PROCEEDINGS OF SCIENCE

Pre-2019 Summaries of α_s

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Summaries of measurements of α_s and determinations of world average values of $\alpha_s(m_Z)$ are reviewed, spanning the time from 1989 to the latest update by the Particle Data Group in 2016/2018.

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Intro

Determinations of α_s, the coupling parameter of the Strong Interaction between quarks and gluons, became available since the early 1980's, based on theoretical predictions of Quantum Chromodynamics (QCD), in next-to-leading or higher order of perturbation theory, and on experimental data at sufficiently large energy scales. Such determinations always were and continue to be challenging, due to the relatively large perturbative and nonperturbative uncertainties which dominate most of the measurements. Determinations of α_s, from different physical processes, energy scales and experiments, therefore do not necessarily agree with each other, within the quoted uncertainties of results. Therefore summaries of α_s results and the determination of one overall "world average" value became mandatory.

One of the earliest and significant of such summaries and extractions was published by Altarelli in 1989, resulting in $\alpha_s(m_Z) \approx 0.11 \pm 0.01$, with an overall uncertainty of about 10% [1]. The latest world summary of α_s, in the 2016 and 2018 Reviews of Particle Physics edited by the Particle Data Group (PDG) [2, 3], quotes $\alpha_s(m_Z) = 0.1181 \pm 0.0011$, with an overall uncertainty of just below 1%. The tenfold reduction of the uncertainty of α_s, achieved over the past almost 30 years, is mainly due – in *reverse* order of importance and impact – to

- higher statistics, multitude and quality of data, and improved experimental methods;
- theoretical predictions and calculations at higher perturbative orders (NNLO, N^3LO, resummation, ...);
- new theoretical developments in lattice gauge theory.

A (personal) selection of the history of summaries of α_s is listed and referenced in Table 1 and displayed in Figure 1. Details of the 2016 world summary of α_s [2] are also presented in Ref. [4]. Note that the overall uncertainty on $\alpha_s(m_Z)$ increased, from its 2014 to the 2016 value, which is mainly due to an adjustment of the procedure to combine systematic uncertainties, as will be discussed below.

In the following, a short recap of procedures used for deriving the most recent world average is given. The first step of summarising results is to define which of (the many) available analyses, measurements and results are to be included:

- the result must be published in a peer-reviewed scientific journal;
- the analysis must be based on at least NLO or higher order QCD perturbation theory (for results of $\alpha_s(Q^2)$ to be included in the running coupling summary plot);
- results entering the world average determination of $\alpha_s(m_Z)$ must be based on at least NNLO or higher order perturbative QCD;
- the analysis must include reliable estimates of experimental, systematic and theoretical uncertainties, based on commonly accepted procedures.

Next, the results are grouped into 6 classes of measurements that are based on similar or identical types of data, calculations or procedures:

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Table 1: World average values of $\alpha_s(m_Z)$ over time.

year	$\alpha_s(m_Z)$	$\Delta\alpha_s(m_Z)$	comment	ref.
1989	0.11	±0.01	NLO (pre-LEP)	[1]
1994	0.117	±0.006	+ LEP + HERA	[5]
1998	0.119	±0.004		[6]
2000	0.1184	±0.0031	at NNLO	[7]
2002	0.1183	±0.0027		[8]
2004	0.1182	±0.0027		[9]
2006	0.1189	±0.0010	+ lattice	[10]
2009	0.1184	±0.0007		[11]
2012	0.1184	±0.0007		[12]
2014	0.1185	±0.0006		[13]
2016	0.1181	±0.0011		[2]

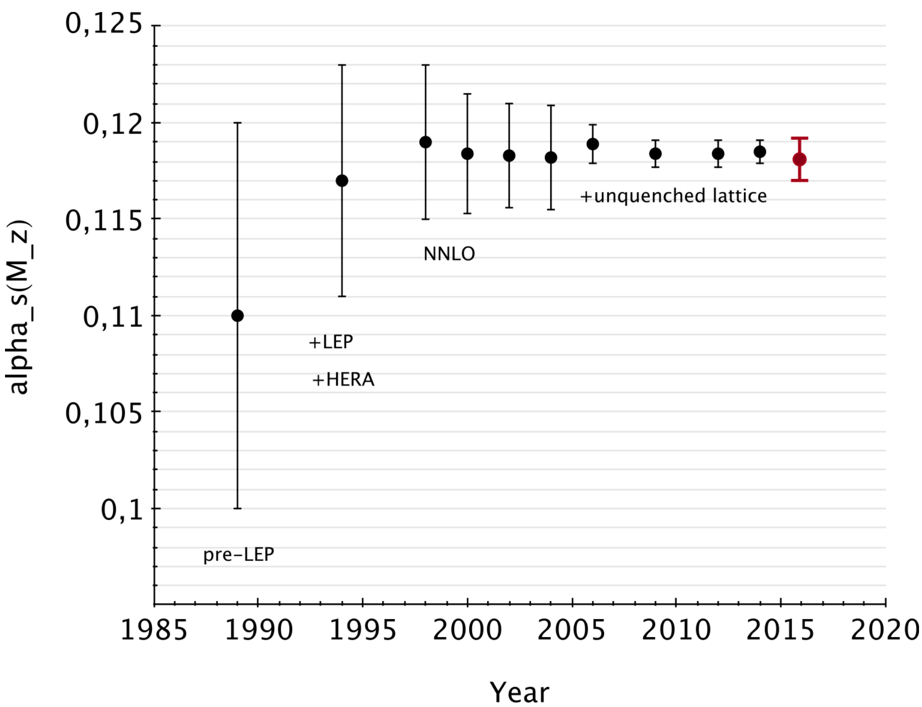

Figure 1: World average values of $\alpha_s(m_Z)$ over time.

- decays of τ-leptons,
- deep inelastic lepton-nucleon scattering (DIS; until recently, only structure functions at NNLO),
- lattice QCD,
- jets and hadronic event shapes in e^+e^- annihilation,
- electro-weak precision fits,
- hadron collider results (so far, only $t\bar{t}$ cross section at NNLO),

and a pre-average is determined for each of these classes. Finally, the world average is then determined from these 6 pre-averages of classes. Pre-averages are determined taking the unweighted mean and average error. This should provide the most unbiased estimator of the average and its un-

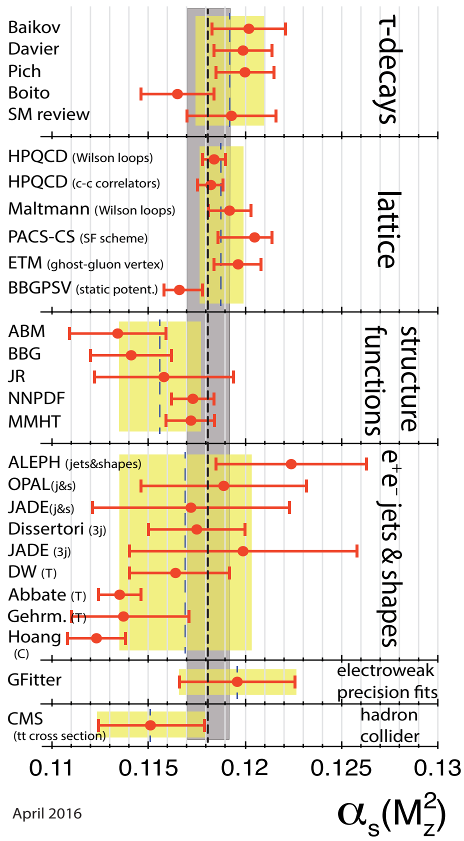

Figure 2: 2016 summary of determinations of $\alpha_s(m_Z)$. The light-shaded bands and long-dashed vertical lines indicate the pre-average values; the dark-shaded band and short-dashed line represents the new overall world average of $\alpha_s(m_Z)$.

certainty in case of largely correlated results, with unknown degrees of correlations and unknown "errors on errors".

The final world average is then determined as the weighted mean of the class pre-averages, initially treating their uncertainties as being uncorrelated and of Gaussian nature. This determines the final world average value of $\alpha_s(m_Z)$. The overall *uncertainty* of the world average is then adjusted according to the following procedure:

If the overall χ^2 is smaller than 1 per degree of freedom (d.o.f.), an overall correlation coefficient is introduced in the error matrix and adjusted such that χ^2/d.o.f. = 1. If the overall χ^2/d.o.f. is larger than 1, all uncertainties are enlarged by a common factor such that χ^2/d.o.f. = 1. Note that in both cases, adjusting a common correlation factor or enlarging all individual uncertainties, the final uncertainty of the average value increases with respect to the initial, "uncorrelated" starting value!

The results included in the 2016 and 2018 world summary of $\alpha_s(m_Z)$, together with the respective pre-averages of classes and the final world average, are displayed in Figure 2. Note that in two of the classes, no pre-averaging has been applied as only one individual result was available in each case, at the time of the analysis (2016).

As shown in Table 1 and Figure 1, the overall quoted uncertainty of $\alpha_s(m_Z)$ increased from 0.0006 (in the review of 2014) to 0.0011 (review of 2016). The reason for this increase was mainly procedural: in 2014, pre-averages were not determined by taking the linear average of individual results and their uncertainties, but by a method called "range-averaging". There, pre-averages were determined by taking the central value of the range of input values and half of this range interval, as central value and its uncertainty, respectively. For the lattice results, which were expected to be essentially uncorrelated with each other, the pre-average was determined using the χ^2 method.

Figure 3 summarises the history and values of pre-averages of $\alpha_s(m_Z)$ for the different classes of measurements. Note that the change in error determinations predominantly affected the class of

$\alpha_s(M_Z)$	2012	2014	2016 & 18
τ	0.1197 ± 0.0016	0.1197 ± 0.0016	0.1192 ± 0.0018
Lattice	0.1185 ± 0.0007	0.1185 ± 0.0005	0.1188 ± 0.0011,
DIS	0.1151 ± 0.0022	0.1154 ± 0.0020	0.1156 ± 0.0021,
e+e-	0.1172 ± 0.0037	0.1177 ± 0.0046.	0.1169 ± 0.0034.
ew	0.1193 ± 0.0028	0.1193 ± 0.0028	0.1196 ± 0.0030
pp	–	–	0.1151 ± 0.0028
world ave.	0.1184 ± 0.0007	0.1185 ± 0.0006	0.1181 ± 0.0011

"range" average and error
unweighted average and error
χ^2 weighted average and error

Figure 3: History and results of pre-averages of $\alpha_s(m_Z)$.

lattice results, whose uncertainty thus increased by a factor of two, using the most recent method of taking the unweighted mean and error instead. This, in turn, affected the overall uncertainty of the world average, which was (and still is, albeit to a lesser extent) dominated by the influence of the lattice results.

The current status of determining the world average value of $\alpha_s(m_Z)$ is rather satisfying, showing consistency and agreement within the quoted overall uncertainty of about 1%. The latter is limited by the fact that, within each class of measurements of α_s, there are issues which prevail since quite some time, and which could not yet be solved in a convincing manner:

- α_s from τ decays: uncertainties between different perturbative calculations (FOPT; CIPT) as well as other technical systematics;
- α_s from lattice calculations: size of quoted uncertainties;
- α_s from DIS: unsolved issues between author groups (PDFs);
- α_s from e^+e^- annihilation: analytic vs. classical treatment of (nonperturbative) hadronisation effects;
- α_s from hadron colliders: so far, only one determination in NNLO (more available recently); in NLO analyses: choice of renormalisation/factorisation scales, treatment of top-threshold, non-perturbative/hadronisation corrections;
- α_s from electroweak precision data: correct in strict Standard Model, very sensitive to many beyond-Standard-Model (BSM) effects if present.

Last not least, the methods applied to select and (pre-)average results might have to be revisited and improved.

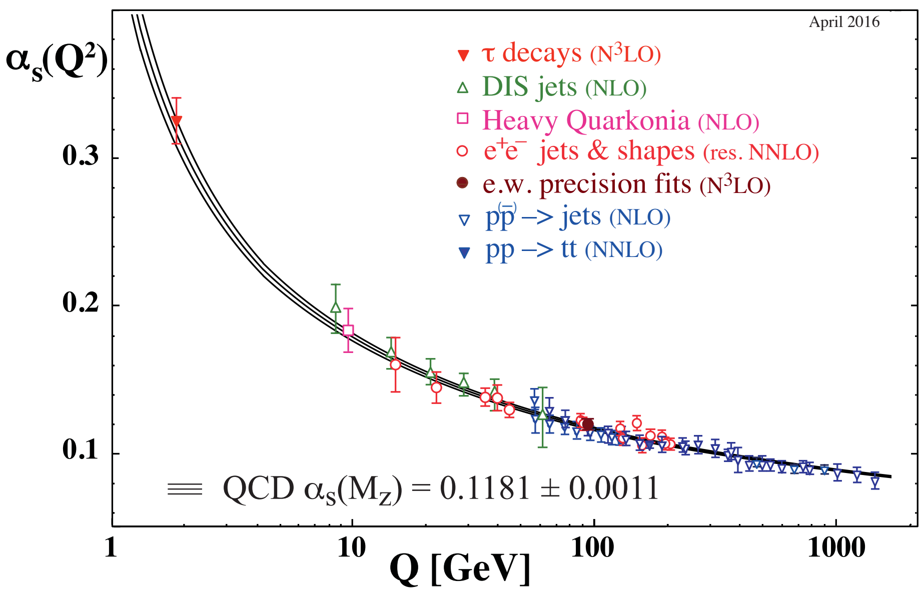

Figure 4: Summary of measurements of α_s as a function of the energy scale Q.

To my personal opinion, significant improvements in the precision of measurements of α_s, below the 1% level, may mainly (only?) be expected from improved lattice calculations, and from high statistics measurements of the Z^0 lineshape (also called Giga-Z or Tera-Z), at future high-energy e^+e^- collider projects.

However, and maybe even more important from the viewpoint of testing the fundamental theory of Strong Interactions, the successful and precise confirmation of the concept of Asymptotic Freedom and thus, the experimental "proof" of *the* key feature of QCD, is regarded to be one of the most remarkable achievements of both, theoretical and experimental particle physics, see Figure 4. My personal thanks and respect go to all those who have taken part and actively contributed to the many measurements and results in this field.

References

[1] G. Altarelli, Annu. Rev. Nucl. Part. S. **39** (1989) 357.

[2] Review of Particle Physics, Chin. Phys. C **40** (2016) 100001.

[3] Review of Particle Physics, Phys. Rev. D **98** (2018) 030001.

[4] S. Bethke, Nucl. Part. Phys. Proc. **282-284** (2017) 149.

[5] S. Bethke, Nucl. Phys. Proc. Suppl. **39** (1995) 198, PITHA-94-30.

[6] S. Bethke, in: Radiative Corrections, Barcelona 1998; hep-ex/9812026.

[7] S. Bethke, J. Phys. G **26** (2000) R27, hep-ex/0004021.

[8] S. Bethke, Nucl. Phys. Proc. Suppl. **121** (2003) 74; hep-ex/0211012.

[9] S. Bethke, Nucl. Phys. Proc. Suppl. **135** (2004) 345; hep-ex/0407021.

[10] S. Bethke, Prog. Part. Nucl. Phys. **58** (2007) 351; hep-ex/0606035.

[11] S. Bethke, Eur. Phys. J. C **64** (2009) 689; arXiv:0908.1135 [hep-ph].

[12] Review of Particle Physics, Phys. Rev. D **86** (2012) 010001.

[13] Review of Particle Physics, Chin. Phys. C **38** (2014) 090001.

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