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α_s from soft QCD jet fragmentation functions

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We present an extraction of the QCD coupling α_s from the energy evolution of the first two moments (multiplicity and peak position) of the parton-to-hadron fragmentation functions at low fractional hadron momentum z. A fit of the experimental jet data, from e^+e^- and deep-inelastic $e^{\pm},\nu$-p collisions, to NNLO*+NNLL predictions yields $\alpha_s(m_Z) = 0.1205 \pm 0.0010\,(\mathrm{exp})^{+0.0022}_{-0.0000}\,(\mathrm{th})$, in good agreement with the current α_s world-average value.

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Introduction

In the chiral limit of zero quark masses and for fixed number of colours N_c, the α_s coupling that determines the strength of the interaction among quarks and gluons is the only parameter of quantum chromodynamics (QCD), the theory of the strong interaction. Starting at an energy scale of order $\Lambda_{\rm QCD} \approx 0.2$ GeV, where the perturbatively-defined coupling diverges, α_s decreases with energy Q following a $1/\ln(Q^2/\Lambda^2_{\rm QCD})$ dependence at leading order. The current $\pm 0.9\%$ uncertainty of α_s at the Z mass pole, $\alpha_s(m_Z) = 0.1181 \pm 0.011$ [1], makes of the QCD coupling the least precisely known of all fundamental constants in nature. Improving our knowledge of α_s is crucial in order to reduce the uncertainties in perturbative-QCD calculations of higher-order corrections of all hadronic cross sections and decays at colliders [2], as well as for precision electroweak fits of the Standard Model in indirect searches for new physics at future e^+e^- machines [3]. The parametric dependence on α_s accounts for a significant fraction of the theoretical uncertainties in e.g. the calculations of the Higgs boson $\mathrm{H} \to b\bar{b}, c\bar{c}, gg$ partial widths [4]. The value of $\alpha_s(m_Z)$ and its evolution have also far-reaching implications including the stability of the electroweak vacuum [5], the existence of new coloured sectors at high energies [6], and our understanding of physics approaching the Planck scales, such as e.g. on the precise energy at which the interaction couplings may unify.

Having at hand new independent approaches to determine α_s, with experimental and theoretical uncertainties comparable to (or, even better, smaller than) those of the methods currently used, is crucial to reduce the overall uncertainty in the combined α_s world-average value [2]. In Refs. [7], we presented a novel technique to extract α_s from the energy evolution of the moments of the parton-to-hadron fragmentation functions (FFs) computed at increasingly higher degree of theoretical accuracy, including up to approximate next-next-to-leading-order (NNLO*) fixed-order and next-to-next-to-leading-log (NNLL) resummation corrections. We review here the latest NNLO*+NNLL theoretical calculations for the jet-energy dependence of the hadron multiplicity and the FF peak position. A fit of the analytical predictions to experimental jet measurements from e^+e^- and deep-inelastic $e^\pm, \nu$-p collisions over $Q \approx 2$–200 GeV provides a new high-precision extraction of $\alpha_s(m_Z)$.

DGLAP+MLLA evolution of the fragmentation functions

The conversion of a quark and gluon (collectively called partons) into a final jet of hadrons is driven by perturbative dynamics dominated by soft and collinear gluon bremsstrahlung [8] followed by the final transformation into hadrons of the last partons produced in the QCD shower at non-perturbative scales approaching $\Lambda_{\rm QCD}$. The distribution of hadrons inside a jet is encoded in its fragmentation function, $D_{\rm a\to h}(z,Q)$, describing the probability that an initial parton a eventually fragments into a hadron h carrying a fraction $z = p_{\rm hadron}/p_{\rm parton}$ of the parent parton's momentum. Starting with a parton at a given δ-function energy Q, its evolution to any other energy scale Q' is driven by a branching process of parton radiation and splitting, $a \to bc$, that can be perturbatively computed. At large $z \gtrsim 0.1$ one uses the DGLAP evolution equations [9], whereas the Modified Leading Logarithmic Approximation (MLLA) [10], resumming soft and collinear singularities, provides the proper theoretical framework at small z. In the latter approach, describing the region of low hadron momenta that dominates the jet fragments, one writes the FF as a function of the log of the inverse of z, $\xi = \ln(1/z)$. Due to colour coherence and interference in gluon radiation (known as "angular ordering"), not the softest partons but those with intermediate energies multiply most effectively in QCD cascades, leading to a final FF with a typical "hump-backed plateau" (HBP) shape as a function of ξ. Such an HBP can be described, without any loss of generality, in terms of a distorted Gaussian (DG, Fig. 2):

$$D(\xi,Y,\lambda) = \mathcal{N}/(\sigma\sqrt{2\pi}) \cdot e^{\left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right]}, \text{ with } \delta = (\xi - \bar{\xi})/\sigma, \tag{1}$$

where $\mathcal{N}$ is the hadron multiplicity inside a jet, and $\bar{\xi}$, σ, s, and k are respectively the mean peak, dispersion, skewness, and kurtosis of the distribution. In Refs. [7], we described a new approach that solves the set of integro-differential equations for the FF evolution combining both DGLAP and MLLA corrections. This is done by expressing the Mellin-transformed hadron distribution in terms of the anomalous dimension γ: $D \simeq C(\alpha_s(t)) \exp\left[\int^t \gamma(\alpha_s(t'))dt\right]$ where $t = \ln Q$. Such an expression leads to a perturbative expansion in half powers of α_s: $\gamma \sim \mathcal{O}(\alpha_s^{1/2}) + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^{3/2}) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^{5/2}) + \cdots$, where integer powers of α_s correspond to fixed-order corrections, and half-integer terms can be identified with increasingly accurate resummations of soft and collinear logarithms, as schematically indicated in the following table:

Order	LL (DLA)	NLL	NNLL	N^{3}LL	N^{4}LL
LO $P_{ac}^{(0)}$	$\mathcal{O}(\sqrt{\alpha_s})$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s^{3/2})$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^{5/2})$
NLO $P_{ac}^{(1)}$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_s^{3/2})$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^{5/2})$
NNLO $P_{ac}^{(2)}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_s^{5/2})$
LO α_s	$\cdots$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s^{3/2})$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^{5/2})$
NLO α_s	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_s^{3/2})$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^{5/2})$
NNLO α_s	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_s^{5/2})$

The full set of NLO $\mathcal{O}(\alpha_s^2)$ terms for the anomalous dimension, including the two-loop splitting functions $P_{ac}^{(1)}$ and the two-loop running of α_s, plus a fraction of the $\mathcal{O}(\alpha_s^{5/2})$ terms, coming from the NNLO expression for the α_s running, have been computed [11]. Upon inverse-Mellin transformation, one can derive the analytical expressions for the energy evolution of the FF, and its associated moments, as a function of $Y = \ln(E/\Lambda_{\rm QCD})$, for an initial parton energy E, down to a shower cut-off scale $\lambda = \ln(Q_0/\Lambda_{\rm QCD})$ for $N_f = 3,4,5$ quark flavors. The resulting formulas for the energy evolution of the moments depend on $\Lambda_{\rm QCD}$ as *single* free parameter. Simpler expressions are obtained in the limiting-spectrum case obtained for $\lambda = 0$, i.e. evolving the FF down to $Q_0 = \Lambda_{\rm QCD}$, motivated by the "local parton hadron duality" hypothesis for infrared-safe observables that states that the distribution of partons in jets are simply renormalized in the hadronization process without changing their shape. Thus, by fitting to Eq. (1) the measured HBP at various energies, one can determine α_s from the corresponding jet energy-dependence of the FF moments $\mathcal{N}$, $\bar{\xi}$, σ, s, and k.

NNLO*+NNLL evolution of the FFs moments

As for the Schrödinger equation in quantum mechanics, the system of equations for the $D_{\mathrm{a}\to\mathrm{h}}(z,Q)$ FFs can be written as an evolution Hamiltonian that mixes gluon and (anti)quark states expressed in terms of DGLAP splitting functions [9] for the branchings $g\to gg$, $q(\bar{q})\to gq(\bar{q})$ and $g\to q\bar{q}$, where g, q and $\bar{q}$ are a gluon, quark, and anti-quark respectively. The evolution Hamiltonian is diagonalized into two eigenvalues $\gamma_{\pm\pm}$ in the new $\mathscr{D}^\pm$ basis. The relevant one for the calculation of the FF moments is γ_{++}. The analytical solution obtained at NLO+NNLL from the Mellin transform of the expressions, including the full-resummed NNLL splitting functions [12], reads (as a function of the energy of the radiated gluon ω and the variables Y and λ):

$$
\begin{aligned}
\gamma_\omega^{\mathrm{NLO+NNLL}} =& \frac{1}{2}\omega(s-1)+\frac{\gamma_0^2}{4N_c}\left[-\frac{1}{2}a_1(1+\mathsf{s}^{-1})+\frac{\beta_0}{4}(1-\mathsf{s}^{-2})\right]\\
&+\frac{\gamma_0^4}{256N_c^2}(\omega\mathsf{s})^{-1}\Big[4a_1^2(1-\mathsf{s}^{-2})+8a_1\beta_0(1-\mathsf{s}^{-3})+\beta_0^2(1-\mathsf{s}^{-2})(3+5\mathsf{s}^{-2})\\
&-64N_c\frac{\beta_1}{\beta_0}\ln 2(Y+\lambda)\Big]\\
&+\frac{1}{4}\gamma_0^2\omega\left[a_2(2+\mathsf{s}^{-1}+\mathsf{s})+a_3(\mathsf{s}-1)-a_4(1-\mathsf{s}^{-1})-a_5(1-\mathsf{s}^{-3})-a_6\right], \qquad (2)
\end{aligned}
$$

where $\gamma_0=\sqrt{4N_c\,\alpha_s/(2\pi)}$ is the LL anomalous dimension, $\mathsf{s}=\sqrt{1+4\gamma_0^2/\omega^2}$, a_1 and a_2 are hard constants obtained in [7], and a_3, a_4, a_5 and a_6 are new constants resulting from incorporating the full-resummed NNLL splitting functions. The different moments of the DG can be finally derived from the anomalous dimension via:

$$
\mathscr{N}=K_0,\quad \bar{\xi}=K_1,\ \sigma=\sqrt{K_2},\ s=\frac{K_3}{\sigma^3},\ k=\frac{K_4}{\sigma^4}\,; \qquad (3)
$$

where

$$
K_{n\geq 0}(Y,\lambda)=\int_0^Y dy\left(-\frac{\partial}{\partial\omega}\right)^n\gamma_\omega(Y+\lambda)\Big|_{\omega=0}, \qquad (4)
$$

Currently, beyond the analytical result given by Eq. (2), we have incorporated all $\mathscr{O}(\alpha_s^{3/2})$ contributions and also added a few of the $\mathscr{O}(\alpha_s^2)$ and $\mathscr{O}(\alpha_s^{5/2})$ ones, reaching NNLO*+NNLL accuracy. The expressions are too long to be provided here but will be given in [11]. The full inclusion of all $\mathscr{O}(\alpha_s^{5/2})$ terms is work in progress. Figure 1 shows the energy evolution of the zeroth (multiplicity) and first (peak position) moments of the FF, computed at an increasingly higher level of accuracy (from LO up to NNLO*). The FF hadron multiplicity and peak increase exponentially and logarithmically with energy respectively, and the theoretical convergence of their evolutions appears robust as indicated by the small changes introduced by adding higher-order terms.

Data-theory comparison and α_s extraction

In the phenomenological analysis, we first start by fitting to Eq. (1) all existing jet FF data measured in e^+e^- and $e^\pm,\nu$-p collisions over $\sqrt{s}\approx$ 2–200 GeV (Fig. 2), and thereby derive the corresponding FF moments at each jet energy. The overall normalization of the HBP spectrum ($\mathscr{K}_{\mathrm{ch}}$), which determines the average charged-hadron multiplicity of the jet (i.e. the zeroth moment

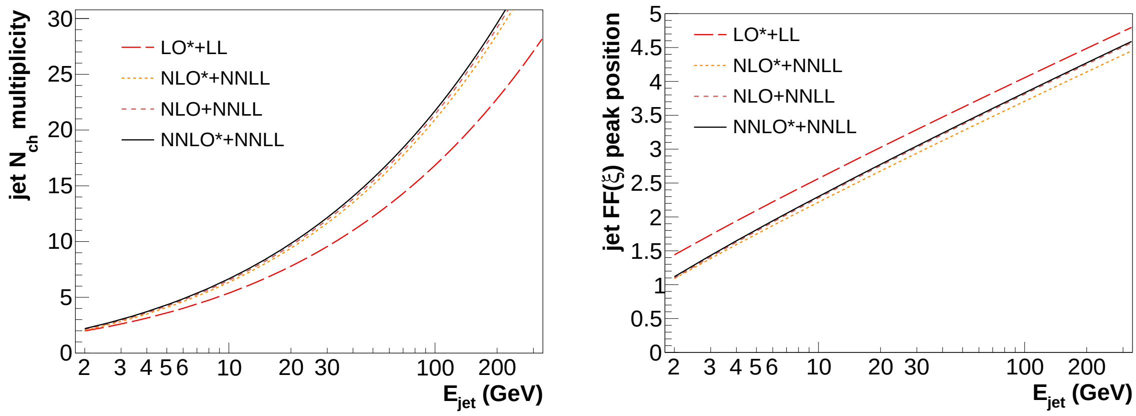

Figure 1: Theoretical energy evolution of the jet charged-hadron multiplicity (left) and FF peak position (right) at four levels of accuracy, from LO+LL up to NNLO*+NNLL.

of the FF), is an extra free parameter in the DG fit that, nonetheless, plays no role in the finally derived $\Lambda_{\rm QCD}$ value that is *solely* dependent on the evolution of the multiplicity, and not on its absolute value at any given jet energy. The impact of finite hadron-mass effects in the DG fit are taken into account through a rescaling of the theoretical (massless) parton momenta with an effective mass $m_{\rm eff} \approx m_\pi$. Varying such an effective mass from zero to a few hundred MeV, results in small propagated uncertainties into the final extracted $\Lambda_{\rm QCD}$ value, as discussed in Refs. [7].

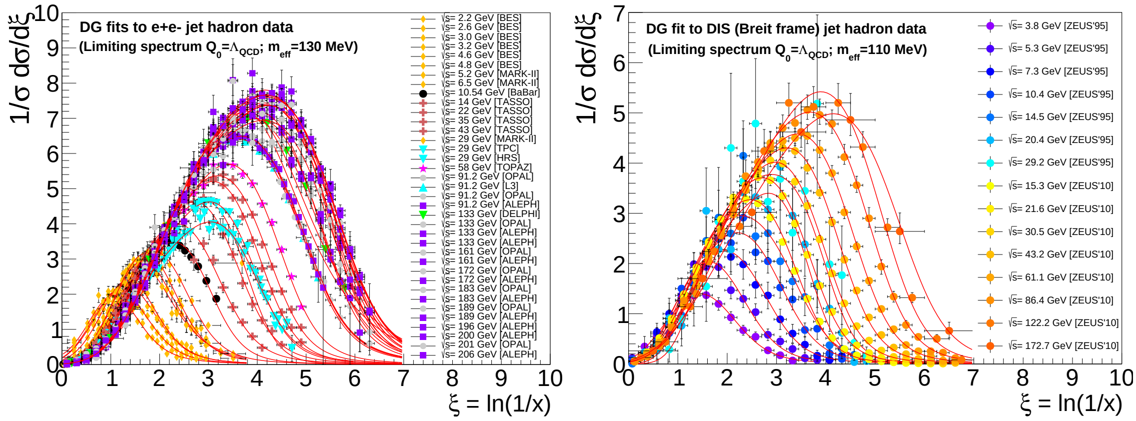

Figure 2: "Hump-backed plateau" charged-hadron distributions in jets as a function of $\xi = \ln(1/z)$ measured in e^+e^- at $\sqrt{s} \approx$ 2–200 GeV (left) and $e^\pm, \nu$-p (Breit frame, scaled up by $\times 2$ to account for the full hemisphere) at $\sqrt{s} \approx$ 4–180 GeV (right), fitted to the DG given by Eq. (1).

Once the FF moments have been obtained, one can perform a combined fit of them as a function of the original parton energy. In the case of e^+e^- collisions, the latter corresponds to half the centre-of-mass energy $\sqrt{s}/2$ whereas, for DIS, the invariant four-momentum transfer $Q_{\rm DIS}$ is used. The experimental and theoretical evolutions of the hadron multiplicity and FF peak position as a function of jet energy are shown in Fig. 3. The hadron multiplicities measured in DIS jets appear somewhat smaller (especially at high energy) than those from e^+e^- collisions, due to limitations in the FF measurement only in half (current Breit) $e^\pm$p hemisphere and/or in the determination of the relevant Q scale [7]. The NNLO*+NNLL limiting-spectrum ($\lambda = 0$) predictions for $N_f = 5$

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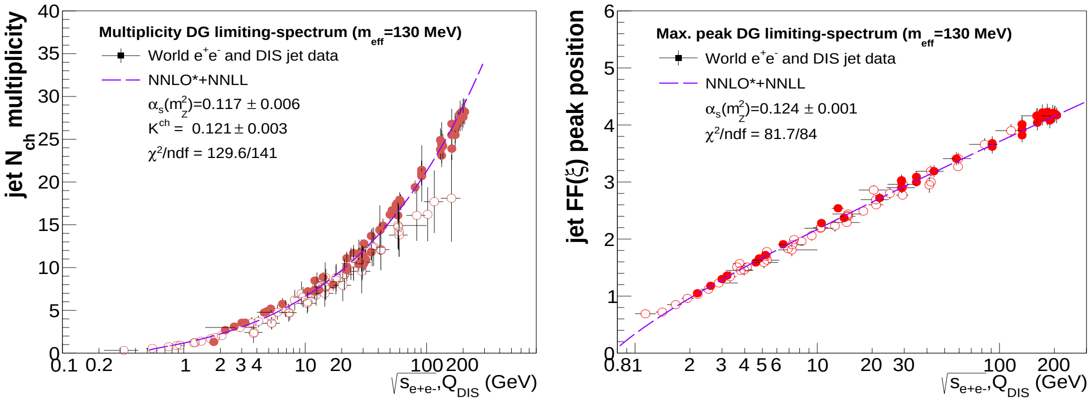

Figure 3: Energy evolution of the charged-hadron multiplicity (left) and of the FF peak position (right) measured in e^+e^- and DIS data fitted to the NNLO*+NNLL predictions. The obtained $\mathscr{K}_{\rm ch}$ normalization constant, individual NNLO* $\alpha_s(m_Z)$ values, and the goodness-of-fit per degree-of-freedom $\chi^2/$ndf, are quoted.

active quark flavours[1], leaving $\Lambda_{\rm QCD}$ as a free parameter, reproduce very well the data. Fit results for the rest of the FF moments can be found in [7]. Among FF moments, the peak position $\xi_{\rm max}$ appears as the most robust one for the determination of $\Lambda_{\rm QCD}$, being relatively insensitive to most of the uncertainties associated with the extraction method (DG fits, energy evolution fits, finite-mass corrections, ...) as well as to higher-order corrections (Fig. 3 right).

The QCD coupling obtained from the combined fit of the energy evolution of the multiplicity and peak position is $\alpha_s(m_Z) = 0.1205 \pm 0.0010^{+0.0022}_{-0.0000}$, where the first uncertainty includes all experimentally-related sources discussed in Refs. [7], and the second one is a theoretical scale uncertainty derived at NLO by stopping the parton evolution of the FFs at $Q_0 = 1$ GeV rather than at the limiting spectrum value $Q_0 = \Lambda_{\rm QCD}$. As shown in Fig. 4, our extracted $\alpha_s(m_Z)$ value is consistent with all other NNLO results from the latest PDG compilation [1], as well as with other determinations with a lower degree of theoretical accuracy [13]. The precision of our result ($+2\%, -1\%$) is competitive with the other extractions, with a totally different set of experimental and theoretical uncertainties.

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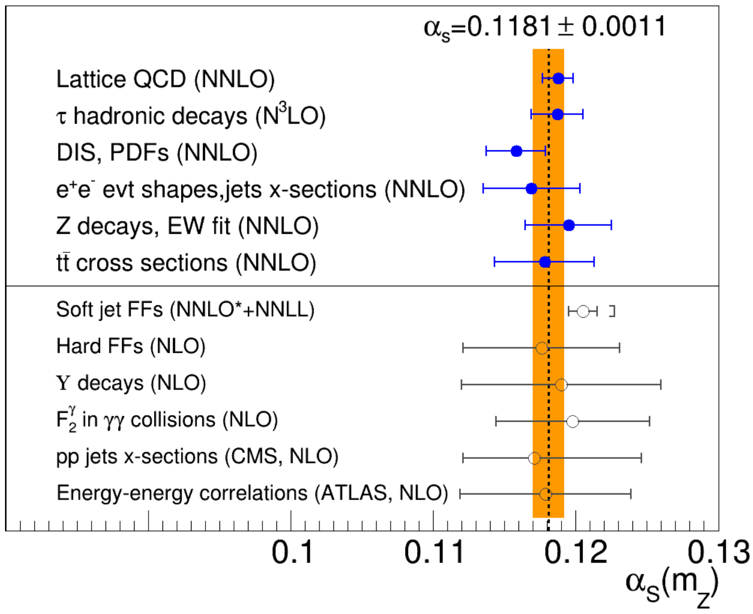

Figure 4: Summary of α_s determinations using different methods. The top points show $N^{2,3}LO$ extractions currently included in the PDG [1], the bottom ones shown those obtained with other approaches at lower degree of accuracy today [13], including the result of our work. The dashed line and shaded (orange) band indicate the current PDG world-average and its uncertainty.

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