \]

<table>
<thead>
<tr>
<th>( (2M_{WW}, 200 \text{ GeV}) )</th>
<th>8 TeV</th>
<th>13 TeV</th>
<th>8 TeV</th>
<th>13 TeV</th>
<th>8 TeV</th>
<th>13 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (200, 500 \text{ GeV}) )</td>
<td>0.763(2)</td>
<td>0.769(2)</td>
<td>0.582(4)</td>
<td>0.591(4)</td>
<td>0.586(1)</td>
<td>0.601(2)</td>
</tr>
<tr>
<td>( (500, 1000 \text{ GeV}) )</td>
<td>0.812(2)</td>
<td>0.831(2)</td>
<td>0.659(3)</td>
<td>0.691(3)</td>
<td>0.673(2)</td>
<td>0.705(2)</td>
</tr>
<tr>
<td>( (1000, 2000 \text{ GeV}) )</td>
<td>0.838(7)</td>
<td>0.873(5)</td>
<td>0.702(12)</td>
<td>0.762(8)</td>
<td>0.697(5)</td>
<td>0.763(6)</td>
</tr>
<tr>
<td>full range</td>
<td>0.782(1)</td>
<td>0.799(1)</td>
<td>0.611(2)</td>
<td>0.638(2)</td>
<td>0.617(1)</td>
<td>0.646(1)</td>
</tr>
</tbody>
</table>

Table 2: Average rapidity gap survival factors: \( S_{R,SD}(|\eta^{\text{ch}}| < 2.5) \), \( (S_{R,SD})^2 \), \( S_{R,DD}(|\eta^{\text{ch}}| < 2.5) \) related to remnant fragmentation for single dissociative and double dissociative contributions for different ranges of \( M_{WW} \).

Finally we wish to show also similar results for \( pp \rightarrow t\bar{t} \) reaction. In Table 3 we show integrated cross sections for different categories of processes. Rather small cross sections are obtained. It is not clear at present whether such a process can be identified experimentally.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>No cuts</th>
<th>( \gamma_{\text{jet}} ) cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic-elastic</td>
<td>0.292</td>
<td>0.292</td>
</tr>
<tr>
<td>elastic-inelastic</td>
<td>0.544</td>
<td>0.439</td>
</tr>
<tr>
<td>inelastic-elastic</td>
<td>0.544</td>
<td>0.439</td>
</tr>
<tr>
<td>inelastic-inelastic</td>
<td>0.983</td>
<td>0.622</td>
</tr>
<tr>
<td>all contributions</td>
<td>2.36</td>
<td>1.79</td>
</tr>
</tbody>
</table>

Table 3: Cross section for \( t\bar{t} \) production in fb at \( \sqrt{s} = 13 \text{ TeV} \) for different components (left column) and the same when the extra condition on the outgoing jet \( |\gamma_{\text{jet}}| > 2.5 \) is imposed.

As an example we show \( t\bar{t} \) invariant mass distribution for inclusive case as well as when extra veto on (mini)jet is imposed. The inclusion of rapidity gap veto reduces the cross section. Whether the cross section corresponding to the photon-photon fusion can be measured requires special dedicated studies.

4. Conclusions

Helicity-dependent matrix elements for \( \gamma^* \gamma^* \rightarrow W^+W^- \) (off-shell photons) have been derived and used in the calculation of cross sections for \( pp \rightarrow W^+W^- \) reaction. We have obtained cross section of about 1 pb for the LHC energies. Different combinations of the final states (elastic-elastic, elastic-inelastic, inelastic-elastic, inelastic-inelastic) have been considered. Several correlation observables have been studied. Large contributions from the regions of large photon virtualities \( Q^2_1 \) and/or \( Q^2_2 \) have been found putting in question the reliability of leading-order collinear-factorization approach.

We have discussed the quantity called “remnant gap survival factor” for the \( pp \rightarrow W^+W^- \) reaction initiated via photon-photon fusion. We have calculated the gap survival factor for single
dissociative process on the parton level. In such an approach the outgoing parton (jet/mini-jet) is responsible for destroying the rapidity gap. We have found that the hadronisation only mildly modifies the gap survival factor calculated on the parton level. We have found different values for double and single dissociative processes. In general, $S_{R,DD} < S_{R,SD}$ and $S_{R,DD} \approx (S_{R,SD})^2$. We expect that the factorisation observed here for the remnant dissociation and hadronisation will be violated when the soft processes are explicitly included. The larger $\eta_{cut}$ (upper limit on charged particles pseudorapidity), the smaller rapidity gap survival factor $S_R$. This holds both for the double and the single dissociation. The present approach is a first step towards a realistic modelling of gap survival in photon induced interactions and definitely requires further detailed studies and comparisons to the existing and future experimental data. We have shown that rather large photon virtualities come into the game for $W^+W^-$ production.

We have also calculated cross sections for $t\bar{t}$ production via $\gamma\gamma$ mechanism in $pp$ collisions including photon transverse momenta and using modern parametrizations of proton structure functions. The contribution to the inclusive $t\bar{t}$ is only about 2.5 fb. We have found $\sigma_{t\bar{t}}^{el-el} < \sigma_{t\bar{t}}^{SD} < \sigma_{t\bar{t}}^{DD}$. We have calculated several differential distributions. Some of them are not accessible in standard equivalent photon approximation. As for $W^+W^-$ production we have shown that rather large photon virtualities come into the game.

**Acknowledgement**

I am indebted to Marta Łuszczak, Laurent Forthomme and Wolfgang Schäfer for collaboration on the issues presented here.

**References**


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[10] M. Łuszczak, L. Forthomme, W. Schäfer and A. Szczurek, “Production of $t\bar{t}$ pairs via $\gamma\gamma$ fusion with photon transverse momenta and proton dissociation”, JHEP 02, 100 (2019).


